Social Welfare Analysis of Income Distributions

Social Welfare, Social Welfare Functions and Inequality Aversion

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by

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## 1.	SUMMARY

This module illustrates the concept of social welfare and the possible ways to define **social welfare functions**. In particular, it deals with how to pass from inequality to social welfare analysis and how social welfare analysis may embody different attitudes with regard to inequality aversion. A step-by-step procedure and numerical examples are also discussed to give operational content to the tool.

## 2.	INTRODUCTION

Objectives

This tool will deal with the passage from the *descriptive approach* to income distribution to the *normative approach*, i.e. from inequality to welfare.

The objective of the tool is to explore the available methods to rank income distributions from a **social welfare** perspective. Different stakeholders may have different views about what is better for a society (different social preferences). This may lead to differently rank two alternative "social states" (different situations of a society) on social welfare grounds. For example, some may prefer a "social state" with more total income even if it is **unevenly distributed**; whereas others may be willing to give up some total income in order to have a "social state" with a more **equitable distribution**. To get a unanimous consensus about the ranking, some common criteria on what is desirable (preferred) for a society need to be found. The aim of this tool is to show how a common consensus about the ranking of two (or more) "social states" can be found in many cases, without imposing too many restrictions on the social preferences of the different stakeholders, i.e. on what they consider desirable for the society.

Use: This tool can be used in an operational context to scrutinize what are the minimum common criteria to agree upon to unanimously rank alternative policy options on welfare grounds, e.g., tax/benefit reforms, infrastructural investment policies, sectoral policies, etc. After identifying the impacts of the different options on income generation and distribution, the analyst must be able to rank the resulting alternative income distributions to be able to select the most preferred, based on welfare grounds. This tool therefore helps users identify under what conditions a unanimous consensus about the ranking is assured.

Target audience

This module targets current or future policy analysts who want to increase their capacities in analysing the welfare effects of development policies. On these grounds, economists and practitioners working in public administrations, in NGOs, professional organisations or consulting firms will find this helpful reference material.

Required background

The audience should be familiar with basic notions of mathematics and statistics.

To find relevant materials in these areas, the reader can follow the links included in the text to other EASYPol modules or references[1]. See also the list of useful EASYPol links included in section 6 of this module.

3.   CONCEPTUAL BACKGROUND

3.1   Inequality and social welfare

There are important conceptual differences between inequality measurement and social welfare analysis.

▪   Social welfare analysis is mainly concerned with how total income ***should* be divided** among different individuals. Inequality measurement is mainly concerned with **how total income *is* divided** among different individuals. This distinction embodies the conceptual difference between the *descriptive approach* and the *normative approach* to the analysis of income distribution[2].

▪   Social welfare analysis takes into consideration the amount of *total income available* in a society (or equivalently, the mean level of income) as well as its degree of inequality. On the other hand, inequality measurement, in general, does not tell us anything about the mean income[3]. The example reported in Table 1 clearly shows this difference. This implies that ranking alternative distributions on social welfare grounds is different to ranking alternative distributions on inequality grounds .

▪   Social welfare analysis requires SWFs and indexes measuring the level of welfare embodied in a given income distribution. Inequality measurement requires only inequality measures[4].

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[1] EASYPol hyperlinks are shown in blue, as follows:
   a)   training paths are shown in **underlined bold font**
   b)   other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
   c)   links to the glossary are in **bold**; and
   d)   external links are in *italics*.

[2] See EASYPol Modules 001, ***Social Welfare Analysis of Income Distributions: Ranking Income Distributions with Lorenz Curves***, 002 ***Ranking Income Distributions with Generalised Lorenz Curves*** and, 003 ***Ranking Income Distributions with Crossing Generalised Lorenz Curves***.

[3] You may recall that many inequality measures are *scale invariant*, i.e. not sensitive to proportional changes of incomes. Yet, one might be less concerned with a certain degree of inequality if it occurs in a society of richer people than if it occurs in a society of poorer people. Ranking income distributions on welfare grounds and on inequality grounds would be equivalent only in those cases where mean incomes of alternative distributions are equal. See Sen (1995).

