Manual on agricultural price index numbers
FOREWORD

Price observations of individual agricultural commodities and input items represent important information for data users in government, business management, or in other areas, conducting economic analysis. However, individual commodity price data alone do not always provide sufficient guidance for studying general price trends. The means serving this purpose are price indexes.

Although commodity price data constitute the basis for the construction of price indices, many countries do not benefit from this source. Indeed, only about half of the countries collecting producer price data use them to compute the corresponding index numbers. This fact was revealed in FAO's report: "National Methods of Agricultural Price Data Collection" (FAO 1986/a, p. 10).

The main objective of this manual is to help countries in starting the construction of agricultural price indices, or improving the methodology of existing ones, if needed. It complements the manual: "Farm and Input Prices: Collection and Compilation" (FAO 1980), therefore the description of the collection, compilation, treatment and dissemination of price data are not repeated here. However, definitions of the basic categories of agricultural price statistics are provided, where appropriate, for convenience. In addition to the discussion of price indices, the text covers the concepts, computation and interpretation of certain derived indicators, such as the parity ratio and the terms of trade.

It is hoped that this publication will be a useful instrument for developing countries in the training of national staff as well as a practical reference for the statisticians in charge of index construction. It may also assist the data users in the interpretation and application of index numbers. For the purpose of revising and improving this first edition all comments and suggestions will be greatly appreciated.

This document was prepared in the Statistical Analysis Service of the Statistics Division by Dr. G. Parniczky who worked as a consultant. His service is gratefully acknowledged.
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CHAPTER 1. INTRODUCTION

The analysis of price data implies comparison of past and current prices. Comparison over time is required to study the price movements in order to understand the history and to indicate future outlook. While price relatives of single commodities can be studied in isolation, general conclusions can only be derived from averages, covering a given set or class of commodities. The indicators of average price changes are the price indices.

1.1 Organization of this manual

Apart from the foreword and the introduction, this publication is composed of four chapters.

The chapter following the introduction is devoted to price and quantity observations, i.e. the fundamental data required for index construction. It provides explanation and guidelines for the compilation and processing of primary data.

The next chapter provides a review of index number theory. Terms, definitions, symbols and formulae are presented here, together with methodological guidance about the construction of price and quantity indices. Background information, complementing this chapter, is presented in the Appendix, for the interested reader. It contains a short historical review, as well as description of the properties of various index formulae.

Chapter 4 deals with practical problems encountered in the process of computing indices; they concern comparability over time, such as quality changes, new products appearing on the market or old ones disappearing from it. Techniques dealing with comparability problems are presented, such as adjustment and imputation.

The last chapter is devoted to the analytical uses of the agricultural price indices. Parity ratios, domestic and external terms of trade, and other measures are presented here. Application of the indices as deflators for converting agricultural accounts at current prices into accounts at constant prices is covered, among other uses.

1.2 Uses and users of agricultural price indices

The construction of agricultural price index numbers may serve various purposes. An exhaustive list cannot be provided, but some of the important goals are listed below:

i) Economic analysis, in particular the estimation of general price trends and their relationship with other pertinent variables, e.g. the study of domestic price changes in relation to prices observed in external markets, or the movement of agricultural commodity prices compared with the purchase prices of the means of agricultural production.

ii) Monitoring the implementation of agricultural price policy decisions, such as the introduction or modification of support prices, intervention prices, etc.

iii) Forecasting price movements in connection with market studies, or business cycle research. Many econometric models feature equations which contain price indices as variables.

iv) Compilation of national accounts at constant prices. In order to estimate the growth of the real product of the agricultural sector, deflator indices are needed. They are appropriately weighted price indices of agricultural commodities or input items.

The basic categories of data users can be identified with reference to the purposes listed above. Government might be mentioned as the prime user, especially ministries or departments in charge of development planning and policy formulation in the agricultural sector. Monitoring of price trends, economic analysis and national accounting are the main applications in this context.
Various business organizations and enterprises are certainly among the data users, including public marketing boards, private or cooperative trading establishments, banks operating in the rural areas, etc. Index numbers are needed for planning trade flows, stock levels, investment and related credit demand. Individual farmers and farmers' cooperatives need the indices for planning the structure of production, investment, etc. in view of the price trends and outlook.

National and international economic research organizations, other academic establishments need the indices for time series analysis, forecasting, model building at national, regional or global level, and related activities.

The government service entrusted with the task of making the index numbers should be aware of the various uses and users of this information. Indeed, it is recommended to establish regular contact with the main users in order to identify the specific needs concerning data dissemination, such as frequency, commodity classification, geographic breakdown, etc. Existing national standards should be of course respected, in particular if the agricultural price indices constitute a component of an interrelated system of price and quantity index numbers within the general national accounting framework (see United Nations 1977).

1.3 Types of agricultural price indices

Since index numbers are based on elementary price data, a typology of the agricultural price indices must follow the price categories. They are defined with reference to the stages of distribution on the one hand, and to the product on the other. According to the stages of distribution producer, wholesale and retail prices can be distinguished on the domestic market. Export and import prices may be recorded in addition. Regarding the product two classes are identified: agricultural commodities and agricultural requisites (means of production). They are also referred to as output and input prices respectively.

A combination of these two aspects yields a number of price subsets, all of which are susceptible for statistical observation and index construction. However, two price categories have special importance:

i) Prices received by farmers represent the producer prices of agricultural products (output prices).

ii) Prices paid by farmers are the purchase prices of agricultural requisites (input prices).

The two classes of prices mentioned above are considered important in the context of economic analysis and agricultural policy decisions. Index numbers based on them show the average changes of these prices and constitute, therefore, information primarily demanded by the data users identified above. Their construction is especially recommended.

It is, of course, desirable to exploit the other agricultural price data sets for making index numbers, such as export prices of agricultural commodities, import prices of agricultural inputs, etc. Indices of this kind usefully complement the main series and serve as basis for comparison (e.g. producer price index versus export price index of agricultural products). In view of this, the construction of various other agricultural price index series is recommended in addition, as the data base permits.
CHAPTER 2. PRICE AND QUANTITY OBSERVATIONS

2.1 The unit of price observation

Elementary price and quantity data are reviewed in this chapter. They are the basis for index construction, and their importance should not be underestimated. No index formula can counterbalance the absence of careful selection and specification of commodities, or the accurate observation of prices and quantities.

The target of price observation should be a homogeneous commodity, so that each unit constitutes a perfect substitute for any other unit on the market. The unit must be specified according to physical and commercial characteristics, affecting the price, such as grade, variety, nutrient content (e.g. fertilizers), performance (e.g. agricultural machinery), etc. The record containing a detailed commodity description belongs to the basic documentation in price statistics.

Crude commodity specification yields heterogeneous units of observation and it may generate unit value bias. This is particularly relevant in external trade statistics, where information is usually provided by the customs administration, based on tariff classification (United Nations 1981). If the unit of price observation is a heading of the standard foreign trade classification, and information is available on values and quantities, it is common practice to use the data for calculating export and import price indices. The value over quantity ratio, computed separately for each tariff heading, substitutes the individual price observations. The index derived from this data base, called unit value index, is accepted as a proxy of the corresponding price index.

While no objection can be raised against this procedure as long as classes are fairly homogeneous, a broad definition of the unit of observation may generate bias. If several individual grades, varieties, qualities, etc. are clustered under a single heading, the unit value index may show both true price changes and effect of shifts within the composition of the heading.

2.2 Primary data processing

Primary data processing, in the present context, means the series of operations performed between recording the individual price quotations and computing the actual price indices, according to the formulae to be presented in the next chapter. Although these operations differ from country to country, due to different conditions and field organization, certain general features can be stated.

As will be seen, all price index numbers are weighted; they are either functions of quantity-weighted price data, or value-weighted price relatives. In any case, the weights must be available for computing the index, therefore quantity and/or value data are needed in addition to prices.

Ideally, the unit of quantity or value observation coincides with the unit of price observation, discussed in the previous section. This might be the case for some agricultural commodities, if the quantity data are available at the level required for the price observation. Even in this case the index formulae are not necessarily directly applicable, because several price quotations might be collected for the same commodity unit at different markets while no data on corresponding quantities can be made available.

In contrast to the abundant literature on the theory of index numbers, not much has been published on the data processing operations susceptible for spanning the gap between the basic price and quantity data. Statistics Canada has issued a report on the subject and the author proposed the following terms (Szulc 1986):

- **basic aggregation level**: the lowest level of aggregation where weights can be associated with price data;
- **micro-indices**: price index numbers below the basic aggregation level (no weights are available);

- **macro-indices**: price indices at or above the basic aggregation level.

This terminology shall be followed in so far as referring to basic aggregation level and micro-indices, defined above. Instead of macro-indices, however, simply indices or index numbers will be used.

Micro-indices will be discussed in the present chapter. Like all index numbers, they are measures of relative changes over time, i.e. current period prices are compared with base period prices. Problems involved in the definition of the two periods (e.g. seasonality) will be discussed later.

