8. PREDICTION MODELS

Future yields and stock biomass levels can be predicted by means of mathematical models which are similar to the ones behind the virtual population analysis, VPA, and the cohort analysis (see Chapter 5). The mathematical formulas used for VPA and cohort analysis, which analyse the history of a fishery, can be transformed in such a way that the knowledge of the past can be used to predict future yields and biomass at different levels of fishing effort. In other words, these models can be used to forecast the effects of development and management measures, such as increases or reductions of fishing fleets, changes in minimum mesh sizes, closed seasons, closed areas, etc. Therefore these models form a direct link between fish stock assessment and fishery resource management.

The prediction models can also incorporate aspects of prices and value of the catch, which make them suitable as a basis for bio-economic analyses, where biological and economic inputs are used to predict future yields, biomass levels and value of the catch under all kinds of assumptions. This chapter contains only a very basic introduction to bio-economic aspects, and for further studies the reader is referred to Sparre and Willmann (1992).

The first prediction models were already developed in the thirties by Thompson and Bell (1934). However, due to the large number of calculations required the so-called "*Thompson and Bell model*" did not reach a high popularity until the introduction of computers.

In the meantime a simpler model, based on rigorous assumptions, but therefore requiring less calculations was developed by Beverton and Holt (1957). Their "*Yield per Recruit*" model was widely used, but now it has been replaced by the Thompson and Bell model in regions where VPA and cohort analysis are being applied.

Beverton and Holt's yield per recruit model can be regarded as a special application of the Thompson and Bell model, which means that any general conclusion derived from it also holds for the Thompson and Bell model.

Although it is not likely that the Beverton and Holt model will be much used in the future, it has been incorporated in this chapter for historical and didactical reasons. The yield per recruit model is suited for calculators and therefore for the demonstration of certain aspects of fish stock assessment. The second part of this chapter (Sections 8.6 to 8.8) deals with the Thompson and Bell model, age-based and length-based and with aspects of gear selectivity related to the model.

The final purpose of the use of the predictive models is to provide those responsible for the management of fishery resources with information on the biological and/or economical effect of fishing on the stock. The managers are then expected to take measures that will lead to a level of exploitation of the resources where the maximum yield is obtained, either in the biological or in the economical sense, on a sustainable basis, i.e., without causing damage to the stocks that would affect future yields.

The managers should try to prevent situations where the fishing pressure becomes too high and where stocks are "*overfished*". An exact prediction of future yields is usually not possible, because stocks are seldom in a "steady state" which is assumed to exist for the application of many models. It has been demonstrated (Sharp and Csirke, 1984) that the abundance of certain stocks, in particular small pelagic species occurring in upwelling areas, usually depends very much on environmental factors which are beyond the control of any human interference. In such cases the predictive value of the models described below is practically nil. However, in the case of some pelagic and most demersal fisheries for fish and shrimp the models have proved to be extremely useful.

Before going deeper into these two models, it is worthwhile to first consider a simpler model and to discuss the concept of overfishing.

The classical model describing a fishery on a particular stock is the one given by Russell (1931). The model is in the form of a "black box" that represents what Ricker (1975) has defined as the "usable stock", the weight of all fish larger than a minimum useful size. The inputs to the usable stock are the weight of the new recruits and the growth of the fish already forming part of the stock. The outputs are the deaths by natural causes and the yield (catch in weight) of the fishery.

In an unfished stock the combined inputs are, on the average, equal to the removal of biomass by natural deaths. When a population is being fished it has an effect on all other factors, viz., there will be a greater rate of recruitment, a faster growth and reduced natural deaths. This is because fishing creates "room" for more new recruits, it removes the large slow-growing fish which are replaced by smaller fast-growing fish, and it removes fish before they can die of old age or other natural causes. A fishery, therefore, has a stimulating effect on the production of fish, as long as the stock is given enough time to adjust to the new situation and as long as the fishing pressure does not become too heavy. When "*overfishing*" occurs, the growth of individual fish cannot keep pace with the deaths caused by fishing and when it becomes very severe it also affects recruitment. Therefore, there are two types of overfishing, viz., "*growth overfishing*" and "*recruitment overfishing*". Growth overfishing occurs when the effort is so high that the total yield decreases with increasing effort. The fish are caught before they can grow to a sufficiently large size to substantially contribute to the biomass. In general, it is reasonable to say that a stock is growth overfished in the biological sense if F exceeds F_{MSY} (cf. Fig. 8.2.3). The term "biological" is used to indicate that only the yield in terms of biomass measured in weight units is considered. Thus, the value of the yield and the cost of fishing are being disregarded here.

