

MApping System & Services for Canal Operation Techniques

MASSCOTE

**FAO Water
Development
and
Management
Unit**



**Sensitivity Analysis
of
irrigation structures**
****** Technical Briefs ******



Daniel RENAULT

DRAFT

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IMPORTANCE of SENSITIVITY for CANAL OPERATION TECHNIQUES

Targets of brief 1:

- how important sensitivity analysis to understand the behavior of the canal?
- what is the use of sensitivity to design appropriate operation techniques?



<p>The issues: fluctuations and low regime</p> <p>Gated canal systems are facing in practice unsteady hydraulic conditions despite having being designed for hydraulic steady state. Water level in canals fluctuates frequently as a result of unscheduled variation of inflows and outflows and changes of the setting of regulators.</p> <p>Another major issue is that some canals are run with water levels much below the designed Full Supply Depth (FSD) for which conditions are optimal.</p>	<p>Consequences for Operation</p> <p>Water level fluctuations and low level regime (below FSD), generate:</p> <ul style="list-style-type: none"> - high discharge variations at sensitive diversion points (offtake); - significant increase of offtake sensitivity due to the lowering of water level in the parent canal; - poor supply conditions to highly elevated offtakes.
<p>Behaviour of canals under perturbations</p> <p>The behaviour of a canal with respect to perturbation is highly dependant on the sensitivity of its structures.</p> <p>Irrigation systems with low sensitive structures, tend to propagate downward and amplify the water fluctuations.</p> <p>Irrigation sub-systems where some structures or reaches are sensitive are likely to absorb the discharge perturbations coming from the upstream point of the canal.</p>	<p>Consequences for Operation</p> <p>Knowing the sensitivity of the structures and of the sub-system allows a better understanding of the perturbation propagation along a canal.</p> <p>Sensitive subsystems will have lower performance results than a non sensitive system if operated with the same rules.</p> <p>Knowing where the irrigation system is sensitive is important to design smart information systems in particular to know where unscheduled changes will be reflected.</p>

Is it compulsory to have a sensitivity approach for COP?	
<p>NO, practical experience of the irrigation system can also lead to similar knowledge of its behaviour and be used for decision making. This implicit empirical knowledge can be sufficient, the problem is related to staff rotation, once the managers left this knowledge almost entirely disappear and the learning process has to start again.</p>	<p>YES in the sense that sensitivity approach is a way to make more explicit the behaviour of canals and sub subsystems through standardized indicators and thus allow any new manager to quickly grasp the constraints and opportunities associated to a particular system.</p>
<p>NO in the sense that the behaviour of canals can be also tackled using hydraulic models provided they are enough powerful to allow unsteady flow simulation.</p>	<p>YES in the sense that hydraulic simulations are not easy to master and there is a need to have much more simplified methods to investigate the behaviour of canals.</p>
<p><i>To summarize one can do of course without a sensitivity analysis however it provides an understandable framework easy to master by managers without having the sophisticated approach of hydraulic simulation and without the risk of losing the accumulated practical experiences.</i></p>	
<p>What is the use of Sensitivity?</p> <p>The sensitivity of irrigation structures allows answering the following questions:</p> <ul style="list-style-type: none"> • What is the propensity of the system to be affected by fluctuations? • What is the propensity of the system to create fluctuations? • How is performance affected by the sensitivity of the system? • How can more appropriate and simplified operational procedures be developed? • Where should managers concentrate efforts in ensuring that no unpredictable deviation affects the water balance? 	
<p>The ability to identify the locations of the sensitive points or sensitive parts of the system is of specific importance for the managers. For these points, precise operation and regular checking are recommended to minimise possible deviations.</p> <p>A sensitivity analysis is fundamental to improve cost effectiveness in operating a canal.</p>	<p>Two major sets of rules can be drawn from sensitivity analysis:</p> <ul style="list-style-type: none"> • a set of modulated rules and guidelines for canal operation to achieve a uniform performance level whatever the distribution of the sensitivity is. • a set of practical rules for the implementation of a selective information system. <p>The first one may imply the need to give more attention to reaches where sensitivity is high, in order to maintain deliveries within an acceptable range of variation.</p> <p>The second set may help to monitor the irrigation system with only a limited number of points. These points should be selected on the basis of their sensitivity.</p>

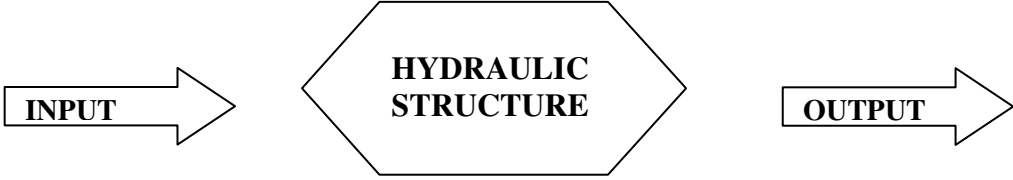
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DEFINITIONS

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Targets of brief 2:

- what are the definitions of the indicators of sensitivity for canal structures?
- =====

<p>“Sensitivity,” is defined as the ability of responding to external stimuli or impressions (Collins Dictionary).</p> <p>From a conceptual point of view, sensitivity expresses the flexibility between output and input variations. Applied to irrigation, it characterises the way structures respond when they are operated or stimulated by an external cause.</p>	<p>An irrigation system is a physical infrastructure composed of numerous structures: offtakes, regulators, canal reaches, and storage facilities. In the whole process leading to water delivery, each structure is meant to transform an input variable into an output variable.</p>
 <p>Principle of Input-Output in an hydraulic structure</p>	
<p>Sensitivity of irrigation structures is addressed by looking at the impacts of perturbations on:</p> <ul style="list-style-type: none"> • delivery to the command area influenced by the structure, and • on flow regime (water depth) in the canal. 	<p>The sensitivity of an irrigation structure is defined as the ratio of the relative or absolute variation of outputs to the relative or absolute variation of the inputs, as follows:</p> <p style="background-color: yellow; padding: 5px;"> $\text{Sensitivity} = \frac{\text{Variation of OUPUT}}{\text{Variation INPUT}}$ </p>
<p>The discharge sensitivity to water depth variations at any structure is defined as the ratio of the relative variation of discharge through the structure ($\Delta q/q$) and water depth deviation (ΔH) upstream of the structure.</p>	$S = \frac{\Delta q / q}{\Delta H} \quad (\text{unit: m}^{-1})$
<p>The water depth sensitivity to discharge variations along the canal, at a regulator or at any other section, is expressed as the variation of water depth (ΔH) resulting from a relative discharge variation in the canal ($\Delta Q/Q$).</p>	$S = \frac{\Delta H}{\Delta Q / Q} \quad (\text{unit: m})$

Relative or absolute water depth variation

Why absolute water depth variation (Δh) in previous equations of sensitivity instead of the relative value ($\Delta h/h$), which would make the indicator dimensionless?

Mathematicians may like relative values, managers prefer the absolute variation for h for practical reasons.

Most of the time, the management variable that is used to define the target for control is (Δh), and rarely ($\Delta h/h$).

For example, an instruction from a manager to a gate operator would probably be: “You should operate the regulator when the deviation in water level (Δh) from the local nominal target exceeds 10 cm.” This instruction is straightforward. The instruction would be much more difficult to handle in relative terms: “You should operate the regulator if the relative deviation of water level from target exceeds 5 percent.”

Proportionality indicator or Hydraulic Flexibility

The proportionality indicator often named hydraulic flexibility aims to characterize the relative variations of discharge in the dependent and parent canals. This indicator has usually been referred to as “flexibility”, and is well adapted to structured systems.

As the term “flexibility” is widely used these days to characterize the degree of freedom in accessing irrigation water (services) we suggest that the term “proportionality” be used instead to avoid confusion.

Dividing both the numerator and the denominator by the variation in water depth in the parent canal allows rewriting F as the multiplication of the two sensitivity indicators [sensitivity for discharge through the offtake (S_{Offtake}) x sensitivity of the regulator in the parent canal ($S_{\text{Regulator}}$)].

$$F = \frac{\Delta q / q}{\Delta Q / Q}$$

$$F = \left(\frac{\Delta q / q}{\Delta H_{\text{US}}} \right) \left[\frac{\Delta H_{\text{US}}}{\Delta Q / Q} \right] = S_{\text{Offtake}} S_{\text{Regulator}}$$

- $F < 1$ (underproportional): a relative change in discharge in the parent canal generates a smaller relative change in the offtaking canal. Fluctuations are diminished in the offtaking canal.
- $F = 1$ (proportional): a relative change in discharge in the parent canal generates an equal relative change in the offtaking canal. Fluctuations are divided uniformly.
- $F > 1$ (hyperproportional): a relative change in discharge in the parent canal generates a larger relative change in the offtaking canal. Fluctuations are exacerbated in the offtaking canal.

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BASICS ELEMENTS in SENSITIVITY/OPERATION

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Targets of brief 3:

- what are the important features of the flowing conditions that needs to be accounted for ?
- =====

The physical elements of the canal that we are considering in the sensitivity approach are:

- a single structure [converting head and flow geometry into a passing discharge]
- a reach or a pool [composed of several in and out structures and bounded by two consecutive cross regulators]
- a sub-system [made of several reaches]

The important aspects we want to characterize are:

- the influence of water level variation (ΔH) on the discharge passing through the offtakes.
- the reach reaction when a change of discharge occurs at its boundaries: What ΔH might result?
- the pace of water level change within a reach ($\Delta H/\Delta t$) as a function of the perturbation.

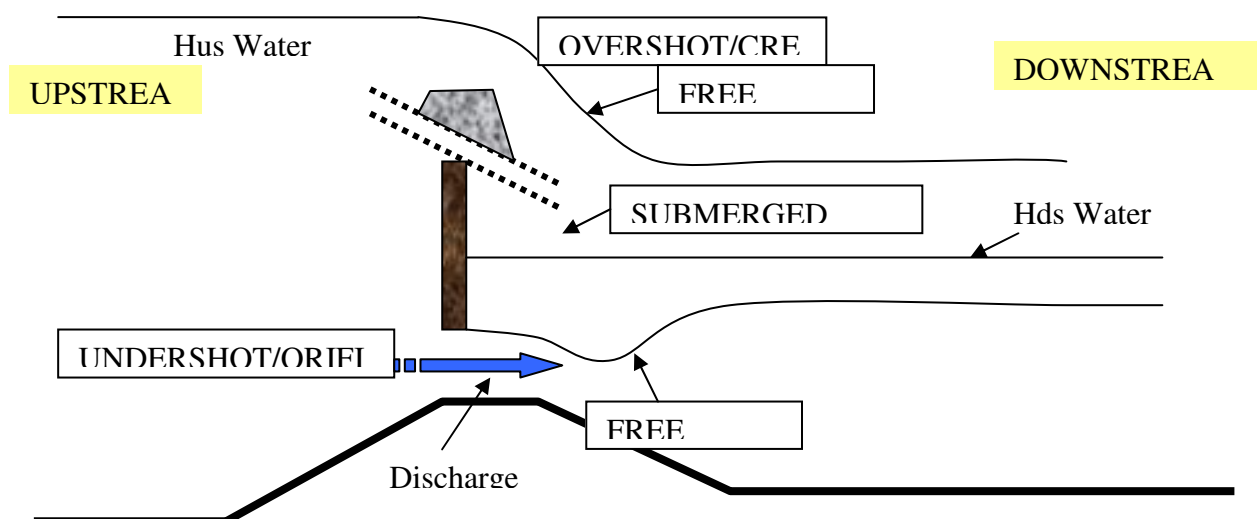
1. Diverting structure [Offtake]

Objective of Canal Operation is the control of diverted discharge through the structure.

2. Cross-Regulator [X-reg]

Objective of Canal Operation is the control of water level in the canal near by the structure

Sketch of a generic irrigation structure



The flowing conditions at an irrigation structure are defined by:

- **the type of flow:** Overshot for a crest - Undershot for an orifice.
- **the downstream conditions:** free flow or submerged.

Submergence can result from various features:

- the presence of a measuring weir immediately downstream of the structure (mostly for offtake)
- the presence of a weir in the canal further downstream
- a normal flow condition
- an influenced flow from a further downstream cross structure.

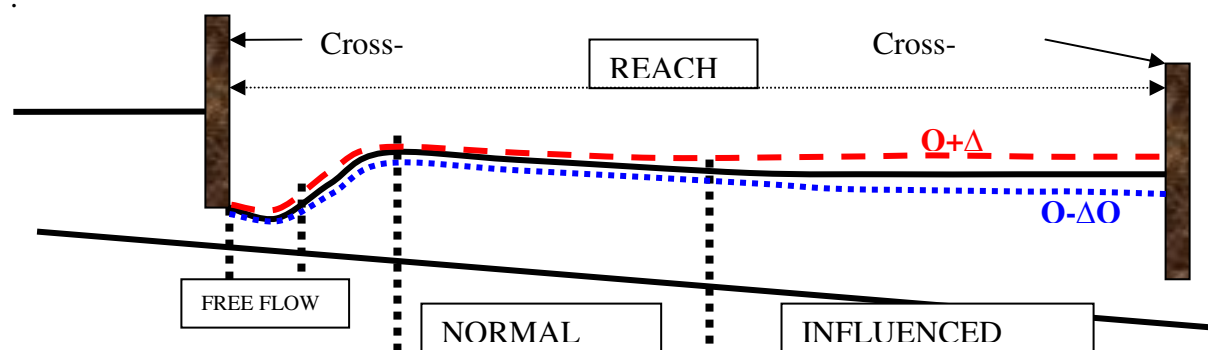
3. A reach

A reach is a pool or section of canal between 2 consecutive X-regs :

Objective of Canal Operation: the reach is controlled by other structures [cross-structures and offtakes] thus no specific operation BUT reach can be used as Storage/buffer capacity when freeboard allows.

The flow within a reach maybe characterized by a sequence of the following types:

- Free-Flow may occur at the downstream part of the upstream X-reg if the flow is supercritical (high velocity).
- Normal Flow occurs in canal section when the flows is steady, undercritical and non influenced
- Influenced Flow when the flow enters the backwater effect of any downstream structure.