Elementary price quotations for the same specific commodity unit collected at different points of time, or in different districts, markets, etc. can be processed first, followed by aggregation at the basic commodity level. In any case it is clear that micro-indices are simple (unweighted or equiweighted) measures. Two different approaches can be adopted for their construction:

i) **Ratio of averages** (RA): prices are averaged first, both for the base period and current period. The current period average is divided by the base period average, as a second step.

ii) **Average of ratios** (AR): first elementary price relatives are calculated from the matching pairs of price observations in the base period and current period. An average of the price relatives is calculated in the second stage.
An example is shown in Table 2.1, using the notation:

\[ o \quad = \quad \text{subscript for the base period} \]
\[ t \quad = \quad \text{subscript for the current period} \]
\[ j \quad = \quad 1, 2, \ldots, m \quad = \quad \text{subscript for the elementary unit of price observation} \quad (\text{e.g. markets}) \]
\[ p_j \quad = \quad \text{price observation for the j-th unit (j-th market)} \]
\[ r_j \quad = \quad \frac{p_{tj}}{p_{oj}} \quad = \quad \text{price relative for the j-th unit (market)} \]
\[ \bar{p} \quad = \quad \text{average price} \]
\[ \bar{r} \quad = \quad \text{average (arithmetic mean of) price ratios}. \]

**Table 2.1**

**PRICE OBSERVATIONS AT TWO MARKETS FOR WHEAT**

(price per metric ton)

<table>
<thead>
<tr>
<th>Market</th>
<th>Price</th>
<th>Price relative (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>base ( (p_o) )</td>
<td>current ( (p_t) )</td>
</tr>
<tr>
<td>A</td>
<td>249</td>
<td>350</td>
</tr>
<tr>
<td>B</td>
<td>179</td>
<td>235</td>
</tr>
<tr>
<td>Average</td>
<td>214</td>
<td>292.5</td>
</tr>
</tbody>
</table>

The results of the two approaches are:

\[
RA = \frac{292.5}{214} = 1.367 = 136.7\%
\]

\[
AR = \frac{1.406 + 1.313}{2} = 1.360 = 136.0\%
\]
The paper by J.B. Saulc, quoted above, provides a full discussion on advantages and disadvantages of the various micro-index formulae. The main findings are summarized below:

i) The selection procedure, and probability sampling in particular, affects the choice, indeed. If the sample design generates selection probabilities proportional to base period values, i.e. every currency unit (e.g. every dollar) has equal probability, AR is preferred, because it is closest to the Laspeyres price index. On the other hand, RA is the best choice if the sample design renders each commodity unit equal selection probability, both in the base period and in the current period (but hardly any sample design can guarantee this condition in practice).

ii) The degree of homogeneity of the units at basic aggregation level should be taken into account. The RA approach is not recommended if the units are heterogeneous, because a few extreme prices (e.g. expensive items) may exercise undue influence on the mean prices, especially if the sample is small.

iii) Transitivity (see Appendix) is a desirable property, since the price series are linked over successive periods. This consideration rules out AR since it is not transitive.

In view of the above, there is no single formula, which is universally applicable, or ideal under any condition. Therefore the choice must be guided by the prevailing conditions, such as sampling design.

Different formulae can be preferred at successive stages of the primary data processing, depending on the selection procedure, degree of homogeneity, and other relevant conditions. For example, within the provinces of a country RA may be calculated and, at the next stage, ARs of the individual provinces can be averaged. It was assumed that no weights were available. However, weights should be employed as and when they are available, even if they are estimated, but provide an approximate measure of the quantity or value share associated with the price data. Transition to the weighted index formulae happens when this level (basic aggregation level) is reached.

In view of the outstanding importance of the data used as weights in the construction of index numbers the next section is devoted to the weighting schedule.

2.3 Coverage and weighting scheme

"Weights" stand for quantities or values used for the construction of index numbers at or above the basic aggregation level. The actual choice between quantity and value data depends of the form of the index, to be discussed in the following chapter. As far as we are concerned in this section, this choice is irrelevant; we shall speak about weights in general, or quantities in particular.

The weights are determined by the type of the price index; e.g. a consumer price index should be weighted by the composition of the consumption of households, and export price index by the composition of exports by commodities. The accurate definition of the weights, however, demands closer examination of the concepts and data sources involved. This will be done in this section, with regard to the two leading agricultural index numbers: the index of prices received by farmers and the index of prices paid by farmers. Both have been defined in the Introduction (see Section 1.3).

Index of prices received by farmers (output price index, or producer price index)

Coverage of this index extends over the full range of agricultural commodities, and all the important items produced in a given country should be included. The relative coverage, in value terms, is recommended to be at least 80 per cent. Component (subgroup) indices might be constructed and published, showing the price trends in particular regions, districts or according to broad commodity groups. As a minimum, price indices should be made available for: total agriculture, crops, livestock and livestock products. A standard list of agricultural products is presented in the Appendix of the Handbook of Economic Accounts for Agriculture.
The first choice to be made when defining the weights is whether they should represent production or marketed production (sales). As price data are associated with commercial transactions, it is logical to relate prices to sales, rather than to total production. In view of this, intra-farm use (such as feed or seed produced and used by the same farm) should be deducted from the total output.

The next question to be considered is the destination of sales: inter-farm use is also deducted by some countries, and only the final output, leaving the agricultural sector, is entered into the weighting schedule of the price index. This corresponds to the "national farm concept" adopted by the European Communities (EUROSTAT 1985, pp. 62-63).

The choice of the weights depends largely on the use to be made by the index number. As a compromise, of course, the data sources available for this purpose shall be taken into account.

If only one price index is calculated, marketed production is recommended for weighting. Alternatively, the final output can be used, if this concept is in line with the national accounting practice of the country concerned and while the gross output (total production) should be considered only as a proxy, if available data sources do not permit the weighting recommended above.

**Index of prices paid by farmers**

Coverage of this index can be defined according to two concepts: in the narrow sense the index covers only agricultural inputs (requisites), including intermediate consumption and gross fixed capital formation. Lists showing the full range of the items covered, are located in the Appendix to the Handbook of Economic Accounts for Agriculture. In any case, the recommendation concerning the relative coverage of the output price index (minimum 80 per cent) is also valid for the inputs. It is also recommended, in addition to a price index for the aggregated production requisites, to calculate price indices for the following sub-groups of input items:

a) goods and services currently consumed (fertilizers, pesticides, feed, seed, energy and lubricants, maintenance and repairs, etc.);

b) investment goods (machinery and equipments, farm buildings, etc.).

While the narrow definition, stated above, is associated with the prices paid by the farmers in their capacity as purchaser of the means of agricultural production, there is a broader interpretation: the coverage may include the household expenditure items (consumer goods and services used by the family), in addition to inputs. This section of expenditure is associated with the retail prices paid by the farmer and his family as consumers.

The weighting schedule depends on the coverage, discussed above. According to the narrow definition weights should be proportional to expenditure on agricultural inputs. On the other hand, if the broader coverage is used, weights should represent the full range of expenditure of farming families, including family living.

Both the narrow and the broad concepts are recommended for countries where resources permit this approach, because the two index series provide different, but equally important information about the price relations affecting the farmers. In the absence of data sources needed for the broad definition the narrower coverage is strongly recommended as a minimum programme.
CHAPTER 3. THE THEORY OF INDEX NUMBERS

3.1 The nature of index numbers

Index numbers are statistical indicators constructed for the purpose of measuring changes of price level or quantity (volume) for a set of commodities, relative to a given base period. The commodities, covered by the index, may represent production, sales, consumption, exports, or other flows.

Index number theory is concerned with concepts and methods of index construction. It provides foundations for the practice, i.e. the calculation of indices, derived from empirical data.

An outline of index theory is presented in this chapter, confined to the main features. It may serve as a convenient reference to basic methods and formulae. Background information, complementing the main text, is available in the Appendix. It contains a summary of the evolution of index theory, as well as discussion of the properties of different formulae, and a guide to choose among them.

3.2 Notation

The following symbols will be used to represent the variables involved in the construction of index numbers:

\[
\begin{align*}
    p & = \text{price per unit of measurement} \\
    q & = \text{quantity (produced, sold, imported, etc.)} \\
    v & = pq = \text{value (of production, sales, imports, etc.)} \\
    o & = \text{subscript for the base (reference) period} \\
    t & = \text{subscript for the current (given) period (}t = 1, 2, \ldots, T) \\
    i & = \text{subscript for a given commodity (}i = 1, 2, \ldots, n) \\
    r_{pi} & = \frac{p_t}{p_o} = \text{price relative for the }i\text{th commodity} \\
    r_{qi} & = \frac{q_t}{q_o} = \text{quantity relative for the }i\text{th commodity} \\
    P & = \text{price index} \\
    Q & = \text{quantity (volume) index} \\
    V & = \text{value index}
\end{align*}
\]

Remark: attention is called to the different interpretation of small and capital letters; e.g. \(p\) means unit price, but \(P\) represents a price index.

The full notation with double subscripts will be used only if its absence may create confusion. Otherwise it will be replaced by an abridged notation, i.e. omitting the commodity subscript, for example:

\[
\sum_{i=1}^{n} p_o q_{ti} = P_0 q_t
\]

is the total value of current year quantities, valued at base prices. The summation runs through \(i = 1, 2, \ldots, n\) commodities, but the limits are indicated at the bottom and top of the summation symbol only in the expression using full notation, on the left side of the equation. They are absent in the simplified formula on the right side.

Using the above notation the \underline{value index} is defined as the ratio of current over base period aggregate value:

\[
V = \frac{\sum P_t q_t}{\sum P_o q_o} = \frac{V_t}{V_o}
\]
Obviously the value index shows the movement of prices and quantities concurrently. The index numbers to be introduced in the following section represent the two components separately.

### 3.3 Computation of indices for two periods

This section is devoted to the definition and computation of price and quantity (volume) index numbers. Given a flow of different commodities, they show the changes of prices and quantities respectively.

Three basic formulae of price and quantity indices are displayed in Table 3.1. Two of them are presented according to both aggregative and weighted average form. The aggregative form shows the index as the ratio of two value aggregates, representing the current and the base year respectively, whereas the weighted average form represents the mean of individual price relatives or quantity relatives.