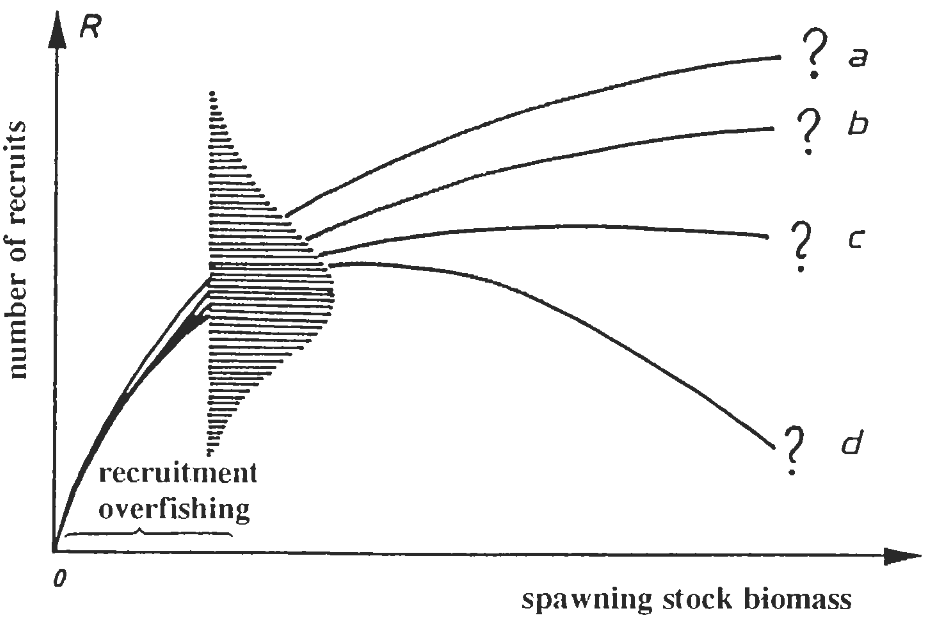

Fig. 8.0.1 **The stock-recruitment relationship in connection with the concept of "recruitment overfishing"**

In order to understand the concept of recruitment overfishing, it is necessary to first consider the relationship between recruitment and the spawning stock biomass as illustrated in Fig. 8.0.1. As indicated by the question-marks, this relationship is not well understood. The only point that is known for sure is (0,0), i.e., when there is no parent stock there can be no offspring. It is then reasonable to assume that at low levels of the parent stock there is a direct positive linear relationship with the number of offspring or recruitment. Under normal conditions such a direct "linear" relationship is not noted, but when it occurs, it means that the parent stock has come down to a very low level and in that case we speak of recruitment overfishing.

8.1 ASSUMPTIONS AND MODELS UNDERLYING THE YIELD PER RECRUIT MODEL OF BEVERTON AND HOLT

The yield per recruit model (Beverton and Holt, 1957) is in principle a "*steady state model*", i.e., a model describing the state of the stock and the yield in a situation when the fishing pattern has been the same for such a long time that all fish alive have been exposed to it since they recruited. There are extensions to the model dealing with the transitory phase after a change in the fishing pattern, but these are seldom used because models of the Thompson and Bell type (Sections 8.6 to 8.8) provide a simpler description of non-steady state situations.

The rigorous assumptions underlying the Beverton and Holt approach are:

1. Recruitment is constant, yet not specified
2. All fish of a cohort are hatched on the same date
3. Recruitment and selection are "*knife-edge*" (see Chapter 6)
4. The fishing and natural mortalities are constant from the moment of entry to the exploited phase
5. There is a complete mixing within the stock
6. The length-weight relationship (Eq. 2.6.1) has the exponent 3, i.e. $W = q*L^3$

One of the models behind the Beverton and Holt model is the exponential decay model that was introduced in Section 4.2, and mathematically expressed in Eq. 4.2.2. The definitions and terminology introduced in Section 4.1 are also applicable to the Beverton and Holt model (e.g. Tc,Tr,R = N(Tr)).