Some reaches exhibit only the last type of flow (influenced flow), while others may have the three types. Also the status of flow might change with the running discharge, for instance free-flow upstream occurring only for the low discharge.

4. A sub-system

A sub-system is a subset of a canal made of several contiguous reaches.

Objective of Canal Operation: Reaches may be connected or separated. For control they are fully separated if free flow occurs, partially separated if there is normal flow and connected when influenced flow.

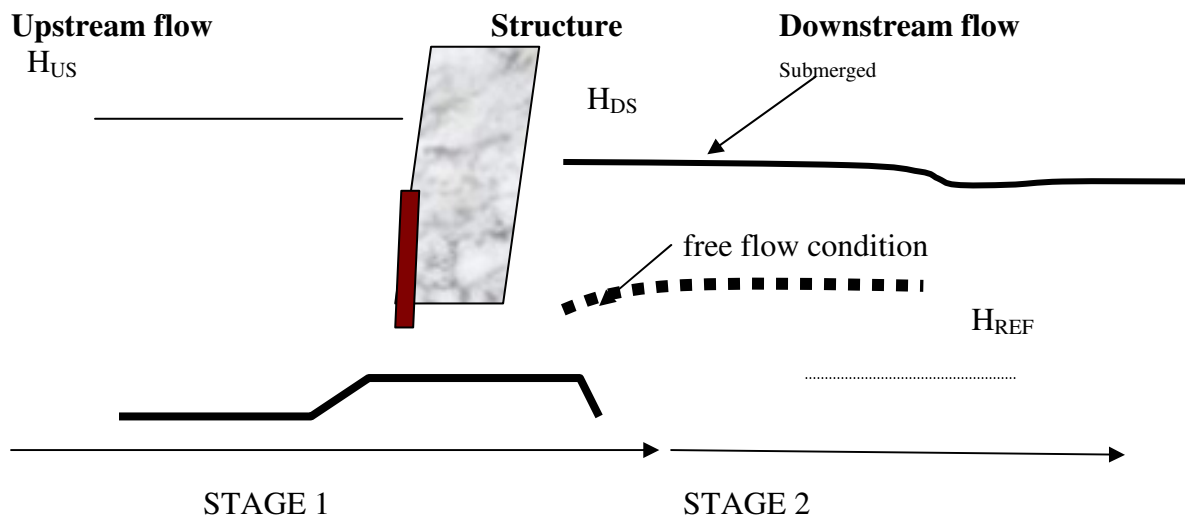
COMPUTATIONS of SENSITIVITY INDICATORS

Targets of brief 1:

- to illustrate the calculation of the sensitivity indicators from the hydraulic flows equations
 - to give the exact and approximate formula to estimate sensitivity indicators
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Hydraulic formulations

In the case of flow submergence downstream of the structure, the exact computation of the flows requires considering two equations, one through the structure itself and one for the flow regime immediately at the downstream side of the structure.



Flowing description through an irrigation structure with different downstream conditions (free-flow or submerged)

The general governing equations of flow through an open-channel structure are:

$$q = a A (H_{US} - H_{DS})^\alpha \quad \text{Stage 1} \quad (1)$$

$$q = a' b (H_{DS} - H_{REF})^\beta \quad \text{Stage 2} \quad (2)$$

where:

- A = flow section parameter through the structure (A = area through the orifice for an undershot flow, and A = the crest length for an overshoot flow);
- a = discharge coefficient equal to $c(2g)^{0.5}$;
- c = flow coefficient function of the shape of the flow ($c \approx 0.5$ for an orifice);
- a' & b = hydraulic parameters of the second stage law;
- α, β = exponent equal to $1/2$ for undershot flow, to $3/2$ for overshoot, and about to 1.6 for normal flow;
- H_{US} = water level upstream of the structure;
- H_{DS} = water level downstream of the structure;
- H_{REF} = a reference level depending on the downstream flow conditions;
- q = discharge through the structure.

H_{REF} is a constant reference level taken at: (i) the crest level of the weir where there is a measurement weir; or (ii) a reference level (bottom bed or a crest level) further down conditioning the flow at the structure (Table below). It is assumed that $dH_{REF} = 0$.

Conditions of flow and reference to be considered in calculations

Specific conditions	H_{REF}	2nd equation	α	β
Undershot free flow	Orifice axis	Not needed	0.5	no
Overshot free flow	Crest level of the weir	Not needed	1.5	no
Undershot submerged by a downstream measurement weir	H crest of the measurement weir	Needed	0.5	1.5
Undershot submerged normal uniform flow	H bed bottom of the downstream canal section	Needed	0.5	1.66

Free-flow conditions

Under free-flow conditions at the offtake, Stage 2 (Equation 2) is irrelevant and the problem reduces to one equation, i.e. Equation 1. Then, H_{DS} is taken either as the crest level of the weir in the case of overshoot, or as the orifice axis in the case of undershoot.

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Solution for the general case with submerged conditions

In a general case, rewriting Equations 1 and 2 produces:

$$\left(\frac{q}{aA}\right)^{1/\alpha} = H_{US} - H_{DS} \tag{3}$$

and

$$\left(\frac{q}{a'b}\right)^{1/\beta} = H_{DS} - H_{REF} \tag{4}$$

Adding Equations 3 and 4 yields:

$$\left(\frac{q}{aA}\right)^{1/\alpha} + \left(\frac{q}{a'b}\right)^{1/\beta} = H_{US} - H_{REF} \tag{5}$$

Then, taking the logarithm derivative with respect to the variable H_{US} gives:

$$\frac{1}{\alpha} \left(\frac{q}{aA}\right)^{1/\alpha-1} \left(\frac{\partial q}{\partial H_{US}}\right) + \frac{1}{\beta} \left(\frac{q}{a'b}\right)^{1/\beta-1} \left(\frac{\partial q}{\partial H_{US}}\right) = \frac{\partial(H_{US} - H_{REF})}{\partial H_{US}} \tag{6}$$

From this can be derived, given that $dH_{REF} = 0$:

$$dH_{US} = \left[\frac{q^{\frac{1}{\alpha}-1}}{\alpha [aA]^{1/\alpha}} + \frac{1}{\beta} \frac{q^{\frac{1}{\beta}-1}}{[a'b]^{1/\beta}} \right] dq \tag{7}$$

which can be rewritten as:

$$dH_{US} = \left[\frac{q^{\frac{1}{\alpha}}}{\alpha [aA]^{\frac{1}{\alpha}}} + \frac{1}{\beta} \frac{q^{\frac{1}{\beta}}}{[a'b]^{\frac{1}{\beta}}} \right] \frac{dq}{q} \quad (8)$$

Replacing Equations 1 and 2 leads to:

$$dH_{US} = \left[\frac{1}{\alpha} (H_{US} - H_{DS}) + \frac{1}{\beta} (H_{DS} - H_{REF}) \right] \frac{dq}{q} \quad (9)$$

which can be rewritten as:

$$dH_{US} = \frac{H_E}{\alpha} \frac{dq}{q} \quad (10)$$

by introducing the term of head-loss equivalent (H_E):

$$H_E = (H_{US} - H_{DS}) + \frac{\alpha}{\beta} (H_{DS} - H_{REF}) \quad (11)$$

The head-loss equivalent, H_E , of a particular structure is equal to the head loss through the structure corrected by a factor expressing the influence of the submergence.

Nota : Head loss equivalent, H_E , of a particular offtake, is equal to the head loss of the same kind of offtake (undershot or overshot), having the same sensitivity value but with free-flow downstream conditions.

For water diversion [Offtake] the sensitivity indicator is the ratio of relative variation of withdrawal (discharge q) to the variation of water level in the parent canal H whereas for water level control [Regulator] sensitivity indicator is the ratio of the variation of water level in the parent canal H to the relative variation of discharge Q in the main canal:

$$S_{\text{Offtake}} = \frac{\Delta q / q}{\Delta H} \quad S_{\text{Cross-regulator}} = \frac{\Delta H}{\Delta Q / Q}$$

Replacing in Eq. 8 leads to identify the exact values of the sensitivity indicators.

S diversion [Offtake]	$S = \frac{dq}{q dH_{US}} = \frac{\alpha}{H_E}$
for S control level [Regulator]	$S = \frac{dH_{US}}{(dq/q)} = \frac{H_E}{\alpha}$
He	$H_E = (H_{US} - H_{DS}) + \frac{\alpha}{\beta} (H_{DS} - H_{REF})$

SENSITIVITY VARIATION and DRIVERS

Targets of brief 5:

- How sensitivity varies with flowing conditions?
- What are the limits of sensitivity?
- What are the drivers of sensitivity?

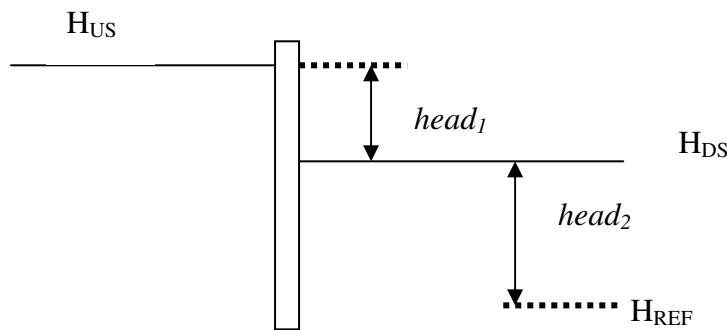
Sensitivity indicators are determined by the following equations (see Brief no 4)....

S diversion [Offtake]	$S_{off} = \frac{dq}{q dH_{US}} = \frac{\alpha}{H_E}$
for S control level [Cross Regulator]	$S_{xreg} = \frac{dH_{US}}{(dq/q)} = \frac{H_E}{\alpha}$
with Head equivalent He	$H_E = (H_{US} - H_{DS}) + \frac{\alpha}{\beta} (H_{DS} - H_{REF})$
and a good proxy for Head Equivalent (not valid for high sensitivity)	$H_E \approx (H_{US} - H_{DS})$

.....with the following Conditions of flow and reference to be considered in calculations

Specific conditions	H _{REF}	2nd equation	α	β
Undershot free flow	Orifice axis	Not needed	0.5	no
Overshot free flow	Crest level of the weir	Not needed	1.5	no
Undershot submerged by a downstream measurement weir	H crest of the measurement weir	Needed	0.5	1.5
Undershot submerged normal uniform flow	H bed bottom of the downstream canal section	Needed	0.5	1.66

Flow variations are conditioned by the type of flows and the head exercised at each flow stage as defined below:



Under the general case of submerged conditions H_{DS} can vary from H_{REF} up to H_{US} , the values of sensitivity obtained for these two levels define the boundaries of the variation of the indicator:

- $H_{DS} = H_{REF}$ H_E is thus reduced to its first term only and we have

$$S_{Off.min} = \frac{\alpha}{(H_{US} - H_{REF})} \qquad S_{Xreg.max} = \frac{H_{US} - H_{REF}}{\alpha}$$

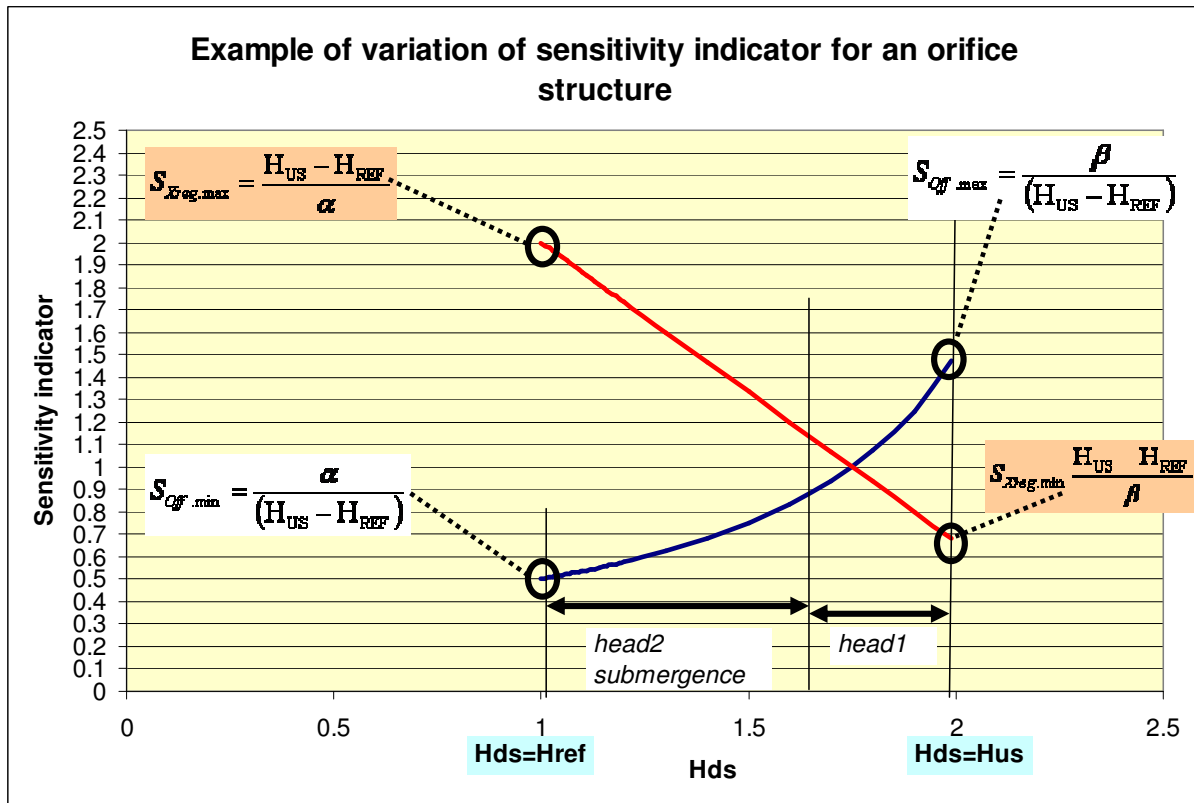
- $H_{DS} = H_{US}$ H_E is reduced to its second term only

$$S_{Off.max} = \frac{\alpha}{\left[\frac{\alpha}{\beta} (H_{DS} - H_{REF}) \right]} = \frac{\beta}{(H_{US} - H_{REF})}$$

$$S_{Xreg.min} = \frac{\alpha/\beta [H_{DS} - H_{REF}]}{\alpha} = \frac{H_{US} - H_{REF}}{\beta}$$

For both sensitivity indicators [discharge (*Soff*) and water level control (*SXreg*)] the maximum value is equal to $\frac{\beta}{\alpha}$ times the minimum.