[4] Inequality measures are discussed in EASYPol Modules 000 and 040, respectively: ***Charting Income Inequality: The Lorenz Curve*** and ***Inequality Analysis: The Gini Index***

Table 1: Inequality measures do not take into account total income

Society 1		Society 2	
Individual	Incomes	Individual	Incomes
1	100	1	900
2	200	2	1,800
3	300	3	2,700
4	400	4	3,600
5	500	5	4,500
Mean income	300		2,700
Gini	0.267		0.267

Mean income in Society 1 and 2 is 300 and 2,700 income units respectively. Yet, the Gini index is equal in both cases, 0.267. However, we might be prone to think that welfare in society 2 is higher as the same degree of inequality is combined with a higher level of mean income. Social welfare analysis, therefore, should take into consideration both "the size of the cake and how the cake is sliced".

3.2 Social Welfare Functions and inequality aversion

Social welfare can be described by means of a so called "Social Welfare Function" (SWF).[5]

An attractive definition of SWF, is put forward by Champernowne and Cowell (1998; 88):

«The generic term for coherent and consistent ordering of social states in terms of their desirability is a *social-welfare function*. We use the term 'social' because it normally refers to the whole community under consideration, but it does not imply that the ordering was somehow chosen by the whole community: there can be as many social-welfare functions as there are opinions held».

In line with the above definition, Deaton (1997) argues that:

«*The social welfare function* should be seen as a statistical 'aggregator' that turns a distribution into a single number that provides an overall judgement on that distribution and that forces us to think coherently about welfare and its distribution. Whatever our view of the policy making process, it is always useful to think about policy in terms of its effects on efficiency and on equity, and *the social welfare function* should be thought of as a tool for organizing our thoughts in a coherent way» (italic added).

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[5] We can take as given, at this stage, the issue related to the Arrow's impossibility theorem. See Arrow (1950, 1951).
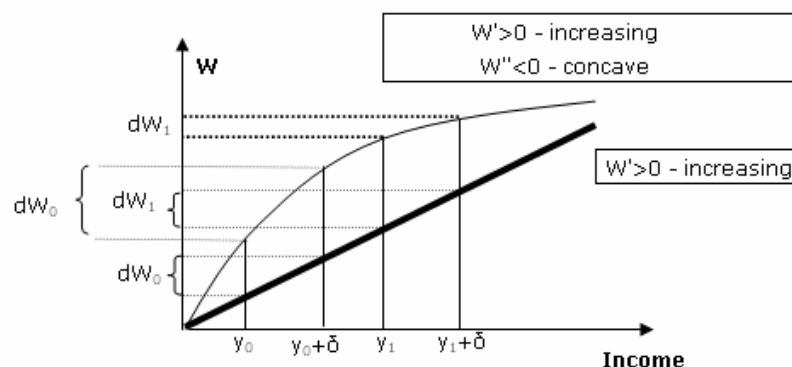
Formally, the SWF can be represented as follows:

[1] $$W = W(y_1, y_2, ..., y_n)$$

where y_i is the income of the i-th individual.

Basic characteristics of the SWF are as follows:

- It is INDIVIDUALISTIC i.e. it only depends on individual incomes,[6]

- It is SYMMETRIC, i.e. the social welfare is not affected if two individuals switch incomes[7].

- It is assumed INCREASING IN INDIVIDUAL INCOME LEVELS: If an individual income increases, other things equal, social welfare must increase (at least not decrease). Formally, this characteristic is expressed by a positive first derivative, $\frac{\partial W}{\partial y_i} \equiv W' \geq 0$. This is sometimes called the *Paretian principle*.

- It may be INEQUALITY AVERSE,. Inequality aversion means that social welfare is more sensitive to a shift in the income of a poorer individual than to the same shift affecting a richer individual (concave line in Figure 1. Mathematically, this characteristic is expressed by a negative second derivative: $W'' < 0$.

- It may be INEQUALITY NEUTRAL. Inequality neutrality means that social welfare is affected equally whenever a shift of income occurs irrespective to the position of individuals affected (bold line in Figure 1). Mathematically, this characteristic is expressed by $W'' = 0$)

Figure 1: SWF and inequality aversion

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[6] Individual incomes are assumed to be the money-metric measure of welfare levels of each individual.

[7] Recall the anonymity principle in inequality measurement: social welfare depends on the *levels* of individual welfare in a society, not on *who* enjoys a particular level of welfare.