**Table 3.1**

**INDEX FORMULAE FOR TWO PERIODS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Aggregative</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sum p_t q_0$</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Laspeyres</td>
<td>$\sum p_o q_0$</td>
</tr>
<tr>
<td>$P_P$</td>
<td>Paasche</td>
<td>$\frac{\sum p_t q_t}{\sum p_o q_t}$</td>
</tr>
<tr>
<td>$P_F$</td>
<td>Fisher</td>
<td>$\sqrt{P_L P_P}$</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>Laspeyres</td>
<td>$\frac{\sum p_o q_t}{\sum p_o q_0}$</td>
</tr>
<tr>
<td>$Q_P$</td>
<td>Paasche</td>
<td>$\frac{\sum p_t q_t}{\sum p_t q_0}$</td>
</tr>
<tr>
<td>$Q_F$</td>
<td>Fisher</td>
<td>$\sqrt{Q_L Q_P}$</td>
</tr>
</tbody>
</table>
The numerical result of the aggregative and the weighted average forms is, of course, the same, if the indices are computed from an identical set of price and quantity data. It is nevertheless important to distinguish the two forms. First, the data processing is different: the aggregative form is adopted if the initial data are individual price and quantity observations, available in both periods. The average form, on the other hand, is preferred if the relatives and the corresponding values are readily available.

The primary data processing of individual price and quantity observations, discussed in Section 2.2, should be recalled at this stage. As a rule, individual price observations are collected below the basic aggregation level, therefore micro price indices are calculated before matching the price and quantity data. In this case the micro price indices are taken as price relatives in the weighted mean formulae.

Apart from the technique of data processing the two forms have different interpretation; the Laspeyres price index will be discussed to illustrate this point. The aggregative form is a ratio, whose denominator is the actual aggregate value in the base year, valued at current year prices. Since the quantities are fixed, but prices move, the ratio should indicate the change of the common price level of the commodities covered by the index. The average form of the same index, on the other hand, represents the arithmetic mean of the individual price relatives, weighted by the corresponding base year values. Thus the index shows the general tendency of price changes.

In view of the above, "weights" have double meaning in index theory and in the construction of price indices;

i) quantities, used in the aggregative index formulae (quantity-weighted prices), or

ii) values, used in the weighted mean formulae (value-weighted price relatives).

Interpretation of the aggregative and average forms of the Paasche formula may follow, mutatis mutandis, the same line. The Fisher index is the simple geometric mean of the corresponding Laspeyres and Paasche indices. It has no direct interpretation, independently from the two others, but it has some desirable properties, which are discussed in the Appendix.

The formulae of the quantity indices are symmetric to the price indices in the p and q variables. Q, can be derived from P, by replacing p,q, with p,q, in the numerator and vice versa. Interpretation of the volume index follows from the interchanging of positions; this time prices are fixed, while quantities move, therefore the index is a measure of the aggregate value change due to the quantity component. The weighted mean form shows the general (average) direction of quantity changes, represented by the quantity relatives.

"Weights" employed for computing a volume index are alternatively

i) prices, used in the aggregative index formulae (price-weighted quantities), or

ii) values, used in the weighted mean formulae (value-weighted quantity relatives).

The computation of indices presented in Table 3.1 will be illustrated by a simple numerical example. Initial price and quantity data, covering two years, are shown in Table 3.2. Intermediate results are available in the worksheet displayed in Table 3.3. They show the base and current year values, as well as the cross-products: p,q, and p,q,. The price and quantity relatives are located in the last two columns. Commodity price data in Table 3.2 represent the results of primary data processing operations performed with individual price observations.
Table 3.2
NUMERICAL EXAMPLE FOR THE COMPUTATION OF INDEX NUMBERS

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Base period</th>
<th></th>
<th>Current period</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>price (per MT)</td>
<td>quantity (million MT)</td>
<td>price (per MT)</td>
<td>quantity (million MT)</td>
</tr>
<tr>
<td></td>
<td>$p_0$</td>
<td>$q_0$</td>
<td>$p_t$</td>
<td>$q_t$</td>
</tr>
<tr>
<td>Wheat</td>
<td>211.1</td>
<td>9.1</td>
<td>286.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Rice, paddy</td>
<td>311.0</td>
<td>0.9</td>
<td>381.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Potatoes</td>
<td>127.7</td>
<td>2.8</td>
<td>146.0</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 3.3
WORKSHEET

<table>
<thead>
<tr>
<th>Commodity</th>
<th>$v_0=p_0q_0$</th>
<th>$v_t=p_tq_t$</th>
<th>$p_0q_t$</th>
<th>$p_t$/$p_0$</th>
<th>$q_t$/$q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>1921.0</td>
<td>2523.8</td>
<td>1857.7</td>
<td>2609.9</td>
<td>1.36</td>
</tr>
<tr>
<td>Rice, paddy</td>
<td>279.9</td>
<td>342.9</td>
<td>279.9</td>
<td>342.9</td>
<td>1.23</td>
</tr>
<tr>
<td>Potatoes</td>
<td>357.6</td>
<td>423.4</td>
<td>370.3</td>
<td>408.8</td>
<td>1.14</td>
</tr>
<tr>
<td>Total</td>
<td>2558.5</td>
<td>3290.1</td>
<td>2507.9</td>
<td>3361.6</td>
<td>.</td>
</tr>
</tbody>
</table>

The final results are tabulated in Table 3.4. They can be verified with reference to the index formulae in Table 3.1 and the worksheet in Table 3.3. The Laspeyres price index according to aggregative and weighted mean forms has been calculated as follows:

\[
P_L = \frac{3361.6}{2558.5} = 1.314
\]

\[
P_L = \frac{1921 \times 1.36 + 279.9 \times 1.23 + 357.6 \times 1.14}{1921 + 279.9 + 357.6} = 1.314
\]
Table 3.4

INDEX NUMBERS
(Percentage, rounded)

<table>
<thead>
<tr>
<th>Formula</th>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laspeyres</td>
<td>131.4</td>
<td>98.0</td>
</tr>
<tr>
<td>Paasche</td>
<td>131.2</td>
<td>97.9</td>
</tr>
<tr>
<td>Fisher</td>
<td>131.3</td>
<td>98.0</td>
</tr>
</tbody>
</table>

The other results can be verified similarly.

The numerical example above has been presented to facilitate understanding. In practice, working with a large amount of data, the procedure can be of course computerized and manually prepared worksheets are not needed.

In our numerical example both the quantity and price Laspeyres indices were higher, than the corresponding Paasche index numbers, although the differences were small. This is the case in general.

Out of the formulae, presented in this section, Laspeyres appears to be most popular in national practice.

3.4 Series of index numbers

If the time span covers three or more periods the consecutive indices constitute a series or run of index numbers. Formulae in this case become more complicated, because, in addition to the index types defined for two periods, a combination of the following alternatives must be chosen:

- indices with fixed or moving weights;
- chained or unchained indices.

Each index number in the series represents a binary comparison between the given (current) period and the base (reference) period, which is usually (but not necessarily) fixed and coincides with the starting period. This means that at least one variable (p in a price index series and q in a volume index run) is always moving. The weights, on the other hand, may or may not change. This is the difference between index numbers with fixed weights and moving weights.

As a rule, each binary comparison is calculated separately. An alternative is the chain index. The chain index run is derived by successive multiplication of "links". These links are calculated by comparing two adjacent periods the aggregates of which should have moving weights.

Table 3.5 is presented to demonstrate the construction of index series. It shows selected types of price index runs, covering the time span of three periods: 0, 1 and 2. All indices are recorded in aggregative form to facilitate interpretation, and "weights" refer to the quantities associated with prices. The indices can be, of course, transformed to weighted average form, using \( v_o \) or \( v_t \) weights, as in Table 3.1.
The series in the first line of Table 3.5 are frequently adopted in practice. It is characterized by fixed weights. The indices in columns A and C are ordinary Laspeyres price indices for the binary comparisons 1/0 and 2/0 respectively. However, the index in column B cannot be considered a true Laspeyres index, since the weights do not correspond to the actual reference period. Indeed, a run of fixed weight indices is composed of Laspeyres index numbers as long as the base of comparison is also fixed and it coincides with the weight-base.

The second line shows a combination of moving weights and chaining, starting again with an ordinary Laspeyres index for the first binary comparison. In this case the second index is also of Laspeyres-type, but the last one is not. It is generated by the chaining operation, being the product of A and B, which are now regarded as links, joined together to produce a chain.

The series located in the third and fourth line start with Paasche indices, which are incompatible with the idea of fixing weights. Therefore the weights are moving in both lines, the difference being that chaining is either adopted or dismissed. As a matter of fact, the Paasche formula is usually combined with chaining, whereas Laspeyres is frequently associated with fixed weights, taken from the base period.