Of course the variations are inverse.



Factors affecting offtake sensitivity: head/water level in the main canal

Practical conclusions for sensitivity assessment

You determine the value of H_{US} - H_{REF} and you bound the sensitivity variation between S_{min} and S_{max} .

$$S_{Offtake} = \frac{\alpha}{\left(Head_1 + \frac{\alpha}{\beta} Head_2 \right)}$$

$$S_{X-reg} = \frac{Head_1}{\alpha} + \frac{Head_2}{\beta}$$

$H_{REF} = H_{DS} = H$ of the orifice if it is an undershot structure or H of the crest if it is overshot

$$H_E = H_{US} - H_{DS} = Head$$

S varies with the head exercise on the structure.

If the water level H_{DS} is known and steady then you

A robust first approximation of sensitivity indicators considering $H_E=$ head

$$\text{Approx. } H_E = Head = (H_{US} - H_{DS})$$

$$S_{OFFTAKE} = \frac{\alpha}{H_{US} - H_{DS}}$$

$$S_{REGULATOR} = \frac{H_{US} - H_{DS}}{\alpha}$$

The sensitivity of the structures depends, as first approximation, on the head exerted on the structures. This may vary as a function of the operation regime of the canal (average height in the parent canal) and of some interventions on the structures.

For instance, in Sri Lanka, a canal that originally was of very low sensitivity (similar to canal B of fig.18), has started to have chaotic operation as a result of the combined effects that moved the sensitivity from 0.5 to 3. The first of these effects is related to the systematic construction of the gauging weir downstream the offtakes, and this resulted in rising the water level by about 40 cm, and moved the average sensitivity around 2 approximately; the second effect is a sub-regime operation of the parent canal (median height of the water level in the

main canal by 10 cm lower with respect to the nominal value FSD), and this further reduces the average head available at the offtakes. The average sensitivity has moved to 3.

The effect of the water line in the canal on the sensitivity indicator is illustrated in figure 21.

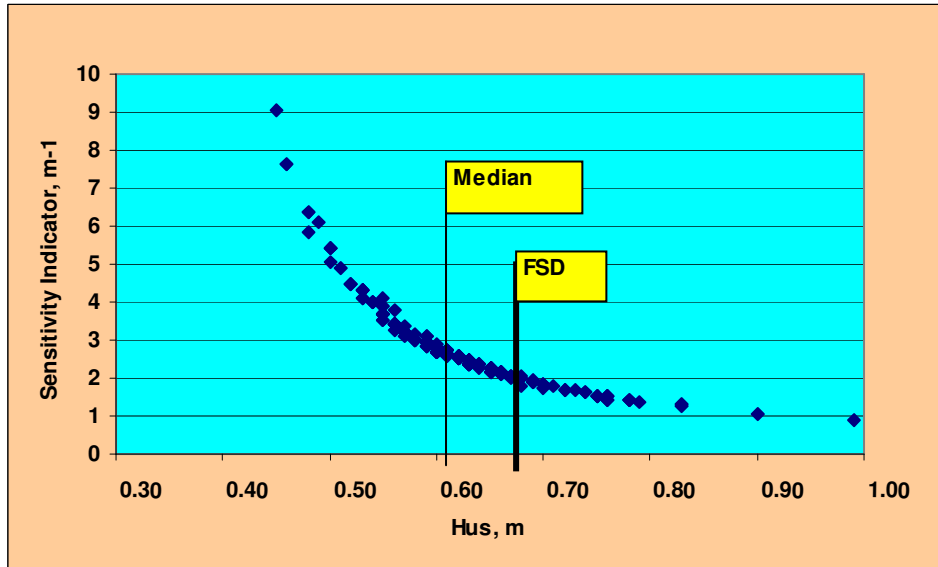


Figure 21. Evolution of sensitivity versus the average level of the water observed in a canal (Sri Lanka). Difference between the nominal value (FSD) and the observed median value.

Influence of downstream submergence of the structure on sensitivity

The exact expression of the hydraulic formulations of indicators involves the back-effect of submergence upon the sensitivity through the expression H_E (equation 17). With a similar head (upstream-downstream difference), submergence tends to reduce the sensitivity of the structure compare to a free flow structure.

For offtake structures, we can initially neglect this effect by calculating the simple indicator (equation 13) with the difference in head ($H_{US} - H_{DS}$). Only the structures having a high sensitivity indicator ($>1,5$) have to be calculated more exactly taking into account the submergence effect.

$$S_{Off.max} = \frac{\beta}{(H_{US} - H_{REF})}$$

$$S_{Xreg.min} = \frac{H_{US} - H_{REF}}{\beta}$$

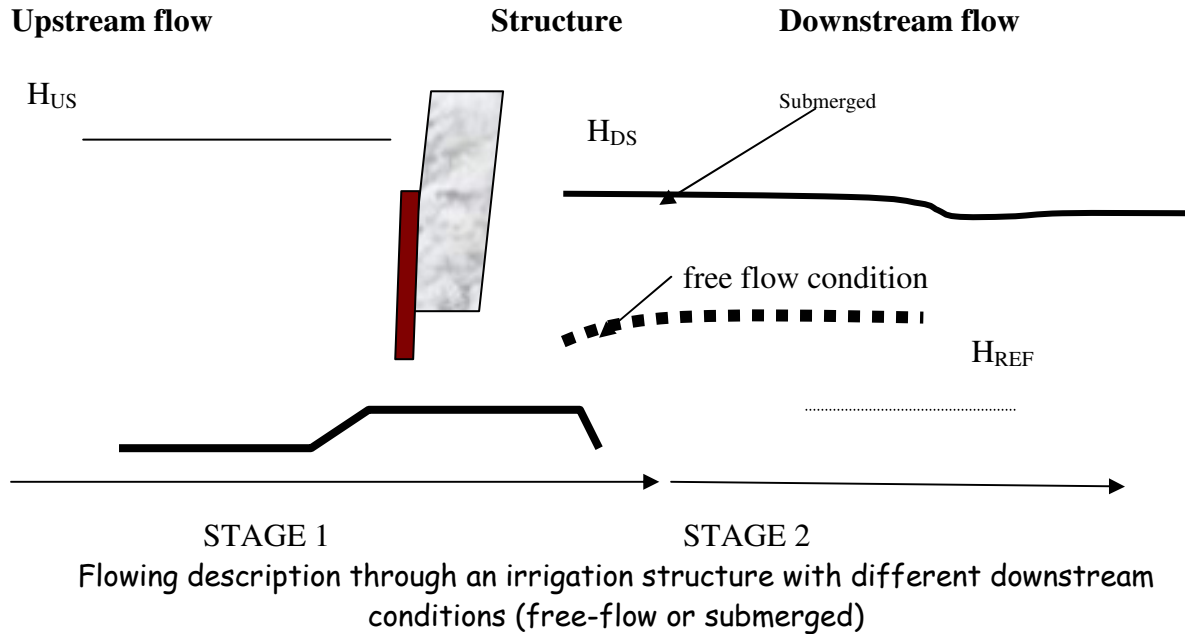
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SENSITIVITY of OFFTAKE

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Targets of brief 6:

- investigate specifically the sensitivity of diverting structures (offtakes)
 - how to reduce the value of sensitivity? how to make operate sensitive offtakes?
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The general governing equations of flow through an open-channel structure are:

$$q = a A (H_{US} - H_{DS})^\alpha \quad \text{Stage 1} \quad (1)$$

$$q = a' b (H_{DS} - H_{REF})^\beta \quad \text{Stage 2} \quad (2)$$

where:

- A = flow section parameter through the structure (A = area through the orifice for an undershot flow, and A = the crest length for an overshot flow);
- a = discharge coefficient equal to $c(2g)^{0.5}$;
- c = flow coefficient function of the shape of the flow ($c \approx 0.5$ for an orifice);
- a' & b = hydraulic parameters of the second stage law;
- α, β = exponent equal to $1/2$ for undershot flow, to $3/2$ for overshot, and about to 1.6 for normal flow;
- H_{US} = water level upstream of the structure;
- H_{DS} = water level downstream of the structure;
- H_{REF} = a reference level depending on the downstream flow conditions;
- q = discharge through the structure.

H_{REF} is a constant reference level taken at: (i) the crest level of the weir where there is a measurement weir; or (ii) a reference level (bottom bed or a crest level) further down conditioning the flow at the structure (Table below). It is assumed that $dH_{REF} = 0$.

Specific conditions	Href	2 nd equation	α	β
undershot free flow	Orifice axis	not needed	0.5	no
overshot free flow	crest level	not needed	1.5	no
Undershot submerged measurement weir	H crest	Needed	0.5	1.5
Undershot submerged normal uniform flow	H bed bottom	Needed	0.5	1.66

Solution for the general case with submerged conditions

S diversion [Offtake]	$S = \frac{\left(\frac{dq}{q}\right)}{dH_{US}} = \frac{\alpha}{H_E}$
with He	$H_E = (H_{US} - H_{DS}) + \frac{\alpha}{\beta}(H_{DS} - H_{REF})$

Type of flow: undershot are best for offtaking.

Overshot structures are too sensitive ($\alpha = 1.5$) compare to undershot ($\alpha = 0.5$). Unless one wants to build a sensitive diversion point for example to divert into a specific command area surplus coming from upstream of to avoid excess water when it rains, it is not recommended to use overshot for offtake.

Depending on the type of offtake and the head exercised on it sensitivity varies between low to very high values. With low sensitive offtakes, the distribution of water is not affected by perturbations which are passing downward. A system aiming at delivering specific service to users (discharge) it is generally desirable that the offtake are low sensitive, whereas for a system that is based on proportional distribution, a low sensitivity is not desirable and sensitivity of the offtake should be adjusted with that of the cross regulator in order to have a flexible indicator of one at partition points.

In some case along gated system it might be interested to have some highly sensitive offtakes for the purpose of diverting the surplus or compensating the deficit.

Offtake sensitivity might varies from as follows:

OFFTAKE	Sensitivity Indicator	Example
HIGHLY SENSITIVE	greater than 2	<ul style="list-style-type: none"> overshot type Offtake undershot type with very low head (0.25 or less)
MEDIUM SENSITIVE	between 1 and 2	<ul style="list-style-type: none"> undershot type with head between 0.25 and 0.5

LOW SENSITIVE	below 1	<ul style="list-style-type: none"> • an undershot gated structure fed with head > 0.5 • specific modulated structure (baffle see below)
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Example of low sensitive offtake: Module or baffle

In a system aiming at delivering specific service to users (discharge), it is generally desirable that the offtakes be low-sensitive. For a system that is based on proportional distribution, a low sensitivity is not desirable, and the sensitivity of the offtake should be adjusted with that of the cross-regulator in order to have a flexible indicator of 1 at partition points.

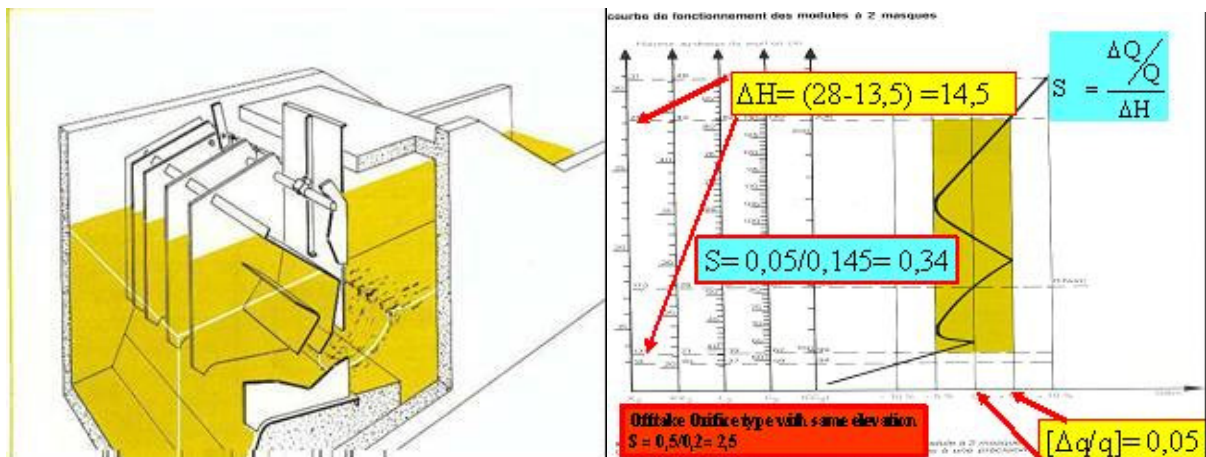
In some case along a gated system, it might be useful to have some highly sensitive offtakes for the purpose of diverting the surplus or compensating the deficit. Offtake sensitivity might vary, as summarized in Table A2.3.

An example of low-sensitive offtakes

Neyrpic distributors (baffles) are designed to control discharge when available head is low and variable. The control is obtained by changing flow regimes. At low head, the regime is overshot free flow on the sill. For greater head, baffles create undershot orifice-type flow regimes with contracted veins.

In the example of the baffle distributor shown in Figure A2.2, the discharge is controlled within 10 percent for water level fluctuating between 13.5 and 28 cm above the sill level. Thus, the sensitivity is equal to $S_{Baffle} = 0.1 / (0.28 - 0.1345) = 0.68$, which is quite low.

A classic undershot offtake placed in the same situation would have experienced sensitivity as follows: $S = 0.5 / (\text{average head}) = 0.5 / 0.2 = 2.5$.