The degree of *Concavity of the SWF reflects the degree of inequality aversion.* The more concave the SWF, the lower is the value attached to additional incomes accruing to richer individuals, i.e. the lower is the increase in social welfare. In the case of the linear SWF there is no strict concavity, which means, in this context, *inequality neutrality*. This is the case where by the social point of view is «a dollar is a dollar» whoever receives it.[8]

Figure 1 illustrates that a given increase δ of income gives the same increase in welfare if such increase is evaluated on the bold straight line ($dW_0 = dW_1$ in this case). When evaluated on the concave SWF, instead, an increase of δ at lower income levels gives more utility than the same increase at higher income levels ($dW_0 > dW_1$ in this case).

3.3   Abbreviated SWF

There is an alternative way of representing an SWF, which is often referred to as the ABBREVIATED SOCIAL WELFARE FUNCTION methodology, as follows:

[2]
$$W = W(\bar{y}, I)$$

Expression [2] explicitly tells us that social welfare W depends on mean income $\bar{y}$ and inequality I. Being an alternative representation of [1] we should expect some link between the two.

A useful way to look at this argument is by following Deaton (1997). Multiplying [1] by mean income and dividing all individual incomes by mean income would yield a new equivalent expression:

[3]
$$W = \bar{y} W\left(\frac{y_1}{\bar{y}}, \frac{y_2}{\bar{y}}, ..., \frac{y_n}{\bar{y}}\right)$$

In this way, mean income and its distribution (relative to the mean) are clearly distinguished. Choosing a normalisation of [3], by which $W(1,1,...,1) = 1$, social welfare will be equal to mean income $\bar{y}$ when all incomes are equal.

We are now in a position to specify a suitable functional form for [3]. We can show that the abbreviated SWF may take the form:

[4]
$$W = \bar{y}(1 - I)$$

where I is an inequality index ranging from zero to one. When all incomes are equal, $I=0$ and $W = \bar{y}$. When incomes are unequal, $I>0$ and $W < \bar{y}$. When all individuals have zero incomes except for one, having total income, $I=1$ and $W = 0$.[9]
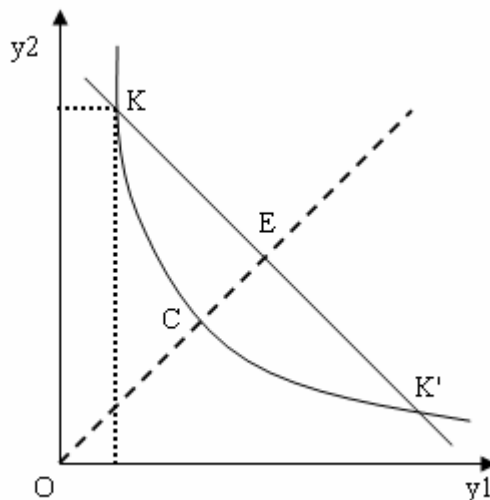
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[8] Deaton (1997).
[9] Inequality indexes not ranging from zero and one are therefore unsuitable for the abbreviated social welfare function, as they could cause negative levels of welfare. The issue of the abbreviated welfare

This alternative representation of SWF highlights the following properties:

- It is INCREASING (AT LEAST NON-DECREASING) IN MEAN INCOME. Technically, $\frac{\partial W}{\partial \bar{y}} \geq 0$. This is the analogous of the Pareto principle in [1].

- It is SYMMETRIC (see above);

- It is DECREASING IN INEQUALITY. Technically, $\frac{\partial W}{\partial I} < 0$. It simply says that social welfare decreases if inequality increases. In some way, it reflects the inequality aversion. Higher inequality aversion (to be reflected in higher inequality indexes) reduces social welfare more.

Figure 2 shows how the abbreviated SWF works in a two-person economy. The axes report the level of individual incomes for two hypothetical individuals.

Figure 2: Trading mean income with inequality

Let us suppose that the initial distribution of welfare is in K, with individual 2 having more than individual 1. By symmetry, an opposite distribution in K' must have the same level of social welfare, i.e. it must lie on the same curve. Now, by construction, the points with the same total (mean) income as in K and K' lie on the straight line KEK'.[10]

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functions is rather technical. In the text, we will only provide a simplified version, which is enough to understand the meaning and the use of the tool. A good advanced reference is Lambert (1993).

[10] It is a geometric property that a negatively sloped curve at 45° passing through a point has the same total as the original point.