**Table 3.5**

**SERIES OF PRICE INDEX NUMBERS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Weights</th>
<th>Chain Index</th>
<th>Comparison: current period over reference period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1/0 = A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2/1 = B</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2/0 = C</td>
</tr>
<tr>
<td>( P_L )</td>
<td>fixed</td>
<td>not</td>
<td>[ \frac{\sum p_1 q_0}{\sum p_0 q_0} ] [ \frac{\sum p_2 q_0}{\sum p_1 q_0} ] [ \frac{\sum p_2 q_0}{\sum p_0 q_0} ]</td>
</tr>
<tr>
<td>( P_L )</td>
<td>moving</td>
<td>yes</td>
<td>[ \frac{\sum p_1 q_0}{\sum p_0 q_0} ] [ \frac{\sum p_2 q_1}{\sum p_1 q_1} ] [ A \times B ]</td>
</tr>
<tr>
<td>( P_P )</td>
<td>moving</td>
<td>not</td>
<td>[ \frac{\sum p_1 q_1}{\sum p_0 q_1} ] [ \frac{\sum p_2 q_2}{\sum p_1 q_2} ] [ \frac{\sum p_2 q_2}{\sum p_0 q_2} ]</td>
</tr>
<tr>
<td>( P_P )</td>
<td>moving</td>
<td>yes</td>
<td>[ \frac{\sum p_1 q_1}{\sum p_0 q_1} ] [ \frac{\sum p_2 q_2}{\sum p_1 q_2} ] [ A \times B ]</td>
</tr>
</tbody>
</table>
In order to facilitate understanding of the practical calculations, the construction of index runs will be illustrated by a simple hypothetical example. The starting data are furnished in Table 3.6, while intermediate (semi-processed) results are located in Table 3.7. Each value aggregate entered in this matrix is computed from the price and quantity data in Table 3.6, e.g.

$$\sum p_i q_i = 1 \times 18 + 2 \times 6 + 3 \times 2 = 36$$

Table 3.6

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Period 0</th>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_0$</td>
<td>$q_0$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 3.7

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$q_t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18</td>
<td>36</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>54</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>36</td>
<td>46</td>
<td></td>
</tr>
</tbody>
</table>

Note: entries show the values $p_t q_t$ based on the data in Table 3.6.
Final results are presented in Table 3.8. They can be easily verified, using the data in Table 3.7, with reference to the formulae in Table 3.5. The series in the last line (Paasche-type, moving weights, chained), for example, was calculated as follows:

\[ P_{1/0} = \frac{54}{36} = 1.50 \]

\[ P_{2/1} = \frac{46}{74} = 0.62 \]

\[ P_{2/0} = 1.5 \times 0.62 = 0.93. \]

All series show an increase of the price level between the period 0 and 1, a decrease between 1 and 2, consequently little or no change between 0 and 2. Note, however, that in period 2 all three commodity prices were identical with the initial prices. It is therefore expected that any price index of period 2, based on period 0, should be 100 per cent. Contrary to this expectation the chained indices show deviations: the Laspeyres-type is higher and the Paasche is lower than 100 per cent. Chaining is practiced nevertheless, since chained indices feature certain desirable properties, discussed in the Annex.

### Table 3.8

<table>
<thead>
<tr>
<th>Type</th>
<th>Weights</th>
<th>Chained</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1/0</td>
</tr>
<tr>
<td>PL</td>
<td>fixed</td>
<td>not</td>
<td>167</td>
</tr>
<tr>
<td>PL</td>
<td>moving</td>
<td>yes</td>
<td>167</td>
</tr>
<tr>
<td>PF</td>
<td>moving</td>
<td>not</td>
<td>150</td>
</tr>
<tr>
<td>FF</td>
<td>moving</td>
<td>yes</td>
<td>150</td>
</tr>
</tbody>
</table>

While fixed weight Laspeyres-type series are very popular, it is clear that such series cannot be continued indefinitely. Weights might become out of date and, in such cases, comparability of prices is rendered imperfect. In addition, over a long period the disappearance of old products and appearance of new ones on the market may create similar problems. In view of this, a revision of the commodity regime and rebasing of the series is performed. At this point a new run starts with weights adjusted accordingly. There is a consensus that the frequency of revision should be between five years to ten years (United Nations 1979).

There is a need, however, to furnish comparable data for the users covering some years before and after rebasing. There are two techniques to achieve this target, viz. reweighting and splicing. The first method involves revision of some years preceding the new base period and supplies unbroken runs on each side of the new base. According to the second method the historical data are not revised backward, but simply switched to the new base. In practice, this procedure creates a chain index, whose links are composed of 5-10 year periods, instead of a single year. Let us note, that only splicing is feasible if the quantity and price data of the new series do not match the previous observations, due to important changes in the product basket. A numerical example illustrating the two methods is given in Table 3.9.
Table 3.9

NUMERICAL EXAMPLE OF REWEIGHTING AND SPLICING

<table>
<thead>
<tr>
<th>Year (t)</th>
<th>Quantities (q)</th>
<th>Prices (p)</th>
<th>Aggregates</th>
<th>Price indexes (per cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>2</td>
</tr>
</tbody>
</table>

Index numbers, under reweighting column were arrived at by using the aggregates

\[
\frac{\Sigma p₄q₄}{\Sigma p₀q₀}.
\]

Index numbers under the column splicing were arrived at by extending backwards the relationship between the two indexes in the overlap period (142.5, 100) which is the new reference period.

Making a recommendation concerning the formula of index numbers is rendered difficult by the fact, that there is no universally ideal index, satisfying every requirement. Index numbers have various properties; some of them are especially desirable for a given purpose, whereas others are not. These properties and the strategy for choosing index numbers are discussed in the Appendix, and the interested reader is encouraged to consult it.
CHAPTER 4. PRACTICAL PROBLEMS ENCOUNTERED

4.1 The choice of the base period

Published series of price index numbers are usually calculated with fixed reference base (base of comparison), which coincides with the weight base (the period of quantity observations) according to the concept of the Laspeyres index. This is due to the fact that the fixed-weight Laspeyres index is by far the most popular formula (see Appendix). However, the reference base and the weight base do not always coincide, and the term "base period" refers to the reference base in this case.

The base period is usually one year. Monthly or quarterly indices may have other bases, such as the corresponding period of the preceding year. This, and the associated problem of seasonality will be discussed in the following sections.

Although calendar years are used in most countries, crop years (split years) are also adopted for constructing the index of prices. Whatever concept is adopted, the next question is which year should constitute the base. It is suggested that the base should be a normal year (FAO 1980, p. 34), when prices are more or less stable and the volume of sales (used as weights) maintains a fairly regular level, not affected by boom, depression, or catastrophes, wars, etc. In practice, however, it is not always easy to distinguish between normal and abnormal years, especially in agriculture, where meteorological conditions and market forces may generate high fluctuations. Moreover, waiting for a normal year to come along may conflict with the regular pattern of rebasing and revision of the Laspeyres index, recommended in Section 3.4 and practiced by many countries (e.g. a five years cycle is decided by the members of the European Community, see EUROSTAT 1985, p. 65).

Extension of the base period may help to resolve the problem stated above. An average of two or more years, instead of a single year, may furnish the desired stability, and facilitate the regular up-dating at the same time, especially if centred on the target year, such as the three years period 1979-1980-1981 is centred on 1980.

Ideally, the price observations and the associated quantities should both refer to the extended base. However, for practical reasons it may happen that a single year constitutes the base for the prices, and a longer period for the weights or vice versa. In this case the reference base and the weight base do not coincide, but no objection can be raised if the two periods overlap (e.g. the weight base is the average of 1979-1980-1981 and the reference base for prices is 1980).

4.2 Monthly and quarterly price indices

While annual series of index numbers satisfy the demand for historical analysis, long term planning and national accounting, there is a need for indices representing subperiods of the year, such as months or quarters; they are used mainly for the study of current changes, short term planning and forecasting. Monthly price indices are published regularly by many national statistical services in charge of index number construction, in addition to the annual series; quarterly series are less frequently available. In view of this, we shall henceforth refer to month as the subperiod normally used, although quarters will be presented in the numerical example to reduce the data set and thereby facilitate understanding.

This section is devoted to the methodology of the monthly price index and to the relationship between monthly and annual index numbers. The closely connected problem of seasonality is the subject of the next section.
Monthly price indices are usually calculated as weighted averages of commodity price relatives. Many different formulae can be proposed, depending on the choice of the base period for the price relatives and the definition of the weights, even if the Laspeyres principle is adhered to. Concerning the reference base of the relatives the following alternatives can be considered:

i) weighted average price of the base year;
ii) price of the corresponding month in the base year;
iii) price of a fixed month in the base year (e.g. January or December).

The weights associated with the commodity price relatives might be:

i) values (or value shares) of the base year (fixed weights);
ii) values (or value shares) of the corresponding month in the base year;
iii) quantities of the corresponding month in the base year combined with the average annual prices in the base year.

The annual index can be computed either independently of the monthly series, or defined as an average of the monthly indices. A numerical example will be presented, showing a simple approach, consistent with the regular annual Laspeyres index.

The notation introduced in Section 3.2 will be used, with the following symbol in addition:

\[ j = 1, 2, \ldots, 12 \] subscript for the month.

The annual Laspeyres price index can be defined in aggregative and weighted average form as:

\[
P = \frac{\sum_{i} p_{ti} q_{oi}}{\sum_{i} p_{oi} q_{oi}} = \frac{\sum_{i} r_{ti} v_{oi}}{\sum_{i} v_{oi}}
\]

where

\[
p_{oi} = \frac{\sum_{j} p_{oji} q_{oji}}{\sum_{j} q_{oji}} \text{ weighted average base year price of commodity } i;
\]

\[
\bar{p}_{ti} = \frac{\sum_{j} p_{tji} q_{oji}}{\sum_{j} q_{oji}} \text{ average annual price of commodity } i \text{ in year } t, \text{ weighted by the monthly quantity distribution of the base year;}
\]

\[
q_{oi} = \sum_{j} q_{oji} \text{ total quantity of commodity } i \text{ in the base year;}
\]

\[
v_{oi} = p_{oi} q_{oi} = \sum_{j} p_{oji} q_{oji} \text{ total value of commodity } i \text{ in the base year;}
\]

\[
r_{ti} = \frac{p_{ti}}{p_{oi}} \text{ annual price relative for commodity } i.
\]
The first monthly price index is:

\[ P_j^* = \frac{\sum_i P_{tji}q_{oji}}{\sum_i P_{oji}q_{oji}} = \frac{\sum_i r_{tji}v_{oji}}{\sum_i v_{oji}} \]

where

\[ r_{tji} = \frac{P_{tji}}{P_{oji}} \]

is the price relative of commodity \( i \) in month \( j \), based on the corresponding month of the base year.