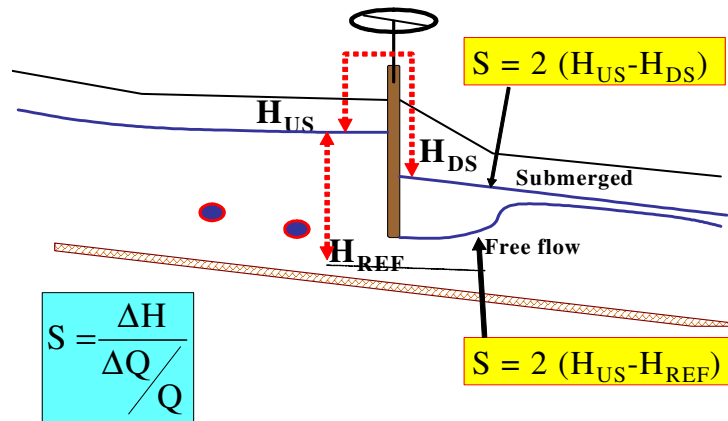


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SENSITIVITY of CROSS-REGULATORS

Targets of brief 7:

- investigate specifically the sensitivity of water level control structures (offtakes)
 - how to reduce the manage the sensitivity? how to make operate sensitive regulators?
- =====



Orifice type regulator.

Regulator sensitivity

Cross-regulators are irrigation structures controlling the water depth along a canal, by adjusting local head losses. The degree of control, and the magnitude of variation of water depth between zero and full discharge, vary with the type of structure. A fixed weir, for example, has no control on water depth, but the magnitude of resulting variation with respect to discharge change is very low. Inversely, gated regulators have a high control level, but improperly managed the magnitude of variation of water depth can be high.

For cross-regulators the distinction between delivery and conveyance makes no sense, however the upward effect as well as the downward effect have to be considered.

Downward Cross-Regulator sensitivity

The equations governing the flow of an orifice type regulator are similar to those of an offtake of the same type (equations 11 and 14-15). By definition, it is known that the value of the sensitivity indicator is the inverse of the offtake one (equation 7 and 8), then we have that:

$$S = \frac{H_E}{\alpha} \tag{20}$$

In the case of a free orifice, H_E is equal to the difference between the upstream level H_{US} and the axis of the opening below the gate ($H_{REF} = H_{OR}$). In general, the regulators are submerged downstream, and the difference in head to be considered is the value of H_E in its complete formulation (equation 17). For an orifice type structure $\alpha=0,5$ and we obtain:

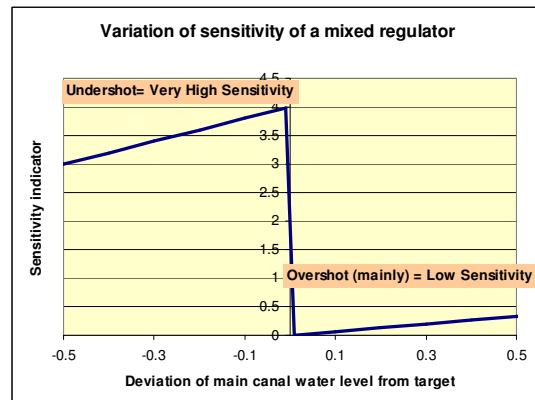
$$S = 2.H_E \tag{21}$$

The orifice type regulators are generally equipped with gated offtakes and their control function is obtained by operating these gates.

Differential variation on mixed regulators

Some level regulators include orifice type gates in the central part and overshoot laterally. The crest of these sills generally defines the target level to be controlled at this spot. These structures thus differ in their behaviour according to the sign of perturbation. For a negative perturbation, the level lowers below the crest of the sill and the sensitivity is the one of the central gates (orifice type). For a positive perturbation, the level increases above the crest of the sill and sensitivity equally depends on the lateral sills that generally are poorly sensitive. For this type of mixed work, the sensitivity to positive perturbations is considerably reduced with respect to the one of the negative perturbations. Two sensitivity indicators should be defined for a composite structure, depending on whether there is a spill or not (S+ and S-).

The mixed regulator shown in the picture below has a very different sensitivity. For water below the crest, the regulator is highly sensitive S certainly be greater than 4 (head estimated at more than 2 meters). For water level above crest the sensitivity drops dramatically to very low sensitive. This type of regulator should always have a spill to avoid high sensitive regime.



Example of a mixed regulator with very different sensitivity indicators depending on the sign of the deviation.

The case of a weir type level regulator (duckbill)

A duckbill weir type regulator is a poorly sensitive regulation work. By design, its function is to transform an important variation in the flowing discharge into a close variation of the flow height, without any intervention on the work.

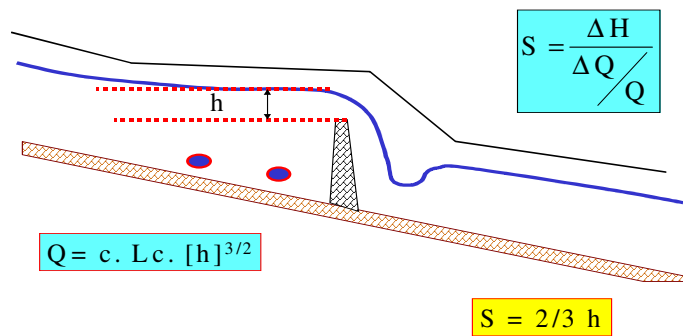
Assuming a free weir (with no influence downstream), the overshoot type flow equation is the following :

$$Q = c \cdot Lc \cdot [h]^{3/2} \tag{22}$$

- c = flow coefficient on the weir depending on the shape of the crest [c≈1,5 to 1,7]
- Lc = Length of the crest
- h = Water head on the weir (measured at a given distance upstream the weir)
- Q = discharge flowing on the weir.

From the logarithm derivative expression of equation 22, we determine the sensitivity indicator, which is equal to :

$$S = 0,66 h \tag{23}$$



with h limited to 0.5m then S < 1/3

Figure 12. Duckbill weir.

Influence of downstream submergence of the structure on sensitivity

The exact expression of the hydraulic formulations of indicators involves the back-effect of submergence upon the sensitivity through the expression H_E (equation 17). With a similar head (upstream-downstream difference), submergence tends to reduce the sensitivity of the structure compare to a free flow structure.

For offtake structures, we can initially neglect this effect by calculating the simple indicator (equation 13) with the difference in head ($H_{us} - H_{ds}$). Only the structures having a high sensitivity indicator (>1,5) have to be calculated more exactly taking into account the submergence effect.

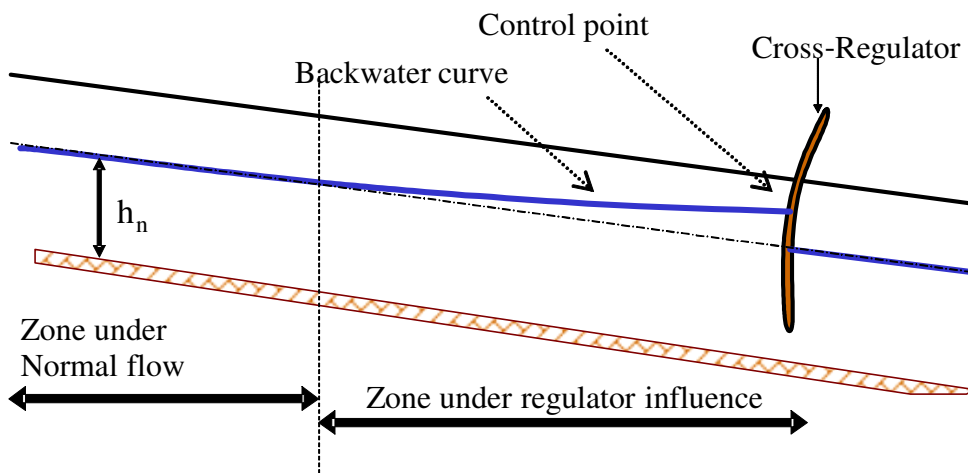
Influence of a downstream regulator on sensitivity

SENSITIVITY in a CANAL SECTION under NORMAL FLOW

Targets of brief 8:

- how sensitive a canal section is when it runs under normal flow?

Normal flow occurs in canal section when there is no change of discharge as a function of time and there is no influence from a downstream control point such as a cross-regulator as illustrated in the figure below. In normal flow, the free surface of the flow is parallel to the bottom bed.



Flow regime along a canal with normal flow upstream of an influenced flow.

Under normal flow the standard Manning-Strickler equation applies:

$$Q(x) = KAR^{2/3}S_o^{1/2} \tag{1}$$

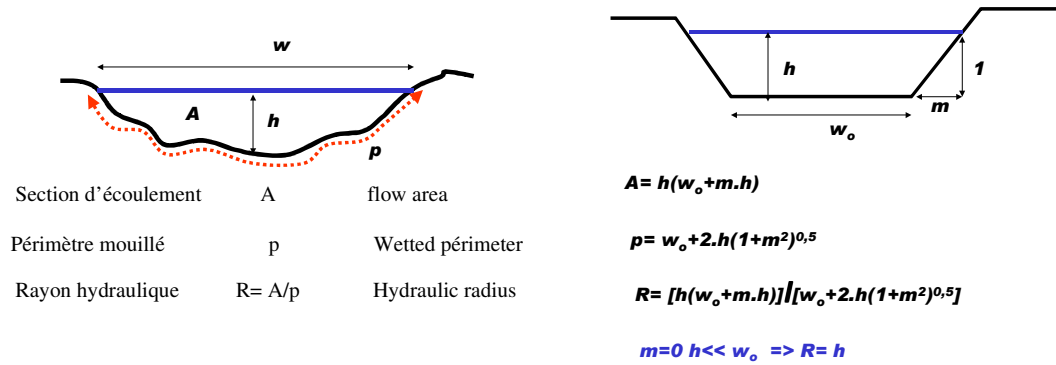
with

- Q(x) discharge
- K roughness coefficient
- A Flow section
- R Hydraulic radius
- So bottom bed slope

When the canal is rectangular and large, the hydraulic radius R is equal to water height h and the equation simplifies to :

$$Q(x) = Kw_oh^{5/3}S_o^{1/2} \tag{2}$$

Geometrical characteristics of a canal section



Geometric variables and Geometric parameters of a trapezoidal canal flow

Sensitivity indicator for uniform flow canal section

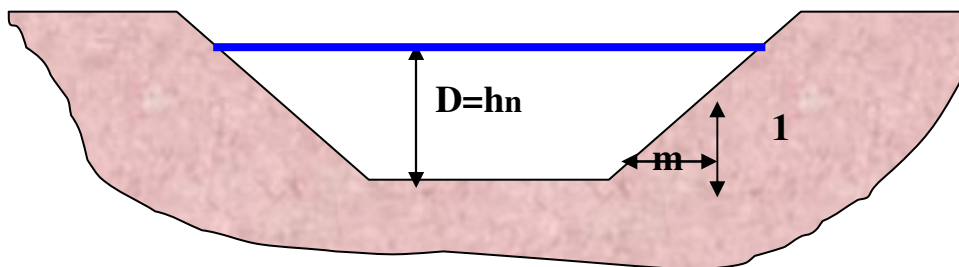
In the general case of any canal section, there is no analytical expression of the sensitivity indicator, and this has to be calculated numerically. In the case of a large rectangular canal in uniform flow, the exponent of the water head in the discharge equation (see equation 2) is then equal to 5/3 (Manning, Chow, 1959), and the logarithm derivative expression of equation (2) leads to the following value of the indicator:

$$S = \frac{3}{5} D \tag{3}$$

where D = the water head in the canal.

In this case, we can usefully express the sensitivity indicator as relative value on H , i.e.

$$S = \frac{\frac{\Delta H}{H}}{\frac{\Delta Q}{Q}}, \text{ that is then a dimensionless value equal to } 3/5.$$



Canal Section with uniform flow

In the case of any trapezoidal canal (figure 13), the sensitivity value decreases as the canal widens, ranging between a value between 0.7 D and 0.8 D for a rectangular canal ($m=0$) to about 0.5 D for a trapezoidal canal ($m=2$).

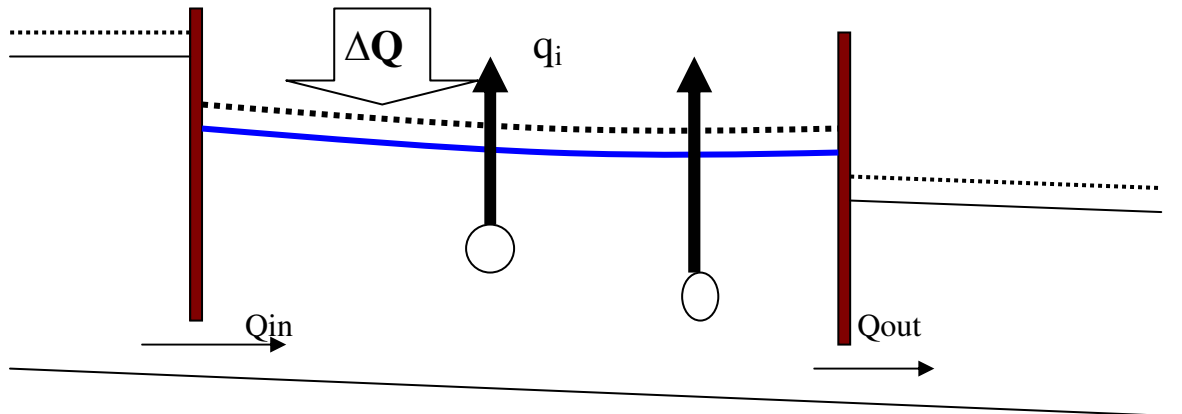
REACH SENSITIVITY & BEHAVIOUR

Targets of brief 9:

- importance of reach sensitivity for operation?
- how to calculate the sensitivity at reach level ?

Importance of reach sensitivity for COP

We consider a reach as a pool between 2 consecutive cross-regulators.