Therefore, at C, mean income must be lower. Yet, the level of social welfare is the same as in K. Why? Because, in K we have higher $\bar{y}$ but also higher inequality I. In C, we have lower $\bar{y}$ but also lower inequality I. The two have conflicting effects on social welfare and, hypothetically, they end up compensating. The curve KCK', therefore, represents the way in which mean income and inequality can be traded to have the same level of social welfare.

3.4   Types of SWF

The graphical intuition of Figure 2 proves useful to identify typical SWF. In particular, we can distinguish two polar cases:

- The **Utilitarian (or Benthamian) SWF**;
- The **Rawlsian SWF**. [11]

The **Utilitarian SWF** simply identifies average social welfare by the unweighted sum of individual incomes. Its functional form is:

[5]
$$W = \frac{1}{N} \sum_{i=1}^{n} y_i \rightarrow \text{Utilitarian}$$

This polar case of SWF does not allow for any trade between the size of the cake and the way it is sliced, i.e. between total income and inequality. Expression [5] tells us that social welfare is increased by the same amount regardless of the fact that the individual is poor or rich. Utilitarians only care about total income. This means that the way incomes are traded between individuals is constant. Again «a dollar is a dollar whoever receives it». **This is equivalent to a zero inequality aversion**.

The **Rawlsian SWF**, instead, identifies social welfare with the welfare of the least well-off individual in the income distribution. Its functional form is:

[6]
$$W = \min(y_i) \rightarrow \text{Rawlsian}$$

where W is equivalent to the minimum income in the income distribution. This polar case of SWF allows for maximum trade between the size of the cake and the way it is sliced. In particular, social welfare cannot increase unless the income of the poorest individual is increased. Income increases associated to any other individual do not increase social welfare. **This is equivalent to an infinite inequality aversion**.

Between these two polar cases, there is a standard way of representing an SWF. For example, aggregation of incomes, instead of being unweighted as in the utilitarian case,

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[11] From Rawls (1972).

might be weighted. A standard way to represent this concept is by using an iso-elastic functional form as follows:[12]

[7] $$W = \frac{1}{N} \left\{ \frac{1}{1-e} \sum_{i=1}^{n} \left[(y_i)^{1-e} \right] \right\} \rightarrow \text{Iso} - \text{elastic}$$

It is worth noting that [7] encapsulates the two polar cases. Indeed, if $e=0$, [7] collapses to [5], while if $e \rightarrow \infty$ it collapses to [6]. Intermediate levels of e give rise to various iso-elastic forms. In this sense, the parameter e can be thought of as an **inequality aversion parameter**. We are now in a position to say that if the inequality aversion is zero, the utilitarian SWF comes up, while if the inequality aversion is very high, the Rawlsian SWF comes up. A standard SWF will come up, instead, for $e \in (0, \infty)$.

Therefore, for the moment, it is worth concentrating on the fact that the type of SWF may be selected simply by choosing a value for e into expression [7].
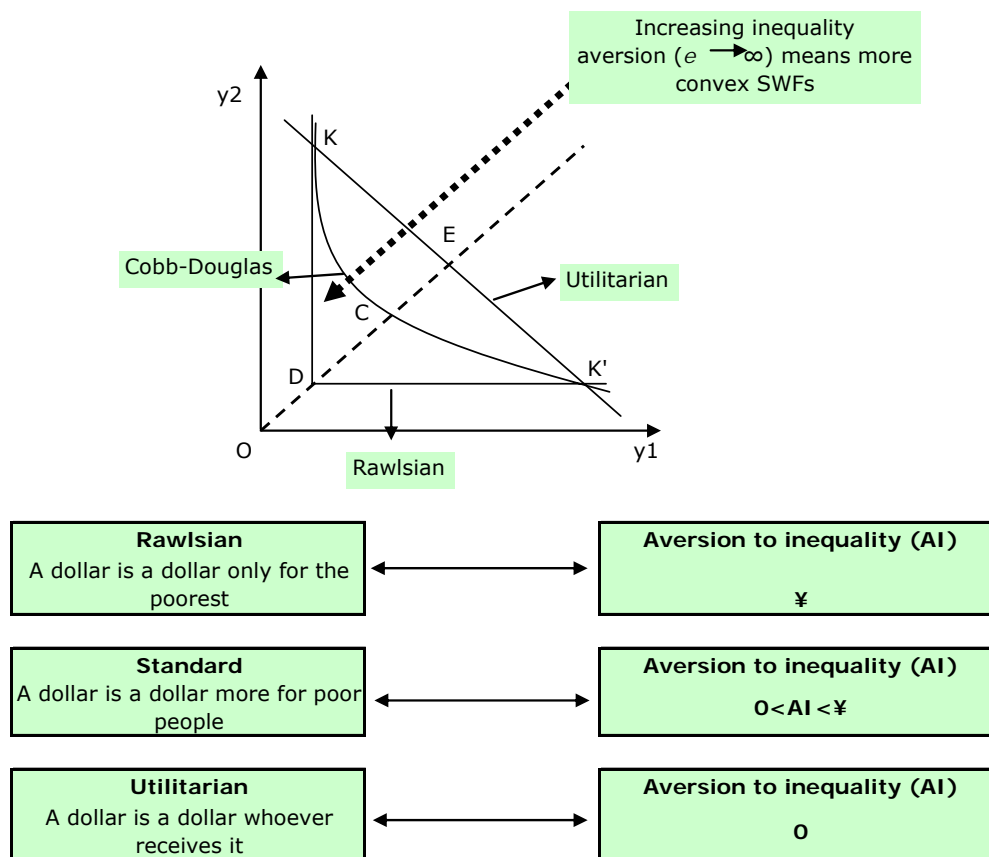
Figure 3 illustrates the various types of SWF in a two-person society. The solid line negatively sloped at 45° line is the Utilitarian SWF (see also Figure 2). This SWF tells us that social welfare is the same on the line KEK' and that social welfare maximise total income in the society (recall discussion of Figure 2). In this case, there is no price society wants to pay to have equally distributed incomes.