\( P_j^* \) is an index with moving reference and moving weight. Combining the full series of \( P_j^* \) through \( j = 1, 2, \ldots, 12 \), using the monthly values \( v_{oji} \) of the base year as weights, yields annual Laspeyres price index numbers corresponding to:

\[ P = \frac{\sum_j P_j^* v_{oj}}{\sum_j v_{oj}} \]

A numerical example illustrating the calculation of \( P \) and \( P_j^* \) is presented in Tables 4.1 and 4.2. The first table contains the initial (hypothetical) price and quantity data, together with the primary calculations. (Months are replaced by quarters in this example, in order to reduce the volume of data needed for illustrative purposes therefore the subscript \( j \) runs through 1, 2, 3, 4 only. Note further that the quantity data \( q_{oji} \) are simply repeated in the current year; viz. we use only Laspeyres formula runs and there would be no need to furnish figures for the current period at all). Taking the summary data from Table 4.1 the annual index:

\[ 2580 \]
\[ P = \frac{2580}{2320} = 1.112, \text{ or } 111.2 \text{ per cent.} \]

**Table 4.1**

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>1st commodity</th>
<th>2nd commodity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( P ) ( q ) ( v = pq ) ( P ) ( q ) ( v = pq ) ( v )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>24 10 240</td>
<td>10 16 160</td>
<td>400 2320</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>22 10 220</td>
<td>10 14 140</td>
<td>360 2320</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>19 20 380</td>
<td>8 30 240</td>
<td>620 2320</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>19 40 760</td>
<td>6 30 180</td>
<td>940 2320</td>
</tr>
<tr>
<td>Annual</td>
<td></td>
<td>20 80 1600</td>
<td>8 90 720</td>
<td>2320 2320</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>26 10 260</td>
<td>12 16 192</td>
<td>452 2320</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>24 10 240</td>
<td>12 14 168</td>
<td>408 2320</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>19 20 380</td>
<td>10 30 300</td>
<td>680 2320</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20 40 800</td>
<td>8 30 240</td>
<td>1040 2320</td>
</tr>
<tr>
<td>Annual</td>
<td></td>
<td>21 80 1600</td>
<td>10 90 900</td>
<td>2560 2320</td>
</tr>
</tbody>
</table>

**Remark:** base year quantities are repeated in the current year.
Table 4.2

WORKSHEETS

<table>
<thead>
<tr>
<th>Quarter: j</th>
<th>i = 1</th>
<th></th>
<th>i = 2</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r_tji</td>
<td>r_tji Voji</td>
<td>r_tji</td>
<td>r_tji Voji</td>
<td>E_r_tji Voji</td>
<td>P_j</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.0833</td>
<td>260</td>
<td>1.2000</td>
<td>192</td>
<td>452</td>
<td>1.130</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.0909</td>
<td>240</td>
<td>1.2000</td>
<td>168</td>
<td>408</td>
<td>1.1333</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>380</td>
<td>1.2500</td>
<td>300</td>
<td>680</td>
<td>1.0968</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.0526</td>
<td>800</td>
<td>1.3333</td>
<td>240</td>
<td>1040</td>
<td>1.1064</td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>.</td>
<td>1600</td>
<td>.</td>
<td>900</td>
<td>2580</td>
<td>1.1121</td>
<td></td>
</tr>
</tbody>
</table>

The computation of the four quarterly indices can be checked with reference to the worksheet in Table 4.2. The first price relative in column 2:

\[ r\_111 = \frac{26}{24} = 1.0833 \]

and the others have been calculated similarly. The quarterly indices are located in column 7. The first index is:

\[ P\_1 = \frac{452}{400} = 1.13 \]

Weighted by the data \(v\_o\_j\) from Table 4.1, the four quarterly indices yield an annual index identical to \(P\) as defined above:

\[ P = \frac{1.13 \times 400 + 1.133 \times 360 + 1.097 \times 620 + 1.106 \times 940}{400 + 360 + 620 + 940} = \frac{2580}{2320} = 1.112 \]

4.3 Seasonality

Prices and quantities of many agricultural commodities show seasonal variations. Agricultural inputs (requisites), on the other hand, are less likely to follow a seasonal pattern. In any case, the problems created by seasonality merit discussion, especially if both annual and monthly (quarterly) price indices are constructed.

The conceptual issue, involved in the construction of the annual index, concerns the identity of a commodity sold or purchased in different seasons, i.e. whether or not it should be treated as the same unit of price observation. Adding the "season" to the commodity specification generates separate observation units, even if the products sold throughout the year are identical according to every other physical or commercial characteristic. Tomatoes grown under cover, for example, can be considered as a different commodity from those grown in the open.
The specification of the unit of observation affects the decomposition of value changes into price and quantity components, explained in Section 3.1. Regarding the particular problem of seasonality, United Nations' recommendations are in favour of the "separate approach" (United Nations 1977, p. 8), and national accounting practices follow this guidance in most countries. Consequently the annual price index of commodities exhibiting seasonal variations should be the weighted average of seasonal price indices. The annual index $P$, defined in the previous section satisfies this condition, since it is the weighted arithmetic mean of the monthly indices $P_i$. Indeed, the observation unit of the monthly index with variable weights is defined according to the "separate" concept, since the price recorded in each month is associated with the corresponding quantity. In contrast, the average of monthly indices with fixed weights, are not in line with the separate commodity approach, because the same weight is used irrespective of the season. As a result, the seasonal distribution of the quantities or sales values, used as weights, should be available, if the "separate" concept is adopted, at least for the base year.

Apart from the conceptual problem concerning the relationship between monthly and annual index numbers, explained above, two practical problems are frequently encountered in the construction of monthly series:

- certain commodity items may entirely disappear from the market for a number of months, (e.g. certain fruits and vegetables), therefore price observations are non-existent;
- the month-to-month changes, indicated by the index with fixed base, reflect seasonal fluctuations together with other nonseasonal changes, which may render the interpretation rather difficult for data users.

The missing price data do not create any problem if the monthly indices are constructed with variable weights, since the quantities associated with the seasonally absent prices are anyhow zero. Fixed weights monthly price indices, on the other hand, require continuous price information. There are various techniques of imputation for supplying fictitious prices:

i) the last recorded price or price relative is repeated (carried forward);

ii) using imputed prices equal to the average of the last season’s prices;

iii) extrapolating prices, based on the group index of similar commodities, whose prices are available in the current month.

While the first method is the simplest, it can be contested on the grounds that the last recorded price (at the end of the season) is likely to be based on low level transactions, therefore hardly representative. The two other techniques seem to be preferable, especially the last one, which has a dynamic element built in, and it is therefore recommended.

Concerning the problem of disturbing seasonal variation in the measurement of month-to-month changes, the following methods can be considered:

i) using a monthly index formula whose reference base is the corresponding month of the base year, and computing month-to-month measures as the ratio of the subsequent monthly indices;

ii) using moving averages (preferably 12 months averages) instead of single price relatives;

iii) using seasonally adjusted price relatives (adjustment procedures are presented in most textbooks covering time series, and computer programmes are available on the software market);

iv) excluding the seasonal items from the commodity regime of the regular monthly index. An index covering the seasonally fluctuating items can be prepared separately.
The first solution appears to be attractive, but it can be criticized on two grounds: firstly it is equivalent to a seasonal adjustment based on a single year. Secondly, the interpretation of the ratio \( \frac{P_{j+1}^*}{P_j^*} \) is rather confusing (see Balk 1980/a).

Moving averages are certainly susceptible of smoothing the seasonal fluctuations (indeed, some of the adjustment methods are based on moving averages), but 12 months moving averages should be mid-year centred. This means that the publication of the index for a given month should wait until all data are available for the calculation, including those six months ahead. Moving averages based on the last month circumvent this delay, but they do not truly represent the month to which they are assigned.

Seasonally adjusted series are good tools for index construction, especially if the seasonal pattern is rather stable. The only problem is that historical series, covering at least 5-6 years, are needed to produce the seasonal coefficients. Both the original and the adjusted series can be published, provided the data users receive clear guidance to distinguish between the two measures.

Construction of separate indices for the regular and seasonal commodities is a good strategy. There is hardly any restriction for the first index run; no objection can be raised against fixed annual reference and weight-base period, if monthly weights are not available. The separate index for the seasonal items, on the other hand, requires moving weights and some kind of smoothing for the month-to-month comparisons, as explained above. If an overall index is needed in addition, covering all items, the two series should be combined. The aggregation procedure depends of course on the respective formulae. The only disadvantage of this separate strategy is the increasing complexity of data processing, documentation and dissemination.

In view of the above, the choice depends inter alia on the resources available for data processing. If resources permit, the separate construction of price indices for regular and seasonally fluctuating items is recommended. Otherwise a seasonal adjustment procedure appears to be the preferable technique.

4.4 Quality changes

A price index is supposed to show the average price changes of carefully specified, strictly comparable products. The items selected for pricing must be therefore identical, according to every important technical and commercial characteristics, during the subsequent periods of observation (see commodity specification in Section 2.1).