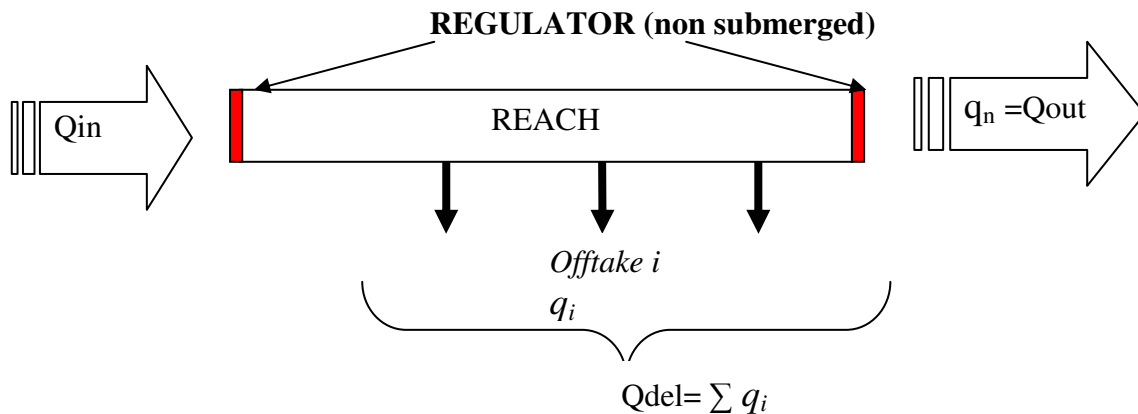


As a consequence of a variation of discharge within the reach (ΔQ) whatever it might originate from (in; out or delivery), the water level in the reach will vary (ΔH) and the entering, delivered and outing flows will be modified.

In the general case of reaches that are hydraulically dependant through submerged conditions, there is no simple solution leading to identify the new equilibrium for water level and distribution of flows. Water level will be changed not only in the reach where the perturbation is active but in the upstream as well as downstream ones.

SOLUTION for a hydraulically isolated REACH

A solution can be found for an isolated reach which is defined by no upward effect at both regulators. The downstream reach does not influence the flow through the regulator. This is when free flow occurs at the regulator for instance on DBW of non submerged orifice type.



Elements important to know about the reach	Interest for COP
Sensitivity of the reach: amplitude of the reach reaction for a given variation of discharge, i.e. ΔH as a function of $\Delta Q/Q$.	Determine the value of on going discharge variation that is compatible with the tolerance of water level variation $TOL(H)$ given by the performance objective.
Pace of reaction , i.e. $\Delta H/\Delta t$ [e.g. how many centimetres per hour] <i>[this point is addressed in BRIEF 10]</i>	Give the frequency of necessary checks and operation at regulators for a given variation of discharge.
Sensitivity of the reach for Conveyance and for Delivery: the fraction of the discharge variation which is transferred downstream and the one absorbed within the reach.	Indicates whether the reach absorbs or propagates perturbations.
The volume stored in or released from the reach resulting from variation of H . <i>[this point is addressed in BRIEF 10]</i>	On line storage and the capacity of storing surplus (e.g. night time) can be useful for canal management.

The system can be analyzed as follows:

1. Starting with a reach balanced for discharge, we have

$$Q_{in} = Q_{del} + Q_{out}$$

2. We consider a perturbation of discharge (ΔQ) affecting one of the terms of previous equation. This yield to a variation of water level within the reach (ΔH) which depends on both regulator and offtake sensitivities. Without operating the structures the situation stabilizes when the variation of discharge compensate each other, i.e.:

$$\Delta Q_{in} + \Delta Q_{del} + \Delta Q_{out} = 0 \quad 1$$

3. Assuming we have (n-1) offtakes and the “n” structure being the downstream regulator. We consider here all structures as diverting structure (including the downstream regulator). The sensitivity for discharge for each structure is given by the following definition:

$$S_i = \frac{\Delta q_i / q_i}{\Delta H_i} \quad 2$$

4. we can then write:

$$\Delta Q_{del} + \Delta Q_{out} = \sum_{i=1}^{i=n} \Delta H_i \cdot S_i \cdot q_i \quad 3$$

5. Assuming that all structures are under close influence of backwater generated by the regulator, they all experience a similar variation of head of (ΔH), this allows to solve for the implicit value of (ΔH) from 1 and 3.

AMPLITUDE of REACH REACTION

$$\Delta H = \frac{\Delta Q}{\sum_{i=1}^{i=n} S_i q_i}$$

After a period of transition due to the variation of water level within the reach, the entering perturbation will be equal to the fraction absorbed by the reach itself and diverted at offtakes (ΔQ_{del}), and that of transmitted downward at the downstream regulator (ΔQ_{out}). The share between the two parts, depends on:

- the regulator sensitivity reflected through the variation of water depth (ΔH_{US})
- the sensitivity of the offtakes within the reach
- and the ratio of discharge withdrawn within the reach compare to main discharge.

REACH SENSITIVITY for WATER LEVEL

The definition of the sensitivity indicator S_{RH} for a reach with respect to water level is:

$$S_{RH} = \frac{\Delta H}{\left(\frac{\Delta Q}{Q}\right)} \quad \text{replacing } \Delta H \rightarrow \quad S_{RH} = \frac{\frac{\Delta Q}{\sum_{i=1}^{i=n} S_i q_i}}{\left(\frac{\Delta Q}{Q}\right)}$$

which simplifies to

$$S_{RH} = \frac{1}{\sum_{i=1}^{i=n} S_i q_i / Q}$$

The sensitivity of a reach for water level is inverse to the weighted sum of sensitivity indicators for discharge for all structures in the reach (including the downstream cross regulator taken as an offtake), the weighting factor being the relative discharge.

The reach behaviour results from the product of sensitivity with discharge at the diverting structures. The higher the product the higher the absorption effect of the structure.

Analysis: Reach 3 is highly reactive (sensitive) for water level even a slight change of discharge 1% or 60 l/s within the reach is likely to be reflected and detected “4 cm”. This is explain by the very sensitivity of the offtakes, thus even a small change of discharge requires high variation of head to reach another equilibrium.

Numerical example:

	REACH1	REACH 2	REACH3
Q _{in} m ³ /s	10	7	6
Q del m ³ /s	3	1	4
Q _{out} m ³ /s	7	6	2
Offtake 1 m ³ /s	0.5	0.3	1
S1	2	1	0.5
Offtake 2 m ³ /s	1	0.5	0.5
S2	0.25	0.5	0.25
Offtake 3 m ³ /s	1.5	0.2	0.5
S3	3	0.50	0.33
Regulator Sensitivity for discharge (1) S_{X-reg}^o	1.5	1	0.33
Regulator Sensitivity for water level	0.66	1	3
$\sum_{i=1}^{i=n} S_i q_i / Q$	[2*0.5+0.25*1+3*1.5 + 1.5*7]/10 = 1.625	0.95	0.24
Sensitivity S _{RH}	0.62	1.05	4.14
	Variation of discharge [ΔQ/Q]		
Variation of water level within the reach ΔH= S _{RH} ΔQ/Q	1 %	0.006 m (ΔQ =100 l/s)	0.011 m (ΔQ =70 l/s)
	2 %	0.012 m (ΔQ =200 l/s)	0.022 m (ΔQ =140 l/s)
	5 %	0.03 m (ΔQ =500 l/s)	0.055 m (ΔQ =350 l/s)
	10 %	0.06 m (ΔQ =1000 l/s)	0.11 m (ΔQ =700 l/s)
		0.041 m (ΔQ =60 l/s)	0.082 m (ΔQ =120 l/s)
		0.21 m (ΔQ =300 l/s)	0.41 m (ΔQ =600 l/s)

(1) We specifically note the sensitivity indicator for discharge at the cross regulator as S_{X-reg}^o to avoid confusion with the usual X-reg indicator for water level (they are inverse).

Reach 1 inversely is low sensitive, therefore even 5 % of discharge variation which means a great 500 l/s might go unnoticed. This results from highly sensitive offtakes that absorbs most of the perturbations.

Reach 2 is sort of intermediate between the previous extremes.

REACH SENSITIVITY for CONVEYANCE

The **reach sensitivity for Conveyance** S_{RC} is defined as the ratio of perturbation absorbed ΔQ_{out} vs the input one ΔQ.

$$S_{RC} = \frac{\Delta Q_{out}}{\Delta Q}$$

S_{RC} can be computed as follows:

Starting from

$$\Delta Q_{Out} = \Delta H \cdot S_{X-reg}^O \cdot Q_{out}$$

and then inserting

$$S_{RC} = \frac{\Delta Q_{out}}{\Delta Q} = \frac{S_{X-reg}^O \cdot Q_{out}}{\sum_{i=1}^{i=n} S_i \cdot q_i}$$

REACH SENSITIVITY for DELIVERY

Similarly we can define a **reach sensitivity for Delivery** S_{RD} as the ratio of perturbation absorbed ΔQ_{del} vs the input one ΔQ .

$$S_{RD} = \frac{\Delta Q_{del}}{\Delta Q}$$

This indicator can be deduced from the previous by the equation

$$S_{RD} = 1 - S_{RC}$$

$$S_{RD} = 1 - \frac{S_{X-reg}^O \cdot Q_{out}}{\sum_{i=1}^{i=n} S_i \cdot q_i}$$

$\sum_{Offtakes} S \cdot q_i$	5.75	0.65	0.79
Sensitivity Srd	0.35	0.1	0.35
Sxreg	0.67	1	3

Numerical example: Reach

Reach 1 absorbs a great deal of the discharge variation affecting the reach almost 40 % while Reach 2 mostly transfer the change downward, only 7 % of the change is absorbed within the reach.

It means for reach 1 that if one offtake of the reach is closed, 40 % of the generated extra flow is likely to remain within the reach and be shared by others offtakes unless the downstream regulator is operated to correct the change on water depth. For reach 2 perturbations are mainly forwarded downstream.

REACH DYNAMIC

Targets of brief 10:

- understanding the dynamic of flow level variation within a reach?

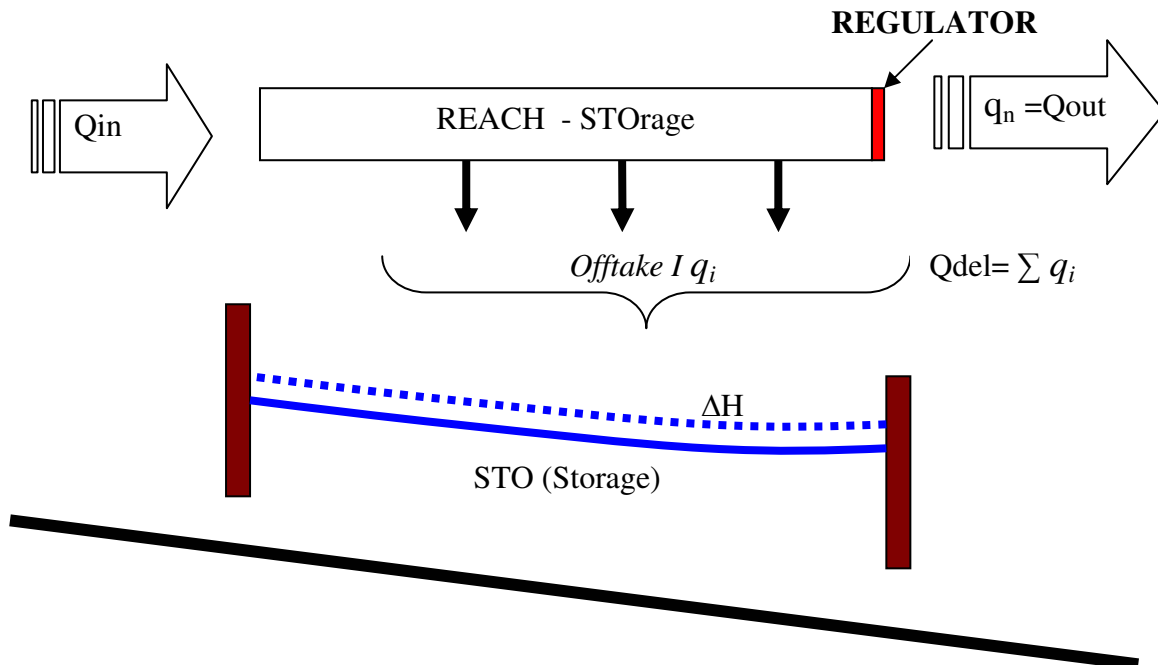
We consider a reach as a pool between 2 consecutive cross-regulators.

As a consequence of a variation of discharge (ΔQ) within the reach, the water level H in the reach vary. Managers are interested to know the dynamic of this change:

Elements important to know about the reach	Interest for COP
Pace of reaction , i.e. $\Delta H/\Delta t$ [e.g. how many centimetres per hour]	Give the frequency of necessary checks and operation at regulators for a given variation of discharge.
The volume stored in or released from the reach resulting from variation of H .	On line storage and the capacity of storing surplus (e.g. night time) can be useful for canal management.

Note: Sensitivity of the reach for H and Sensitivity for delivery are addressed in BRIEF 9

We start first with a reach under perfect equilibrium for discharge. We then consider a variation of on going discharge (ΔQ) being the results of inflow or outflow sudden change.