The curve KCK', instead, is the Cobb-Douglas SWF. In this case, society is ready to pay a price in terms of total income to have equally distributed incomes. As already observed in Figure 2, point C is a point where each individual has the same income, but total income is less than in the Utilitarian case. It means that, in the case of Cobb-Douglas, society is inequality averse, i.e. it prefers not to have inequality even though it costs a bit in terms of total income.

The curve KDK', finally, is the Rawlsian SWF. As far as society is incline to lose greater shares of total income to have equally distributed incomes, the SWF becomes more and more convex. It means that society becomes more and more inequality averse. The extreme case is that society is so inequality averse that it does not allow for any income increase that is not focused on the least well-off individual of the society.

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[12] It is worth noting that what follows is a common representation of the SWF but not the unique way incomes can be aggregated.

Figure 3: Types of Social Welfare Functions

Rawlsian A dollar is a dollar only for the poorest	⟷	Aversion to inequality (AI) ¥
Standard A dollar is a dollar more for poor people	⟷	Aversion to inequality (AI) 0<AI<¥
Utilitarian A dollar is a dollar whoever receives it	⟷	Aversion to inequality (AI) 0

4. A STEP-BY-STEP PROCEDURE TO BUILD AN SWF

At this stage, it may be useful to understand how to build an SWF and how to calculate social welfare. Figure 4 illustrates the steps.

Step 1 asks us, as usual, to sort income distributions by income levels. Step 2 only asks us to choose a value for e in formula [7]. It is worth recalling that, at this stage, the choice of e corresponds to a choice on the degree of inequality aversion. If we are less inequality averse, we will choose values very close to zero. If we are more inequality averse, the choice will fall on relatively higher values. Once e has been selected, Step 3 asks us to apply formula [7] to calculate the level of social welfare that can be associated to the initial income distribution.

Figure 4: A step-by-step procedure to build SWF

STEP	Operational content
1	Sort the income distribution by income level
2	Choose the value of the parameter *e* in the functional form
3	Apply formula [7] to each income of the income distribution

5. AN EXAMPLE OF HOW TO BUILD AN SWF AND HOW TO CALCULATE SOCIAL WELFARE

Table 2 exemplifies the procedure to build an SWF. Step 1 ranks incomes in ascending order. Step 2 defines the appropriate level of *e*=0.5. Step 3 applies the formula and calculates average social welfare. Its value, as reported in the bottom row of Step 3, is 17.7. By itself, this number does not provide very much information. Its informational content is more evident when we compare, for example, income distributions.