It is, of course, easier to declare the above principle, than to preserve the complete identity of a commodity in practice. While agricultural commodities normally maintain their specification over a long period, agricultural requisites (inputs), such as machines and chemical products, undergo more frequent changes due to technological progress and other conditions of production. Quality changes in this process should be distinguished from entirely new products, entering the market first time. "Quality change" means that certain characteristics of a given commodity are modified, but enough specific characteristics are retained, so that the commodity can be reasonably considered identical (United Nations 1979, p. 50). A new tractor model, e.g., replaces an existing one, offering certain improvements, such as more powerful engine and lower fuel consumption, is considered quality change, not a new product. The methodology of dealing with quality changes is discussed in this section, and the next section is devoted to new products.
The simplest technique of rendering the series of price observations comparable is substitution, called also "splicing", because it is similar to the procedure described in Section 3.4 for linking the fixed weight index series preceding and following a revision and rebasing. Substitution means replacement of the original ("old") item selected for pricing by the matching ("new") product in case of quality changes. The schedule of calculations is exhibited in Table 4.4. The symbol \( p \) stands for price, as usual, the first subscript represents the time period \( t \) and the second one represents the product: \( 1 = \) old and \( 2 = \) new. Substitution does not affect the weighting pattern of a Laspeyres index, since the weight assigned to the original price relative is simply attached to the substitute item.

**Table 4.4**

SUBSTITUTION METHOD
(Numerical example given in parenthesis)

<table>
<thead>
<tr>
<th>Period ( t )</th>
<th>Price observation</th>
<th>Price relative over-base period</th>
<th>New price series</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>old quality</td>
<td>new quality</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>( P_{01}(160) )</td>
<td>.</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>( P_{11}(165) )</td>
<td>.</td>
<td>( P_{11}/P_{01}(165/160)=1.03 )</td>
</tr>
<tr>
<td>2</td>
<td>( P_{21}(180) )</td>
<td>( P_{22}(204) )</td>
<td>( P_{21}/P_{01}(180/160)=1.125 )</td>
</tr>
<tr>
<td>3</td>
<td>( . )</td>
<td>( P_{32}(210) )</td>
<td>( (P_{32}/P_{22})x\times=(210/204)(1.125) = 1.158 )</td>
</tr>
<tr>
<td>4</td>
<td>( . )</td>
<td>( P_{42}(220) )</td>
<td>( (P_{42}/P_{22})x\times=(220/204)(1.125) = 1.213 )</td>
</tr>
</tbody>
</table>

Technically speaking, the method requires only a single overlapping period, when both the old and the new versions are on the market (\( t = 2 \) in our example). However, an extended period of concurrent availability on the market is desirable, so that prices can be considered as equilibrium prices. Otherwise the introduction of the new model or variety by the producer may conceal a price increase, especially if the producer or importer has a monopoly position. In this case the new features may serve as tools of a marketing strategy, which forces the buyer to pay a higher price for essentially the same utility. The statistician should be aware of this possibility and choose the technique accordingly.

Other techniques should be used if the new model replaces the old one without overlapping, or the suspicion of disguised price increase exists, as explained above, therefore substitution cannot be applied. In this situation the statistician must either drop the price record (discontinue the series), thereby reducing the sample of observed prices, or make an adjustment (correction) which renders the price of the new model comparable with the base price. Price adjustment is the preferred solution; various techniques are briefly reviewed below.
A simple approach to the price adjustment is estimating the extra utility or consumer satisfaction due to quality changes. Unfortunately, there are only few cases, when this can be done directly. If only the packaging of the commodity is changed, e.g. 20 kilogramme fertilizer bags are replaced by 25 kilogramme units, the extra utility can be estimated by the ratio 25/20. Similar estimates are feasible if only one and well measurable characteristic changes; e.g. durability. Suppose that, according to reliable tests (not just advertisement) a new model of truck or tractor tire lasts for 40 per cent longer distance, therefore it provides 40 per cent more "satisfaction", which in turn justifies 40 per cent price increase. This is equivalent to the assumption that if both the old and the new quality (more durable) were concurrently available on the market, the new model would really sell for proportionally higher price. The statistician should examine this proposition and decide whether or not a price adjustment of this kind is reasonable.

Adjustment based on the production cost is frequently practiced; the idea is "costing out the quality change", i.e. estimating the marginal cost associated with the modified specification. The cost data are then used to adjust the new price by removing the additional cost and the price thus adjusted is deemed to be comparable with the base price. (Alternatively, the base price can be adjusted, by adding the extra cost to it). It should be noted that quality change is not necessarily improvement. Occasionally it can happen that the inferior quality replaces the superior one. The extra cost, associated with the quality change, is likely to be negative in this case and the adjustment is performed accordingly.

There are two problems involved in the cost adjustment method. The first is finding a reliable source of information. The best source is the producing establishment, since the calculation can be based directly on the accounts. However, the producer may not be willing to cooperate, or cannot be trusted to supply accurate data for this particular purpose. The alternative in this case is employment of an independent expert, who is familiar with the relevant technology and capable of estimating the cost data.

The second problem is more fundamental; the assumption implied in this procedure is that the extra utility incorporated in the new model is proportional to the extra cost of production. As long as this is valid, the cost data are true indicators for the price adjustment. Often, however, the extra utilities exceed the extra cost of production. In this case the adjusted price relative tends to overstate the price increase. This point will be illustrated now. Returning to our tire example above, we shall operate with the following hypothetical data.

\[ p_0 = 20 \text{ (old model)} \]
\[ p_1 = 30 \text{ (new model, 40 per cent more mileage), estimated extra cost = 3} \]

Unadjusted price relative:
\[ p_1/p_0 = 30/20 = 1.5 \]

Adjustment based on the extra utility concept:
- adjusted base price = 20x1.4 = 28
- adjusted price relative = 28/20 = 1.40

Adjustment based on the extra cost:
- adjusted new price = 30 - 3 = 27
- adjusted price relative = 27/20 = 1.35

Alternative procedure:
- adjusted base price = 20 + 3 = 23
- adjusted price relative = 23/20 = 1.15.
Looking at the methods of dealing with quality changes, presented above, we may conclude that none of them is perfect. Changing quality does create a problem, and each method has its own conditions and limitations. Substitution can be recommended, as the simplest solution, provided the conditions of application prevail. Price adjustment is also recommended, especially if the required information is available from reliable sources. Nevertheless, the best strategy is to reduce the likelihood of disturbing changes as much as possible.

The chain index is of course a superior formula from this point of view, because the weight-base is revised every year. Fixed weight Laspeyres price index, on the other hand, is a poor choice in this context, since the likelihood of quality changes increases with the passage of time. Nevertheless, the Laspeyres index is much more popular in practice, because other considerations are predominant in choosing the formula (see Appendix). Problems created by the changing quality can be taken into account when decision about the timing of regular revision and rebasing is made. The recommendation, cited in Section 3.4 is 5-10 years. Quality changes constitute a powerful argument in favour of the shorter period, such as five years, at least for the input price index, which is especially affected by this phenomenon. The output price index (index of prices received by farmers) is of course less affected by quality changes, due to the more stable character of agricultural commodities.

4.5 New products

In contrast to quality changes, new goods are not comparable with any product existing in the base period. It may happen, for example, that in a country no herbicides were used in the base period but relatively high quantities are introduced during the current period.

The first method for dealing with new product follows from the above; since the weight attached to the new good is anyhow zero, and no observable price change happened, the new items are simply neglected in the current run of the fixed weight index series. When the next revision comes all new items, "born" during the period between the two revisions, are examined and admitted into the commodity basket, if the volume of sales justifies this operation. The associated weights should be, of course, proportional to their share in the relevant commodity flow.

It must be added, that the strategy, described above, is valid for a price index, but not for a quantity index. There is no justification for ignoring the new products by excluding them from volume measures, notwithstanding the comparability problem. Since they exist in the current period, and contribute to the production (or sales, imports, etc.) they must be taken into account. Admission of new items into the Laspeyres volume index is usually done by means of imputed prices (see United Nations 1979/a, p. 32).

Returning now to the price index, the alternative strategy is introducing the new items into the index as they appear on the market (United Nations 1979/b, pp. 62-64). This is naturally much more complicated than the first strategy, i.e. waiting with the admission until the next revision. Two procedures have been proposed:

i) revising the series backwards to the base year, by introducing entirely fictitious data;

ii) neglecting the very principle of the fixed weight Laspeyres index by modifying the weighting pattern every year (even though slightly).

Neither of the above alternatives seems to be very attractive. They can be recommended especially for those countries, which keep the fixed weights during an extended period (e.g. ten years) without revision. As a result, many new items are "born" and some of them may gain market shares of such significance, that omitting them would bias the price index. Here again, we have an argument in favour of more frequent revisions, as it was pointed out in the last section.
CHAPTER 5. ANALYTICAL USES OF AGRICULTURAL PRICE INDICES

5.1 Domestic agricultural terms of trade

Ratios of price indices, showing the relative position of selected price trends, are frequently applied for the purpose of economic analysis. While many different index ratios can be defined, depending on the subject of the study, certain measures are regularly published and quoted. Their construction and interpretation will be explained in this section.

The example in Table 5.1 is presented to facilitate understanding of the parity ratio. In the base period one metric ton of maize was equivalent to 0.4 ton, or 400 kilogrammes of fertilizer. Given the price changes of both commodities, indicated in the table, one unit of maize became equivalent to only 0.360 unit of fertilizer in the current period. This means that the purchasing power of maize, in terms of fertilizer, decreased by 10 per cent. This is also indicated by the ratio of the relevant price relatives:

\[
\frac{112.5}{125.0} = 0.9
\]

Ratios of price relatives, such as the example presented above, are useful for throwing light on the particular price movements of selected pairs of commodities, especially agricultural commodity prices, compared to prices of inputs. However, they cannot indicate the general tendency of the agricultural commodities vis-à-vis agricultural requisites.