The assumed life history of a cohort in the Beverton and Holt model, as shown in Fig. 8.1.1 is as follows:

1) At age Tr all fish belonging to a given cohort recruit to the fishing grounds at the same time: "*knife-edge recruitment*".

2) From age Tr to age Tc the cohort is not exposed to any fishing mortality. (It is assumed that all fish of ages between Tr and Tc escape through the meshes if they enter the gear.) Thus, in that period they suffer only from natural mortality, M, which is assumed to remain constant throughout the life span of the cohort.

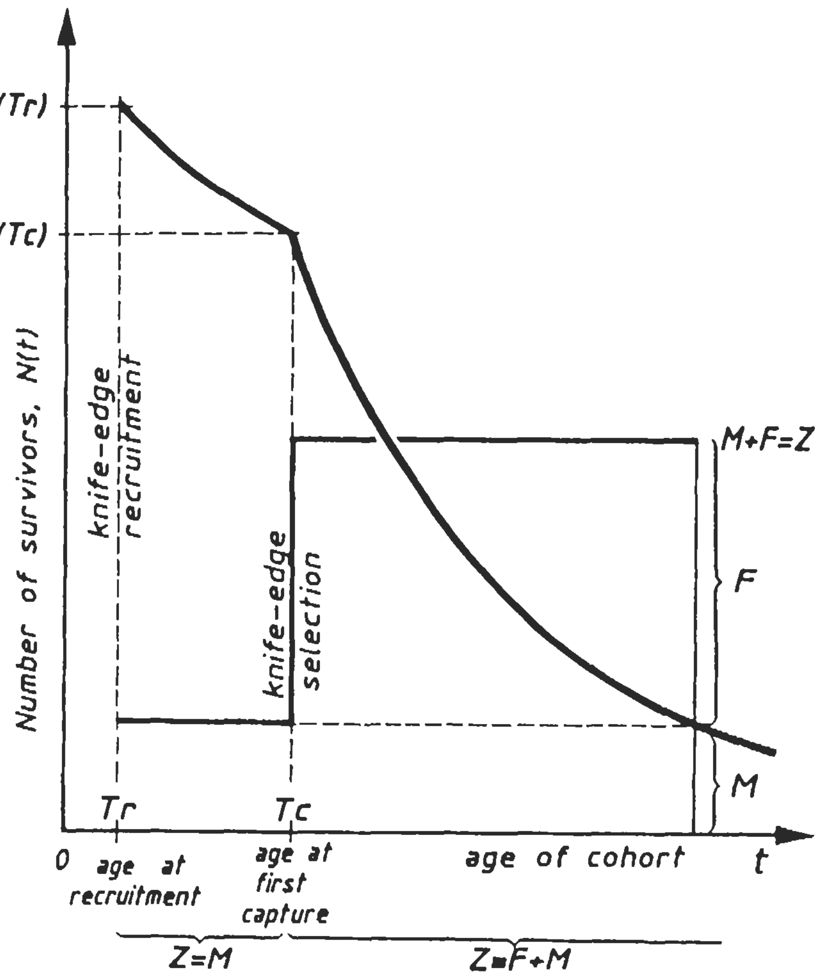

Fig. 8.1.1 **The life history of a cohort as assumed in the Beverton and Holt model**

3) At age Tc, the "*age-at-first-capture*", the cohort is assumed to be suddenly exposed to full fishing mortality, F, which is assumed to remain constant for the rest of the cohort's life. The sigmoid shaped gear selection curve introduced in Chapter 6 is approximated by the so-called "*knife-edge selection*" (see Fig. 6.4.1.1). The catch from the cohort is therefore assumed to be zero until the cohort has attained the age Tc.

The number of survivors at age Tr is the recruitment to the fishery:

$$R = N(Tr) \qquad (8.1.1)$$

The number of survivors at age Tc is:

$$N(Tc) = R*\exp[-M*(Tc-Tr)] \qquad (8.1.2)$$

The number of survivors at age t, where t > Tc, is:

$$N(t) = N(Tc)*\exp[-(M+F)*(t-Tc)] = R*\exp[-M*(Tc-Tr) - (M+F)*(t-Tc)]$$

The fraction of the total recruitment N(Tr) or R surviving until age t is obtained by dividing both sides of the equation by R and becomes:

$$N(t)/R = \exp[-M*(Tc-Tr) - (M+F)*(t-Tc)] \qquad (8.1.3)$$

This means that Eq. 8.1.3 gives the number of fish at time t "per recruit", i.e. as the fraction of each fish that recruited to the fishery.