Table 2: An example of how to calculate social welfare

STEP 1	STEP 2	STEP 3
Sort income distribution by income levels	Choose the value of *e*	Apply formula to calculate social welfare

Individual	Income distribution A
1	1,000
2	2,000
3	3,000
4	4,000
5	5,000

e	0.5

Individual	Income distribution A
1	31.6
2	44.7
3	54.8
4	63.2
5	70.7

Social welfare	17.7

Let us now turn to this case in Table 3, where two income distributions are shown and social welfare is calculated. Income distribution B is only a scaled version of income distribution A, as all incomes are obtained by multiplying those in A by 20 per cent. This means that any relative inequality measure (as the Gini index) will measure the same inequality in A and in B. However, social welfare may be different, because, as discussed above, social welfare takes into account not only inequality but also total income.

Table 3: Same inequality, more total income, more social welfare

Individual	Income distribution A	Income distribution B	Individual	Social welfare in A (e=0.5)	Social welfare in B (e=0.5)
1	1,000	1,200	1	31.6	34.6
2	2,000	2,400	2	44.7	49.0
3	3,000	3,600	3	54.8	60.0
4	4,000	4,800	4	63.2	69.3
5	5,000	6,000	5	70.7	77.5
Total income	15,000	18,000	Social welfare	17.7	19.4

This is shown in the bottom row. Inequality is by assumption the same, but total income is higher in B (18,000 income units) than in A (15,000 income units). Average social welfare is therefore higher in B (19.4) than in A (17.7).

6.   READERS' NOTES

6.1   Time requirements

Time required to deliver the module is estimated at about two hours.

6.2   EASYPol links

Selected EASYPol modules may be used to strengthen readers' background and to further expand their knowledge on welfare analysis.

This module belongs to a set of modules that discuss how to implement a welfare analysis comparing income distributions generated by different policy options. It is part of a set of modules composing a training path addressing **Analysis and monitoring of socio-economic impacts of policies.**

The following EASYPol modules form a set of materials logically preceding the current module, which can be used to strengthen the users' background information:

- ✓ EASYPol Module 000: *Charting Income Inequality: The Lorenz Curve*
- ✓ EASYPol Module 001: *Social Welfare Analysis of Income Distributions: Ranking Income Distribution with Lorenz Curves*

Issues addressed in this module are further elaborated in the following modules:

- ✓ EASYPol Module 002: *Social Welfare Analysis of Income Distributions: Ranking Income Distribution with Generalised Lorenz Curves*
- ✓ EASYPol Module 050: *Policy Impacts on Inequality: Welfare-Based Measures of Inequality*

A case study presenting the use of the social welfare functions to measure the welfare impacts in the context an agricultural policy impact simulation exercise with real data is reported in the EASYPol Module 042: *Inequality and Poverty Impacts of Selected Agricultural Policies: The Case of Paraguay.*

6.3   Frequently asked questions

- ✓ What is the difference between inequality and social welfare analysis?
- ✓ How many types of social welfare functions can be specified?
- ✓ Do results based on a social welfare function hold for alternative social welfare functions?

7.   FURTHER READINGS

Arrow K.J., 1950. A Difficulty in the Concept of Social Welfare, *Journal of Political Economy*, **58**, 328-346.

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Rawls J., 1972. *A Theory of Justice*, Oxford University Press, Oxford, UK.

Module metadata

1. EASYPol Module	041

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3. Subtitle in original language	
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4. Summary

This module illustrates the concept of social welfare and the possible ways to define social welfare functions. In particular, it deals with how to pass from inequality to social welfare analysis and how social welfare analysis may embody different attitudes with regard to inequality aversion. A step-by-step procedure and numerical examples are also discussed to give operational content to the tool.

5. Date

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7. Module type	☐ Thematic overview ☐ Conceptual and technical materials ☒ Analytical tools ☐ Applied materials ☐ Complementary resources
8. Topic covered by the module	☒ Agriculture in the macroeconomic context ☐ Agricultural and sub-sectoral policies ☐ Agro-industry and food chain policies ☐ Environment and sustainability ☐ Institutional and organizational development ☐ Investment planning and policies ☒ Poverty and food security ☐ Regional integration and international trade ☐ Rural Development
9. Subtopics covered by the module	
10. Training path	**Analysis and monitoring of socio-economic impacts of policies**
11. Keywords	capacity building, agriculture, agricultural policies, agricultural development, development policies, policy analysis, policy impact anlaysis, poverty, poor, food security, analytical tool, impacts of policies on poverty, income inequality, inequality, inequality analysis, inequality measures, inequality aversion, income distribution, income ranking, gini index, lorenz curve, social welfare, social welfare functions,