Table 5.1

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Unit of measurement</th>
<th>Price* in national currency</th>
<th>Price relative %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Base</td>
<td>Current</td>
</tr>
<tr>
<td>Maize</td>
<td>MT</td>
<td>320</td>
<td>360</td>
</tr>
<tr>
<td>Urea fertilizer</td>
<td>MT</td>
<td>800</td>
<td>1000</td>
</tr>
<tr>
<td>Urea fertilizer</td>
<td>per unit of maize</td>
<td>0.40</td>
<td>0.36</td>
</tr>
</tbody>
</table>

* Price received by farmer (producer price) for maize, and paid by farmer for the fertilizer.

The ratio which shows the general trend of the agricultural terms of trade is defined below:

\[
\text{Parity index} = \frac{\text{Index of prices received by farmers for agricultural output commodities}}{\text{Index of prices paid by farmers for input commodities}}
\]
Note that this is similar to the R ratio above, but the individual price relatives are replaced by the corresponding index numbers. The indices in the numerator and denominator of the parity index should have a common base period and they should be constructed according to the same formula to maintain comparability.

Parity index above one (100 per cent) means that prices received by farmers (producer prices of agricultural commodities), in relation to prices paid by the farmers to buy agricultural inputs, are on the average higher in the current period, than they were in the base period. This indicates favourable terms of trade for the farmers. Vice versa, ratios below one show that price changes are not favourable for the farmers. Consequently, the parity index is considered an important statistic for the formulation and monitoring of agricultural price policies, and it is regularly computed and published in many countries.

It was pointed out in Section 2.3 that the index of prices paid by farmers can also be defined according to a broad definition. If in calculating the parity index the broad definition of prices paid by the farmers is used, the interpretation changes: it shows the trend of the purchasing power of a given set of agricultural commodities versus all items demanded by the farmer, in his capacity as both producer and consumer.

In addition to the parity index as described above, other price index ratios can be defined, showing various aspects of price relations of interest to the agricultural sector. The index of prices received by farmers can be compared e.g. to the index of wholesale or retail prices of agricultural commodities, provided the indices are available in comparable form (base period, coverage, formula, etc.).

If the ratio e.g.

Index of prices received by farmers

Retail price index of agricultural commodities

is lower than one, it means that the trade margin has increased.

5.2 External terms of trade

The index ratios presented in the previous section are measures of relative price changes on the domestic market. External price relations can be studied in addition if the pertinent foreign trade price indices are available. Various measures of the terms of trade have been proposed (vide Meier 1963, pp. 40-63; Kindleberger 1956, pp. XIX-XX; and Appendix B). Discussion in this Manual will be confined to the simple statistic net barter terms of trade, showing the changes in the purchasing power of a given amount of exports in terms of imports.

Explanation of the terms of trade measure will be provided with reference to Table 5.1, but changing the definition of the data therein. Assume now that a country has a single export commodity: maize, and a single import item: urea fertilizer. Let prices in the first row of the table stand for export price f.o.b. per unit in US dollars, and in the second row for import price c.i.f. in US dollars (see United Nations 1981 for definitions of the foreign trade price categories).

According to this new interpretation the following conclusion can be derived from the data: the purchasing power of one unit of exports in terms of imports declined by 10 per cent between the base and the current period. We can now generalize by dropping the restriction of exports and imports composed of a single commodity, and define the net barter terms of trade as the ratio:

\[
TT = \frac{\text{Export price index}}{\text{Import price index}}
\]
TT above one (100 per cent) means that a given amount of exports in the current period can be exchanged against more imports (a bigger volume of imports) than in the base period. Terms of trade are considered favourable in this case. Vice versa, if TT is below one, the purchasing power of exports is declining.

The TT ratio, defined above, is not a specific indicator of the agricultural sector; however it can be considered as the external counterpart of the domestic parity ratio for developing countries whose exports are mainly agricultural commodities, while agricultural machinery, equipments, chemical products, etc. are heavily represented in the imports.

A ratio closer to this concept is

\[
\text{ATT} = \frac{\text{Export price index of agricultural commodities}}{\text{Import price index of manufactured products}}
\]

which is the main indicator of the external agricultural terms of trade.

5.3 **Deflator price indices**

Price index numbers are frequently used to deflate flows of goods and services, valued at current prices. Estimation of economic accounts at constant prices is an outstanding example of this application. The contents and structure of agricultural accounts and the procedure adopted for calculating the flows at constant prices is described in the FAO Handbook of Economic Accounts for Agriculture. Discussion in this section is confined to the deflation operation and the choice of index formula for this particular purpose.

Although the term "constant prices" does not mean necessarily base year prices, a flow of gross output values at constant prices is usually interpreted as

\[
\sum P_0 q_0, \quad \sum P_0 q_1, \quad \sum P_0 q_2, \quad \ldots, \quad \sum P_0 q_t.
\]

Scaled by the base year value, this yields a series of Laspeyres-type volume indices with fixed weights (see Sections 3.3 and 3.4 for the definition of symbols and concepts used in this section).

If complete information is available about all individual commodity prices and quantities, the series of values at constant prices can be compiled directly. The procedure is called direct repricing, and it is the preferred method, whenever feasible. This is more likely to be the case for agricultural commodities, which maintain stability and comparability over a long period. Agricultural requisites (inputs), on the other hand, par excellence machinery and chemical products, are subject to more frequent changes. Services (e.g. veterinary, communication, etc.) create special problems if the quantities are not available separately. Other methods should be used if the existing information is incomplete and does not permit repricing.

Assume first that complete information is available and consider the identities

\[
\sum P_0 q_t = \sum P_0 q_0 \frac{\sum P_0 q_t}{\sum P_0 q_0} = \left( \frac{\sum v_0}{P_0} \right) Q_L
\]

\[
\sum P_0 q_t = \sum P_t q_t / \frac{\sum P_t q_t}{\sum P_0 q_t} = \sum v_t / P_P
\]
We can drop the assumption of complete information at this stage and consider the Laspeyres volume index \( Q_L \) and the Paasche price index \( P_P \) as derived from a sample of price and quantity observations. Hence the statistics

\[ Q_L, \frac{E_V}{Q_L}, \text{ and } \frac{E_V}{P_P} \]

are estimators of the unknown sum of cross-products \( E_P Q_p \). Application of the first method is called quantity (volume) extrapolation and the second price deflation. We are interested in the second approach, as a special application of the price index.

It is pointed out in the Appendix that Paasche price index is the best formula for national accounting purposes. We can now understand the reason: it is precisely the deflator needed to estimate flows of goods and services at constant prices. However, it is also noted there, that the Laspeyres formula is adopted by most countries for constructing price index numbers. As a result, the price indices, at the disposal of the national accountants in many countries, are of the wrong kind, for the purpose of deflation.

A crude method, commonly used to cope with this problem, is to disaggregate the flows to the maximum detail and to deflate each subaggregate with the available price index (naturally Laspeyres). The resulting flows are added up to the level of the higher aggregate in the accounts (United Nations 1979/a, p. 19). In addition to the sampling error, this method of estimation involves bias, resulting from the application of the wrong formula. However, the bias might be negligible if the disaggregation yielded fairly homogeneous sub-aggregates.

The estimation of volume measures for value added flows, involving the deflation of intermediate consumption, demands other procedures, not discussed in this manual.

We may conclude that agricultural price indices may have various analytical applications, and deflation is one of them. Deflation of agricultural output or sales, however, requires, careful study of the coverage of the flow concerned, as well as the formula of the deflator.
REFERENCES


APPENDIX

INDEX NUMBER PROPERTIES AND CHOICE
AMONG THE FORMULAE

1. Two schools of index theory

There are two schools of index number theory, adopting the statistical and the economic (functional) approach to the index number problem, explained in Section 3.1. Indices proposed by the statistical school are based on actual price and quantity data, which are regarded as independent observations. In contrast, the economic theoretic school assumes that quantities are functions of prices.

History started with the pioneering work of Laspeyres and Paasche more than hundred years ago. They proposed price and volume index numbers, conceived as the average changes of prices and quantities respectively, thereby creating the foundations of the statistical school. Some fifty years later Irving Fisher introduced the test approach, requiring that the index satisfies certain plausible conditions. The axiomatic method, which emerged recently, can be considered as an improvement of the classical test approach (Eichhorn and Voeller 1983).

The origin of the economic school is marked by the works of Konus and Frisch during the twenties and thirties. More refined concepts and mathematical models have been proposed recently (Samuelson and Swamy 1974). According to this approach first the minimum cost of maintaining a constant utility (living standard) in two different periods (price situations) should be established, then the index can be computed as the ratio of the two costs. This ratio is considered as the "true" price index.

No matter how attractive this approach might appear from the theoretical point of view, enormous difficulties are encountered on the road to practical applications. In order to estimate a constant utility index, based on existing price and quantity data, a number of arbitrary assumptions and over-simplifications should be adopted, since neither utility, nor consumer preferences can be observed, directly. As a result, the form and the parameters of the functional relationship, which is of paramount importance, cannot be tested and verified (Jazairi 1983, p. 142; Hill 1982, pp. 25-34). Moreover, under the usual assumptions the traditional statistical indices prove to be quite good approximates of the theoretically "true" index numbers (Allen 1974, p. 69).

It is therefore comprehensible, that regularly published series of indices by national statistical offices are based on the methods proposed by the statistical school. International recommendations support the same approach. Indeed, the International Labour Office rejected the constant utility index as a replacement of the consumer price index (ILO 1962), and all United Nations' manuals on this subject suggest the traditional statistical formulae (UN 1977, 1979, 1981).

In view of the above, this publication is devoted to the statistical approach and all formulae presented in Chapter 3 and in this Appendix are based on actual price and quantity data.

2. Properties of index numbers

Index formulae are presented in Sections 3.3 and 3.4. The choice among them is dictated by the objective of index number construction. Desirable properties of the different formulae are reviewed first, and the formula with the best performance is accepted, the other rejected. This procedure is called test approach or axiomatic method.