The system can be analyzed as follows:

6. A perturbation of discharge (ΔQ) within the reach results in by a variation of water level within the reach (ΔH) which depends on both regulator and offtake sensitivities. The mass equation writes:

$$\Delta Q = \Delta Q_{del} + \Delta Q_{out} + \text{Var}(\text{STO}) \quad 1$$

7. Let's assume that we have the following estimator of the storage volume

$$\text{Var}(\text{STO}) = kA \frac{\partial H}{\partial t} \quad 2$$

with

k a factor between 0 and 1 (close to one)
A the area covered by the backwater

8. Then equation 1 rewrites considering (n-1) offtakes and the “n” structure being the downstream regulator. We consider here all structures as diverting structure (including the downstream regulator). The sensitivity for discharge for each structure is given by the following definition:

$$S_i = \frac{\Delta q_i / q_i}{\Delta H_i} \quad 3$$

9. we can then write the right term of equation 1 as follows Assuming that all structures are under close influence of backwater generated by the regulator, they all experience a similar variation of head of (ΔH), this allows to solve for the implicit value of (ΔH) from 1 and 3.:

$$\Delta Q = \sum_{i=1}^{i=n} \Delta H S_i q_i + kA \frac{\partial H}{\partial t} \quad 4$$

10. The change for H is equal to the change in the deviation ΔH

$$\Delta H = H - H_i^0 \quad \text{then we have } \partial H = \partial \Delta H$$

11. The equation 3 rewrites then in a differential equation for ΔH of the following form

$$ay + by' = c$$

12. which solutions are as follows:

$$\Delta H = \frac{\Delta Q}{\sum_{i=1}^{i=n} S_i q_i} \left[1 - e^{-\frac{\sum_{i=1}^{i=n} S_i q_i}{kA} t} \right] \quad 5$$

Inversing the equation yield the expression of time for a given deviation of H:

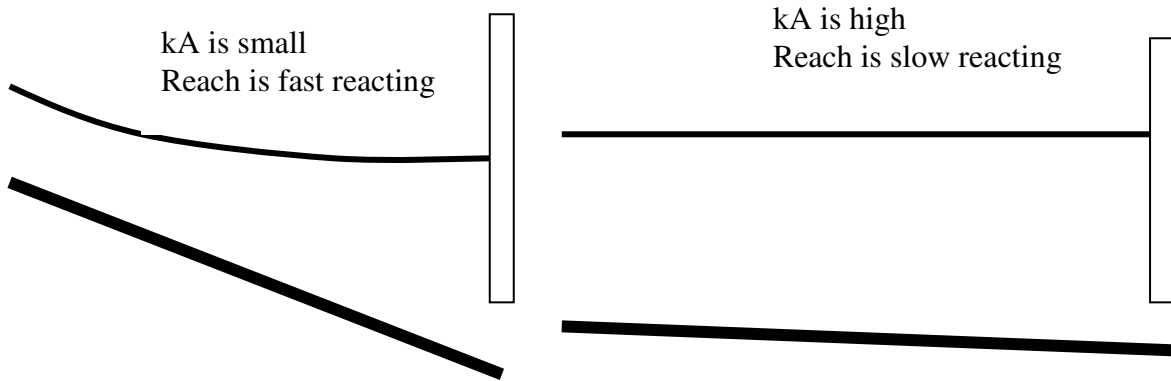
$$t = -\frac{kA}{\sum_{i=1}^{i=n} S_i q_i} \ln\left(1 - \frac{\Delta H}{\Delta H_{\max}}\right) \quad 6$$

with ΔH_{\max} the ultimate amplitude of the deviation (see Brief 9):

$$\Delta H_{\max} = \left[\frac{\Delta Q}{\sum_{i=1}^{i=n} S_i q_i} \right] \quad 7$$

Time to reach half of the maximum deviation is :

$$t = 0.7 \frac{kA}{\sum_{i=1}^{i=n} S_i q_i} \quad 8$$



VOLUME STORED

The maximum volume that can be stored from the perturbation in the reach is determined by ΔH_{\max} and kA , assuming that there is no risk of overtopping and that we can allow water depth to deviate from usual flow by ΔH_{\max} :

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SENSITIVITY in UPSTREAM and DOWNSTREAM CONTROL

Targets of brief 11:

- understanding the specific rationale of sensitivity for upstream control
 - understanding the specific rationale of sensitivity for downstream control
- =====

SENSITIVITY for UPSTREAM CONTROL

Upstream control is the canal control technique that targets the control of water level upstream of cross structures [X-regulators] to maintain levels within acceptable limits for serving the near by offtakes no matter what the main discharge is. With the upstream control technique, discharge [Q] is imposed from upstream and the control is exercised on water level [H] upstream the X-reg.

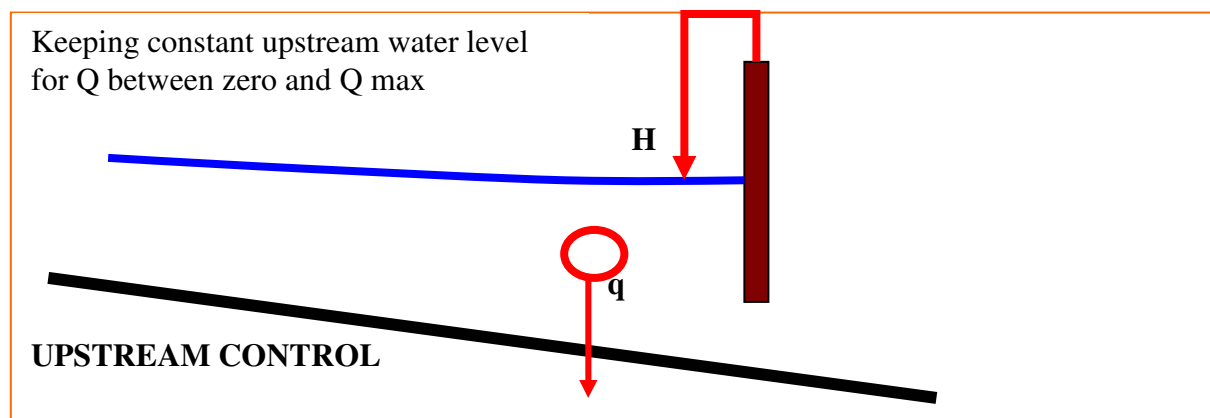
For a canal system under upstream control operation, sensitivity is treated separately for each type of structure from the main headworks down to the farm inlet, alternatively looking at main discharge, water level control and withdrawal. For upstream regulated canals, the rationale starts with the main discharge imposed at headworks. Basically the following sequence is considered:

Main discharge Q → Water level H in main canal → discharge q at offtake to secondary canals → water level h along secondary canals → discharge q at offtake to tertiary canals → ..

The **water depth sensitivity to discharge variations** along the canal, at a regulator or at any other section, is expressed as the variation of water depth (ΔH) resulting from a relative discharge variation in the canal (ΔQ/Q).

The **discharge sensitivity to water depth variations** at any structure is defined as the ratio of the relative variation of discharge through the structure (Δq/q) and water depth deviation (ΔH) upstream of the structure.

sensitivity to discharge variation [Offtake]	sensitivity to water depth variation [X-reg]
$S = \frac{\Delta H}{\Delta Q / Q} \quad (\text{unit: m})$	$S = \frac{\Delta q / q}{\Delta H} \quad (\text{unit: m}^{-1})$

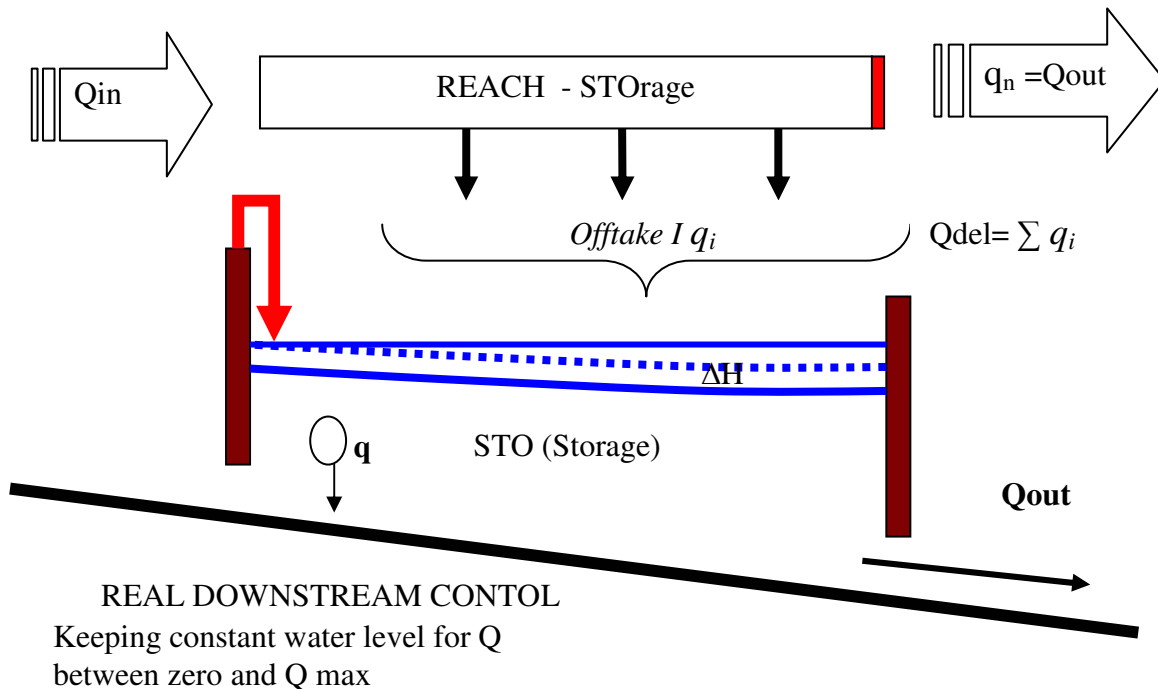


SENSITIVITY for DOWNSTREAM CONTROL

Downstream control is a canal operation technique that targets the control of water level downstream of the cross regulators to maintain levels within acceptable limits for serving the near by offtakes no matter what the main discharge is. In downstream control technique, discharge [Q demand] is imposed from downstream users and the control is exercised on water level [H] downstream of the X-regs.

For a canal system under downstream control operation, the sensitivity rationale is inverted than for the US technique. It starts at the inlet (farm) and move up to the main headworks, alternatively looking at main withdrawal, water level control and discharge along the canal. For downstream regulated canals, the rationale starts at farm level and moves up to headworks, with the following sequence:

Discharge q at offtake \rightarrow Water level along tertiary canal \rightarrow Discharge from secondary to tertiary \rightarrow water level along secondary \rightarrow Discharge from main to secondary \rightarrow Water level in main canal from reach to reach \rightarrow discharge at main headworks.



Considering the basic approach of sensitivity as follows:

$$\text{Sensitivity} = \frac{\text{Variation in output}}{\text{Variation in input}} \quad (1)$$

The INPUT in DS control is the discharge Q as the sum of the withdrawals in a reach including the discharge Q_{out} leaving the reach at the downstream regulator.

The OUTPUT is the variation of water depth within the reach.

Thus by definition the Sensitivity for downstream control is equivalent to the reach sensitivity which is the ratio of the reaction of the reach in terms of water level and of a change in discharge either at offtake or at the next downstream regulator.

From the brief 9 on reach we have:

$$\Delta H = \left[\frac{\Delta Q}{\sum_{i=1}^{i=n} S_i q_i} \right]$$

From which we derive the indicator of sensitivity of the DS Xreg;

$$S_{Xreg DS} = \frac{\Delta H}{\Delta Q} = \left[\frac{1}{\sum_{i=1}^{i=n} S_i q_i} \right]$$

In DS control high sensitivity is usually not a problem. High sensitive cross-structure/reach means frequent check/adjustment of the structure setting this is not a problem with automatic or motorized structure.

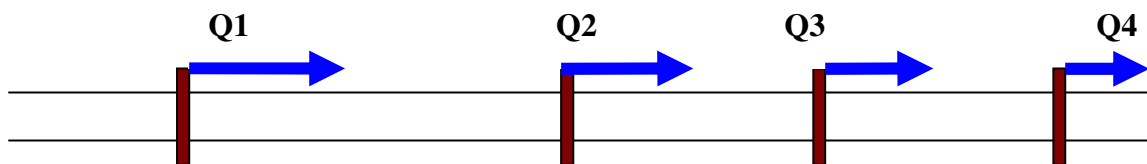
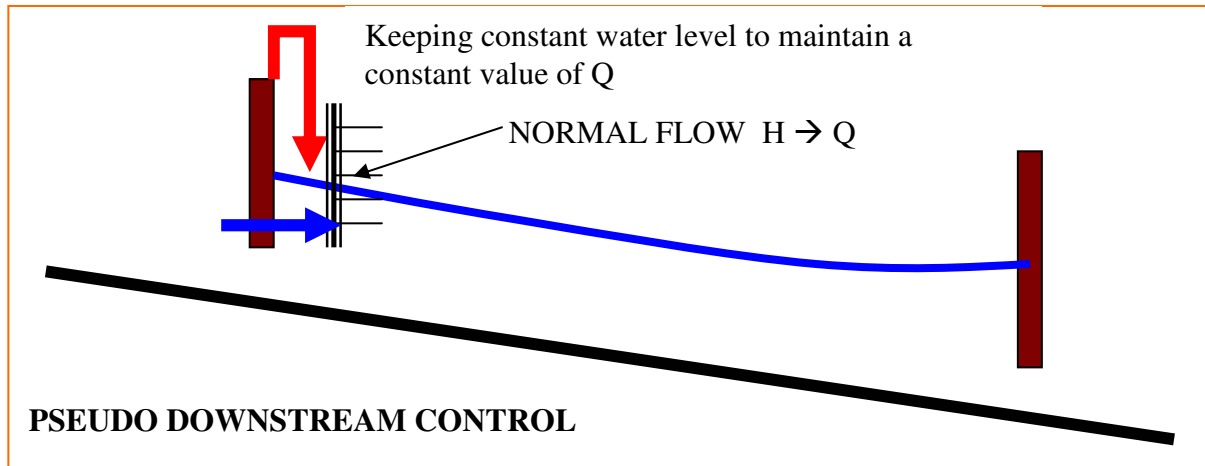
Pseudo downstream control or real discharge control

There are often situations where operation in place looks like downstream control technique but in fact is what we called “pseudo downstream control”. This is characterized by operating each cross regulator to maintain a given discharge downstream through a gauge located downstream of the X-reg. This is not real downstream control, **BUT real discharge control** along the main canal, lateral withdrawals are then automatically adjusted by the variation of water level upstream of the regulator, this allows keeping the downstream discharge close to the setting point.

These two situations (real and pseudo downstream control) might be confused on the field because both are dealing with a downstream target. In **the real downstream control** the downstream reach is under the influence of the downstream regulator, flow is submerged¹.

¹ There are several options for DS control, the most common that we are referring to here is when the control is exercised on the level immediate downstream of the structure. Others are considering other points within the downstream reach.

In the **pseudo downstream control** the flow is often non influenced [normal flow occurs] and there is one relationship between height and discharge therefore targeting a specific height means keeping a discharge about target.



Pseudo DS Control is like starting all over again upstream control at each cross regulator

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SENSITIVITY AND PERFORMANCE

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Targets of brief 15:

- understand how performance is affected by sensitivity of structure?
- =====

In general the overall objective, in controlling a canal with upstream control, is to maintain a constant head on the upstream side of the delivery structures (offtakes - outlets) in order to maintain the required discharge within permissible limits. The control of the head in a canal is enabled by cross-structures, also called cross-regulators, at strategic points along the canal.