Many desirable properties have been proposed (Eichhorn and Voeller 1983, pp. 417-418). The properties discussed below are selected in view of their special relevance in official statistics.
PROPORTIONALITY is a plausible condition, which requires that if the price of all commodities changes in equal proportion (price relatives are constant) the price index should show the same relative change, e.g. if all of the individual prices increased by 6 p.c. the price index must be 106 per cent. Similar condition can be formulated, mutatis mutandis, for the quantity index.

Both Laspeyres and Paasche indices satisfy this condition. This is evident, since they are averages of the individual price or quantity relatives. The Fisher formula passes the proportionality test a fortiori, being the geometric mean of the other two. In contrast, the chain index with variable weights fails to pass this test.

FACTOR REVERSIBILITY, or factor reversal test requires that

$$PQ = V$$

for any specific formula of price and quantity index.

The rationale of this test is that for any given individual commodity the transaction value is the product of the price and quantity data: $$v_i = p_i q_i$$ and it seems obvious to set the same condition for the index numbers representing the price and quantity movements. Neither Laspeyres nor Paasche formula comply with factor reversibility, but the Fisher formula does.

There is a weak version of this test, called simply factor test or product test. This requires only that the ratio of the value index over the price index should produce a volume index or vice versa:

$$V/P = Q \quad \text{or} \quad V/Q = P$$

where the resulting Q or P must be identifiable and acceptable as a quantity or price index respectively. Both Laspeyres and Paasche formulae satisfy the factor test.

The behaviour of the chain index with moving weights regarding the factor test follows from the above: it passes the weak factor test, provided the links are computed according to either Laspeyres or Paasche formula. The chain index satisfies even the strong factor reversibility condition, if the links are composed of Fisher indices.

TRANSITIVITY or circularity (circular test) demands that

$$I_{1/0} \times I_{2/1} = I_{2/0}$$

where $$I$$ represents either a price or a quantity index according to a specific formula and the subscripts stand for the periods compared. The condition involved in this test appears to be plausible, since the individual price or quantity relatives are certainly transitive, i.e. a direct comparison between 0 and 2 yields the same result as an indirect one via 1. Nevertheless, this is a much debated condition; T. Fisher himself, who proposed the circular test in 1911, reconsidered it in his book written in 1922 (Köves 1983, p. 181).

The chain index with moving weights satisfies transitivity by definition, i.e. the comparison $$t/0$$ is not performed independently, but through the chain of intermediate binary comparisons between successive periods $$1, 2, ..., t-1$$. The index with fixed weights, on the other hand, passes the test. Unfortunately, the links in the chain of binary comparisons between adjacent periods do not correspond to any of the recognized formulae. Strictly speaking, fixed weights should be used only for the construction of a series of comparisons over a stable weight-base period, in which case the result is an ordinary Laspeyres run. However, there is always a legitimate demand for measures of year-to-year changes and they are rather irregular if computed with fixed weights. Why should we use e.g. 1980 quantities to measure price changes between 1983 and 1984? Or vice versa why should 1980 prices be applied in a volume index showing the production changes between 1983 and 1984?
The argument presented above demonstrates, that the transitivity condition is rather controversial and open to argument, in spite of its apparent simplicity and desirability.

**ADDITIVE CONSISTENCY** or aggregation consistency is a test especially relevant for data sets arranged in groups or classes, where several sub-aggregates and higher aggregates exist. Indices derived from the sub-aggregates are considered components of the overall index, calculated at higher level of aggregation. The condition demands, that the components should "add up" to the change shown by the overall index. In formal terms let \( P_i \) stand for the price index of the \( i \)-th group, which can be put as a ratio of the corresponding sub-aggregates in the form

\[
P_i = \frac{A_i}{B_i} \quad (i = 1, 2, \ldots, g)
\]

The consistency condition requires that the overall price index, computed from individual price and quantity data according to the same formula as \( P_i \), irrespective of the classification, should be equal to the weighted average of the components:

\[
P = \frac{\sum_{i=1}^{g} A_i}{\sum_{i=1}^{g} B_i} = \frac{\sum_{i=1}^{g} B_i P_i}{\sum_{i=1}^{g} B_i}
\]

It is evident that additive consistency guarantees proportionality at sub-aggregate level, i.e. if all component indices are constant, the overall index is the same constant.

Both Laspeyres and Paasche formulae are consistent in this sense. In contrast, neither Fisher, nor the chain index with moving weights satisfy additive consistency. The fixed-weight index runs, on the other hand, are perfectly consistent, whether or not the reference period (base of the comparison) coincides with the weight-base.

**CHARACTERISTICITY** means that the weights correspond to the current state of the economy, i.e. do not become obsolete or out-dated with the passage of time. In other words, characteristicity requires timeliness; recent data should be used to compute the weights.

This condition clearly favours the Paasche formula in a binary comparison and moving weights in an index run, whether or not chained. On the other hand, fixed weights cannot be considered characteristic, especially if they are maintained in the long run without revision and rebasing.

3. **The choice of the formula**

The table below entitled "Choice of index numbers" shows various types of index numbers versus a list of desirable properties (tests). The symbol X indicates that the index passes the test marked in the box-head of the table. Such a table may serve as an instrument for the selection of the appropriate formula. Obviously, both the typology and the set of requirements can be modified, or enlarged, by accommodating items not listed in this table, although it is hoped that the most important and relevant items are included.
CHOICE OF INDEX NUMBERS

<table>
<thead>
<tr>
<th>Type of index</th>
<th>Proportionality</th>
<th>Factor reversal</th>
<th>Transitivity</th>
<th>Additive consistency</th>
<th>Characteristicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laspeyres</td>
<td>fixed</td>
<td>not</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Laspeyres</td>
<td>moving</td>
<td>yes</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Paasche</td>
<td>moving</td>
<td>not</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Paasche</td>
<td>moving</td>
<td>yes</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Fisher</td>
<td>moving</td>
<td>not</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

The first conclusion we can draw from the table is rather disappointing: there is no single formula which passes all tests. The reason is that some tests are in conflict with others. E.g. proportionality and transitivity seem to be mutually exclusive. Similarly, the strong factor reversibility excludes both transitivity and additive consistency.

In the absence of a perfect formula, satisfying all conditions, the choice must be guided by the objective, and a selective strategy applied. While all the properties discussed in the previous section seem to be desirable, we must nevertheless establish a preference between the conflicting pairs, taking into account the specific objective or users’ demand.

For the purpose of compiling national accounts at constant prices, both factor reversibility and additive consistency are fundamental. Factor reversibility is indispensable since the change in the current value of a given flow of goods must be allocated to quantity and price components. Additive consistency is necessary because the accounts are usually disaggregated according to the national industrial classification of activities, and the entries in the accounts should add up to their totals in constant prices as well as in current prices. Laspeyres formula with fixed weights and Paasche with moving weights (not chained) satisfy both conditions, although only the weak factor condition is satisfied. However, the Paasche formula is preferred if a price index is applied as a deflator, because it yields a Laspeyres volume measure (see United Nations 1979, pp. 17-19, and United Nations 1977, p. 21).

Short or medium-term economic analysis and forecasting generates different requirements. Characteristicity is of primary importance, since obsolete weights may compromise the measurement of current changes. All types of indices with moving weights satisfy this requirement, whether or not chained. If transitivity is added as a condition to render year-to-year changes consistent with comparison over longer time spans, only the chain indices with moving weights remain. The chain index has, indeed other attractions; it can be considered as an empirical approximation of the theoretical Divisia index, which is the limiting process of a chain, composed of discrete links, when the time periods are infinitely small. A practical advantage of chaining is that problems of comparability (new and disappearing products, quality changes, etc.) are less likely to occur if adjacent years are compared.

The construction of historical series for long-term planning and econometric models demands proportionality and additive consistency, especially if the time series represent different levels of aggregation, or classification. The ideal tool is therefore a Laspeyres index with fixed weights.
The examples cited above illustrate that different objectives are associated with different and sometimes conflicting requirements. This makes the position of a national statistical office all the more difficult, because it should satisfy all legitimate user's demand concurrently. A possible solution is of course to calculate specific purpose indices according to the need of the major users, if resources permit this operation. However, the publication of different figures answering apparently the same question may create confusion. The other alternative is to construct a "multi-purpose" or general index, which satisfies every user, if not perfectly, but at least sufficiently. In other words, a compromise formula must be found among the conflicting requirements.

Laspeyres index with fixed weights may be regarded as a good compromise and it is therefore recommended, provided revision and re-basing takes place regularly. Indeed, the more frequent revision, the closer the series approximate a run of chain indices with moving weights. As a result, frequent revision renders the Laspeyres series better for short-term analysis, while a less frequent revision for the long-term use. The 5 to 10 years interval, recommended in Section 3.4 seems to be in line with the multi-purpose character of this index. Another advantage of this choice is cost efficiency; during the period between revisions there is no need to compile and process quantity data, only prices should be recorded. On the other hand, the Laspeyres price index is not the ideal deflator for the current price flows in the national accounts, because it yields a Paasche volume index, whereas the opposite is normally required, as it has been pointed out above. A technique to overcome this problem was recommended by the UN (see United Nations 1979/a, pp. 19-20).

Taking into account the desirable properties of the Laspeyres formula it is not surprising that this index is very popular in national practice. This preference is documented in a recent FAO report: out of 35 countries providing information on the formula of the index of prices received by farmers 30 use Laspeyres, 2 both Laspeyres and Paasche, and only the remaining 3 countries use other formulae (see FAO 1986, pp. 25-28, Table 5).