The extent and the magnitude of control exercised by a particular cross-structure depends both on its property in controlling local water depth (precision) and on the extension (influence) of the backwater curve effect within the controlled reach (upstream in most cases).

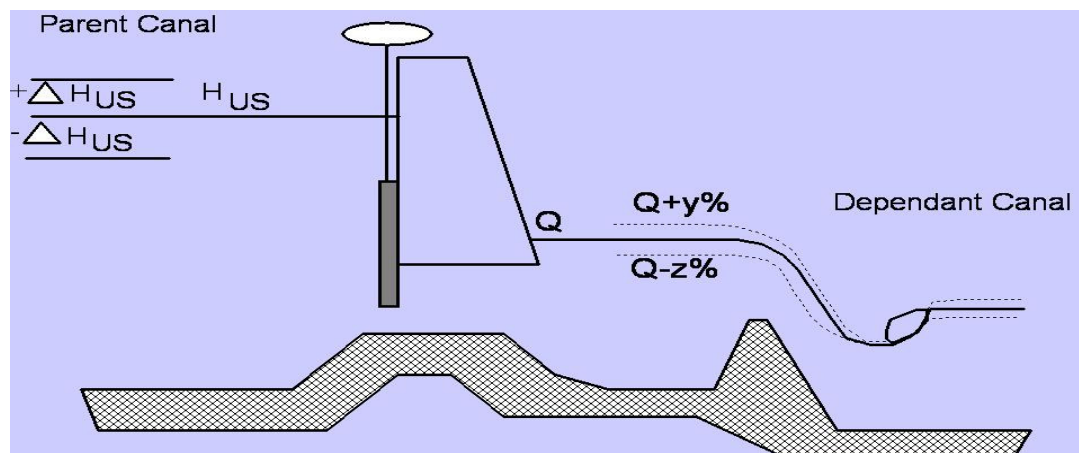
Precision is a parameter under the control of the manager, the precision exerted by the operator can be assessed through the fluctuation of water depth (ΔH_R) experienced at the regulator.

Other structures, offtakes and outlets along a canal, are aiming to deliver the targeted discharge. Their role is to convert an input, the water depth in the parent canal, into an output, a discharge series, feeding the dependant canal. For a delivery structure, the sensitivity of delivery expresses the link between a variation of water depth (ΔH_{US}) in the parent canal and the resulting deviation in discharge (Δq) in the dependant canal. A highly sensitive structure provokes high changes of discharge for slight water depth deviation and vice versa.

The generic relationship is

$$\text{PERFORMANCE} = \text{Function} \left\{ \begin{array}{l} \text{SENSITIVITY} \\ \text{of delivery structures} \end{array} \text{ and } \begin{array}{l} \text{CONTROL} \\ \text{on Water depth} \end{array} \right\}$$

Sensitivity and Performance for an offtake



Consider an offtake with an initial discharge corresponding to the targeted discharge, the consequences of a perturbation of water depth ($\Delta H_{US(i)}$) in the parent canal are examined in terms of the discharge variation (Δq_i) at the offtake.

The performance indicator for adequacy is the ratio of actual vs target discharges, it is then derived for the perturbed state as:

$$p_{A(i)} = \frac{q_i + \Delta q_i}{q_i} = 1 + \frac{\Delta q_i}{q_i} \quad (\text{if } \Delta q_i \text{ is negative and } 1.0 \text{ otherwise}) \quad (1)$$

The performance indicator for efficiency is inverse (target/actual) as:

$$p_{F(i)} = \frac{q_i}{q_i + \Delta q_i} \quad (\text{if } \Delta q_i \text{ is positive and } 1.0 \text{ otherwise}) \quad (2)$$

which simplifies to:

$$p_{F(i)} \approx 1 - \frac{\Delta q_i}{q_i} \quad (3)$$

The sensitivity for delivery for the offtake (i) is defined as:

$$S_{(i)} = \frac{\Delta q_i / q_i}{\Delta H} \quad (4)$$

which combined with previous equations leads to the following local relationship:

$$p_{(i)} = 1 \pm \Delta H_{USf(i)} S_{(i)} \quad (p_{(i)} \leq 1.0) \quad (5)$$

In (5), the minus sign applies for adequacy when ($\Delta H_{US(i)}$) is positive and the plus sign applies for efficiency when ($\Delta H_{US(i)}$) is negative. Equation (5) explicitly expresses the link between the adequacy or efficiency performance indicator, the sensitivity and the control exercised on the water depth at the local level.

Aggregated relationships for a sub-system

The objective is to derive a similar relationship to (5) at an aggregated level to enable the performance of sub-systems, or even whole systems to be compared considering a uniform precision (ΔH_R).

A particular assumption has to be made regarding the way the deviations of water depth affect the system. It is proposed here to refer to a system where positive and negative deviations are somehow balanced. Another possible option would be to consider a constant sign perturbation (ΔH_R) at every regulator.

The aggregation process, with varied sign perturbations, corresponds to a balanced system where it is assumed that the number of offtakes (n) of the unit considered, can be regrouped into two similar subsets of $(n/2)$ offtakes. The similarity between the two subsets is based on the discharge delivery and the sensitivity of the offtakes. Reaches of one subset experience a positive perturbation $(+\Delta H_R)$, while the others a negative one $(-\Delta H_R)$, somehow compensating each other.

Performance indicators for adequacy and efficiency, are aggregated here using a weighted process, **the weight k_i** being the relative offtake discharge $q_i / \sum_{j=1}^n q_j$. Since only instantaneous values are considered here, there is no integration over time in equation. The performance for the whole system is the sum of the two subset indicators derived from (5), as follows:

$$P = \sum_{i=1}^{n/2} k_i \left(1 - \Delta H_{US(i)} S_{(i)} \right) + \sum_{i=n/2+1}^n k_i \quad (6)$$

For adequacy, the first term in (6) corresponds to the set of underfed offtakes and the second term corresponds to the set of overfed offtakes. For efficiency, the first term corresponds to overfed offtakes and the second term to underfed offtakes.

Knowing that the **sum of the weights k_i** over the whole set is equal to 1, by definition, and assuming that the water depth deviation at offtake (i) can be written as a linear relationship of ΔH_R as follows:

$$\Delta H_{US(i)} = m_i \Delta H_R \quad (7)$$

then 6 can be rewritten as

$$P = 1 - \Delta H_R \sum_{i=1}^{n/2} k_i m_i S_{(i)} \quad (8)$$

with

m_i indicator of the regulator control on the offtake (i); $m_i = 1$ when the offtake is very close to the regulator, and becomes zero when the offtake gets far from it (at the upstream limit of the backwater curve when flow becomes normal).

Assuming a similarity in discharge and sensitivity of the two subsets of offtakes enables the rewriting of (8) as:

$$P = 1 - \frac{1}{2} \Delta H_R \sum_{i=1}^n k_i m_i S_{(i)} \quad (9)$$

Equation (8) has been established for a varied sign perturbation with a perfect initial state. It can also be shown, by a similar computation, that this relationship (17) holds true with any initial state $P_{(o)}$, under the restriction that no offtake switches from one condition to the other (overfed/underfed); in that case $P_{(o)}$ replaces 1 in equation (9).

Equation (9) states the relationship between performance, the precision and influence of control, and the sensitivity of delivery structures along the canal.

A **system sensitivity indicator** (S_s) can then be proposed for adequacy and efficiency as follows:

$$S_s = \sum_{i=1}^n k_i m_i S_{(i)} \quad (10)$$

and the performance indicator (9) is written:

$$P = 1 - \frac{1}{2} \Delta H_R S_s \quad (11)$$

The performance expected from an irrigation system is the product of two terms: the capacity of control on water depth (tolerance on water depth H) and the system sensitivity. This allows managers to estimate the control to exercise [tol.(H)] given the performance required for the service and the physical properties of the system. Different global sensitivity indicators at system level have been developed (Renault D. 1999) for adequacy, efficiency and equity performance.

The performance for adequacy and efficiency is related to the precision and influence of control. A system sensitivity indicator along the canal can be proposed as follows:

$$P = 1 - \frac{1}{2} \Delta H_R S_s \quad (9)$$

in which (S_s) is a **system sensitivity indicator**, equal to:

$$S_s = \sum_{i=1}^n k_i m_i S_{(i)} \quad (10)$$

Using the C.V. approach, a **system sensitivity indicator for equity** (S_{se1}) can be proposed as the square root of the arithmetic mean of the product of the square of local sensitivity and the influence factor:

$$S_{se1} = \sqrt{\frac{1}{n} \sum_{i=1}^n m_i S_{(i)}^2} \quad (11)$$

This global sensitivity indicator is related to the performance equity indicator by the following equation:

$$P_E = \Delta H S_{Se1} \quad (12)$$

For the analysis based on the Theil index, a **system sensitivity indicator for equity** (S_{se2}) can also be proposed as:

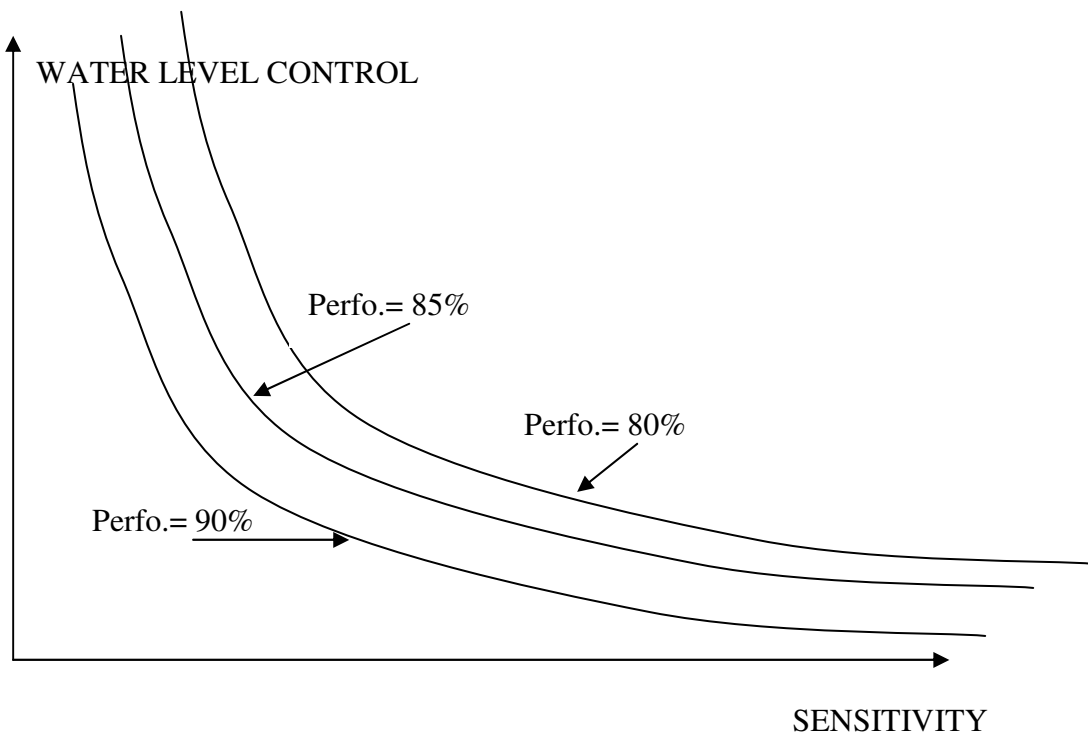
$$S_{Se2} = \sum_{i=1}^n \frac{q_i}{Q_D} m_i^2 S_{(i)}^2 \quad (13)$$

This system indicator is related to the system performance equity indicator by the following relationship:

$$Thi = \frac{1}{2} (\Delta H_R)^2 S_{Se2} \quad (14)$$

The invariant of performance

As expressed previously the invariant for performance is the product of the sensitivity and the tolerance on water level control $S_s \cdot \Delta H_{US}$



SENSITIVITY Analysis for a lift station

Targets of brief 17:

Description of a lift station

The lift station is composed of:

- Inlet line
- pump stage
- Outlet line

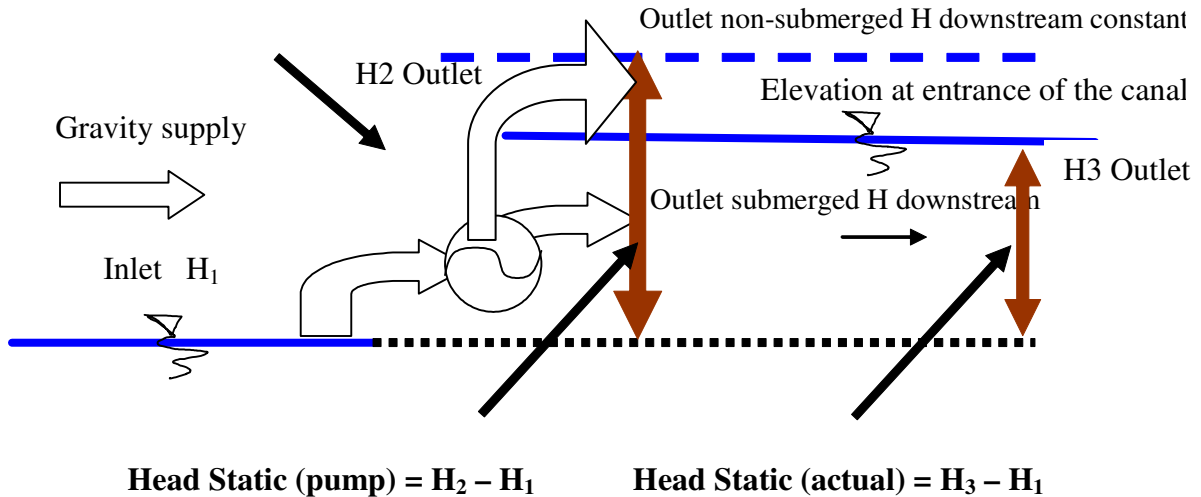


Figure 1 Sketch showing the important variable for a lift station.

Inlet and outlet lines consumed energy through head losses while pump stage insufflate energy into the fluid. Variables that are important for the characteristic of the process are:

- H_1 (head inlet)
- H_2 (head outlet)
- H_3 (head at canal entrance)

Head losses through the pumping station [pumps and inlet outlet lines].
Efficiency of the pump [η].

Pump and system curves

Pumps effectiveness are captured in a Head-Discharge diagram with curves that also gives efficiency of the setting as shown in diagram below.

For a lift station the system around the pump stage is limited to the inlet and outlet pipelines. However it must be considered as a full fledged system where head losses occurred as a function of discharge. Usually head losses are function of square of discharge as follows:

$$\text{Headlosses}(m) = \mu Q^2$$

For instance the two system curves shown in figure below results of a pipeline of a length of approx. 20 meters (a) for 100 mm diameter and (b) for 150 mm diameter. Of course solution (b) generates much less head losses than (a) for the same discharge.

System curve is a function of the following type:

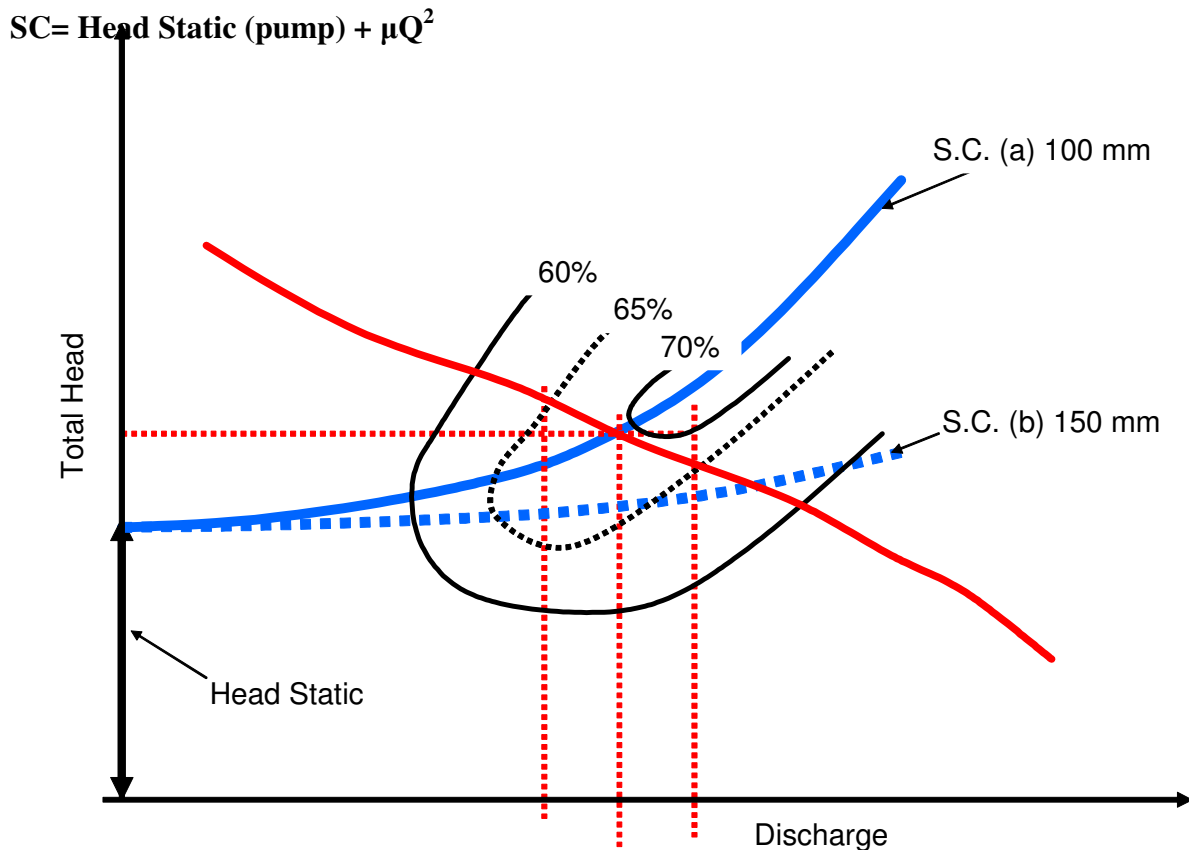


Figure 2 H-Q diagram with Pump curve and system curve at a lift station (pipes of 100 and 150 mm diameter).

Setting pumping point

The ideal set point for a pumping station is defined by matching requirements (Head static, discharge) and the pump' characteristics. Of course ideally one should set the functioning one in the area of high efficiency of the pump as shown in figure above.

Discharge Capacity

The capacity of a lifting structure is defined in terms of discharge (Q) at the outlet of the station or the entrance of the main canal. This capacity depends on the internal characteristics of the station (power & efficiency/losses) and the water level conditions of the supply and of the restitution. These two levels determine head at the lift station.

For a lift station the discharge lifted [Q] into the system at a given elevation will then depends on:

- the water levels (head conditions) at lift station
 - the power and energy input
 - the head losses within the station (inlet and outlet pipes ; pumps)
 - the energy efficiency of the pumps.
- Energy produced by the lift station

The energy produced at a lift station in terms of quantum of water elevated is given by the following:

$$Energy(KWh) = \frac{Volume(m^3) \times Headstatic(actual)(m)}{367} \quad (1)$$

head static (actual) is the difference of water elevation between canal inlet (H_1) and outlet (H_3).

Energy required by the lift station

The energy required at a lift station depends on the total head, the volume pumped (V) and the efficiency of the pumps [η].

$$Energy(KWh) = \frac{Volume(m^3) \times Headtotal(m)}{367} \times \left[\frac{1}{Efficiency} \right] \quad (2)$$

total head is the head static of the pump [$H_2 - H_1$] plus head losses in the inlet and outlet pipes.

Constant and variable head for a lift station

A lift station may have:

a constant head when inlet and outlet elevation does not change much, for instance when both are controlled canals or when their variation remains small compare to static head.

A variable head when inlet level varies for example when it is associated to a river, drainage, a reservoir or a well.

The importance of the head fluctuations on the regime of the station must be looked at in terms of relative variation [$\Delta Head/Head$]. A variation of 2 meters at the inlet might be important for operating the station if the lift is only for 5 meters but neglectable if it is of 50 meters.

OUTPUTS and INPUTS for a lift station

The first stage in the sensitivity approach is to define the INPUTS and OUTPUTS of the process.

INPUTS at lift station are:

water level at inlet which set the head of the lift and defines the energy requirement.

water level at the outlet which together with the previous defines the head of the lift

OUTPUTS are:

The water flow at head of the canal [Discharge, volume]

The energy required and spent to produce the service (lift).

In one way the energy must be considered as an input to the lift system. Here, we hold the energy at the lift station (required and spent) as an output of the head conditions and head losses.

Sensitivity for Discharge to head variation

Sensitivity indicators can be defined in absolute or relative terms. The advantage of relative terms is that the resulting indicator is given dimensionless for instance as follows:

$$\text{Sensitivity} = [\Delta Q/Q]/[\Delta \text{Head}/\text{Head}] \quad (\text{dimensionless})$$

which expresses the ratio of the relative variation of inflow Q (discharge) and that of pump static head. This variation of head might be generated by a variation of water level at the source (H1 figure 3). It might also be at the inlet in the downstream canal (H3) is the pipe if it is submerged.

In practice it might also be worth expressing the sensitivity indicator in absolute terms as follows:

$$\text{Sensitivity} = \Delta Q/\Delta \text{Head} \quad (\text{m}^3/\text{s}/\text{m})$$

Placing ourselves at the set point we assume that the pump curve around the set point can be expressed as a linear relationship with discharge of the following type:

$$\text{Head} = H_0 - \mu Q$$

with μ the slope of the line on above diagram which is defined by:

$$\mu = \Delta \text{Head}/\Delta Q$$

When a deviation of water level occurs at inlet (most probable case for head variation) it generates a shift of the functioning of the pumps along the pump curve. the total head at the station varies, discharge and head losses in the system varies as well.

Variation of head and discharge about set point are illustrated in figure below for a positive or negative variation of static head [$\Delta \text{Head} = H_2 - H_1$]. This results in a shift of the functioning point along the pump curve. For a positive variation of ΔHead discharge reduces and also head losses generated into the system. The new point of functioning F1 is reached when the difference between the pump curve and the system curve is equal to the head change [ΔHead]. The same reasoning applies for a negative variation of static head ΔHead (discharge and headlosses increase).

If the system curve is steep then there is a significant retro effect on the total head at the pump station from head losses variation which reduces the sensitivity.

If the system curve is flat then the sensitivity indicator is maximum and equal to:

$$\text{Sensitivity max.} = 1/\mu \quad (\text{m}^3/\text{s}/\text{m})$$

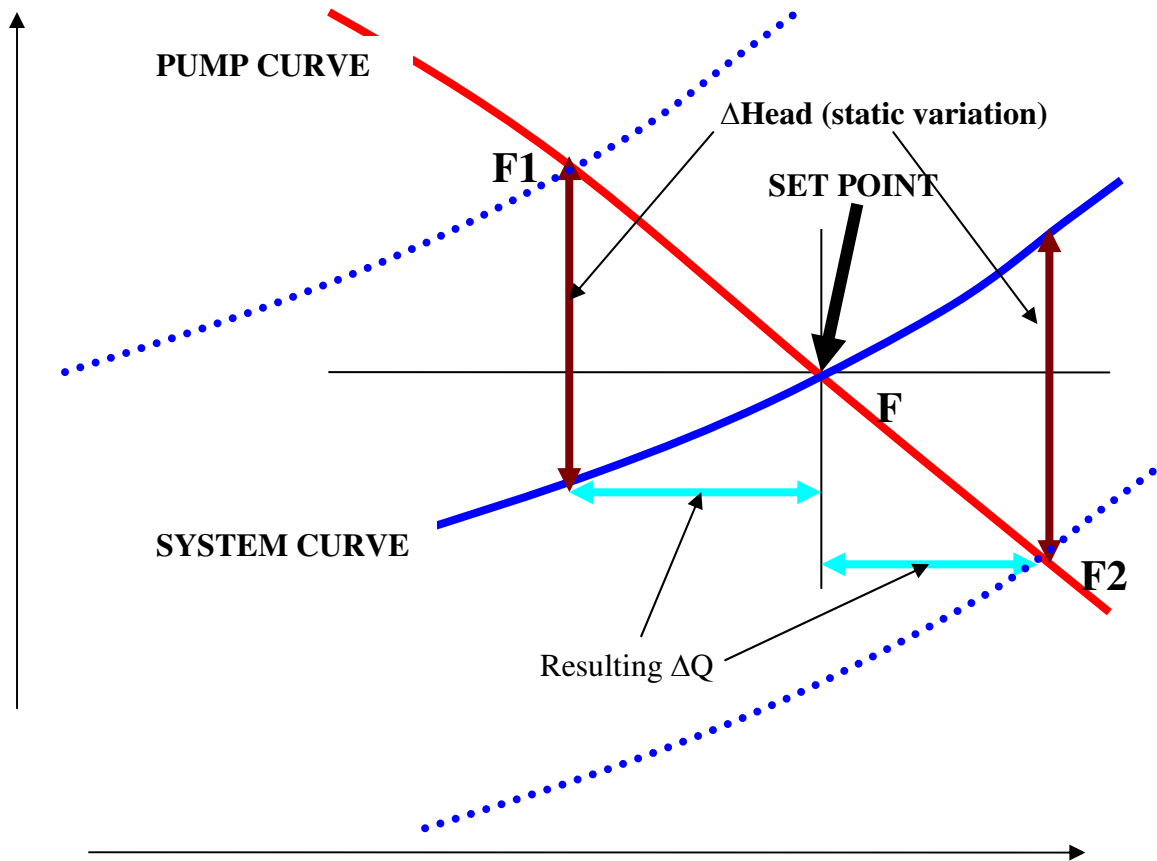


Figure 3. Zoom about set point

Sensitivity for Energy to head variation

As one can see in figure 8, deviating from the set point results in moving away from the most efficient point. Therefore one of the consequences of the variation of INPUT (head) is a variation of the energy spent per unit of energy produced at the lift station (Efficiency).

The energy produced is given by equation 1:

$$Energy(KWh) = \frac{Volume(m^3) \times Headstatic(actual)(m)}{367}$$

The energy spent to produce the service is:

$$Energy(KWh) = \frac{Volume(m^3) \times Headtotal(m)}{367} \times \left[\frac{1}{Efficiency} \right]$$

The definition of sensitivity indicators of a lift structure with respect to energy can be as follows:

$$S = [\Delta Energy/ Energy] / [\Delta H/ H]$$

Values for the example shown in figure 6 are displayed in table below:

Technical briefs on SENSITIVITY

	SET POINT F in figure 3	Positive variation of 1 meter static head considered (Pipe diameter = 120 mm length = 20 meters) F1 in figure 3	Negative variation of 1 meter static head considered (Pipe diameter = 120 mm length = 20 meters) F2 in figure 3
Total Head	15.10 meters	15.81 meters	14.40 meters
Static Head pump	13.56 meters	14.56 meters	12.56 meters
Head losses at this point	1.54 meters	1.26 meter	1.84 meters
SET POINT Discharge	2000 liters minute	1625 liters minute	1970 liters minute
Pump Efficiency at this point	70%	67.5%	67%
Energy spent per hour	6.35 KWh/h	6.22 KWh/h	6.9
Energy produced per hour	4 KWh/h	3.87 KWh/h	4
Global efficiency	63 %	62 %	59 %
Energy spent per m3 per m elevated	4.33 Wh/m3/m	4.38 Wh/m3/m	4.62 Wh/m3/m
Slope of the pump curve at set point	244 m/m3/s		
Discharge Sensitivity Maximum for a flat curve system (relative)	1.8		
Discharge Sensitivity exact for a system curve (Ø 120 mm)		1.3	1.32
Sensitivity for Energy (relative dimensionless)		0.12	0.89
<u>Comments</u>		The system is low sensitive for energy at positive head changes [compensation effect of head losses reduction]	The system is high sensitive for energy at negative head changes [adding effect of head losses increases and loss of pump efficiency]

Table 1. Example of variation analysis from pump characteristic given by manufactures. [it is assumed in this figures that H3=H2 there is no energy losses at the outlet of the lift station]