The other model underlying the Beverton and Holt model is the "catch equation" in the form of Eq. 4.2.10 as explained in the next section.

8.2 BEVERTON AND HOLT'S YIELD PER RECRUIT MODEL

To derive the mathematical expression for Beverton and Holt's yield per recruit model we take (as usual) a starting point in the catch equation in the form of Eq. 4.2.10:

$$C(t,t+\Delta t) = \Delta t*F*N(t) \qquad (8.2.1)$$

Eq. 8.2.1 gives the number of fish caught from a cohort, in the time period from t to t+Δt when Δt is small. To get the corresponding yield in weight, this number should be multiplied by the individual weight of a fish. If Δt is small, then the body weight of a fish will remain approximately constant during the time period from t to t+Δt, and the yield becomes:

$$Y(t,t+\Delta t) = \Delta t*F*N(t)*w(t) \qquad (8.2.2)$$

where w(t) is the body weight of a t years old fish, as defined by the weight-based von Bertalanffy equation (Eq. 3.1.2.1). To get the yield per recruit for the time period from t to t+Δt Eq. 8.2.2 is divided by the number of recruits, R:

$$\frac{Y(t,t+\Delta t)}{R} = F*\frac{N(t)}{R}*w(t)*\Delta t \qquad (8.2.3)$$

where N(t)/R is defined by Eq. 8.1.3.

Eq. 8.2.3 is the "Beverton and Holt model for a short time period". To get the total yield per recruit for the entire life span of the cohort, Y/R, all the small contributions defined by Eq. 8.2.3 must be added up:

$$\begin{aligned}Y/R = {} & Y(Tc,Tc+\Delta t)/R + Y(Tc+\Delta t,Tc+2\Delta t)/R + \\ & Y(Tc+2\Delta t,Tc+3\Delta t)/R + Y(Tc+3\Delta t,Tc+4\Delta t)/R + \\ & Y(Tc+4\Delta t,Tc+5\Delta t)/R + Y(Tc+5\Delta t,Tc+6\Delta t)/R + \\ & \ldots. \\ & \ldots. \\ & + Y(Tc+(n-1)*\Delta t,Tc+n*\Delta t)/R\end{aligned}$$

where "n" is some large number, so large that the number of fish older than Tc+n*Δt, i.e., N(Tc+n*Δt), is so small that it can be ignored.

The next step is to convert the sum written above into a form which can easily be calculated. If the number of terms, n, in the sum is large (and it must be large to make the approximation for w(t) a reasonable one) it will take a long time to do this summation. However, by using a long series of mathematical derivations of which an explanation is outside the scope of this manual, one can show that the sum above can be written in a more convenient way as:

$$Y/R = F*\exp[-M*(Tc-Tr)]*W_{\infty}*\left[\frac{1}{Z} - \frac{3S}{Z+K} + \frac{3S^2}{Z+2K} - \frac{S^3}{Z+3K}\right] \qquad (8.2.4)$$

where:

```
S  = exp[-K*(Tc-t_0)]
K  = von Bertalanffy growth parameter
t_0 = von Bertalanffy growth parameter
Tc = age at first capture
Tr = age at recruitment
W∞ = asymptotic body weight
F  = fishing mortality
M  = natural mortality
Z  = F+M, total mortality
```

Eq. 8.2.4 is the "*Beverton and Holt yield per recruit model*" (1957), Y/R model, written in the form suggested by Gulland (1969). Although the equation looks complicated, it is quite easy to handle with a programmable pocket calculator.

Because Beverton and Holt express yields on a "per recruit basis", the yields are relative, i.e. relative to the recruitment. If, say, a recruitment of 100 million fish gives a yield of 10 000 tonnes then 200 million recruits would yield 20 000 tonnes according to the model. This assumption may appear trivial, but it is not, as one could well imagine that the more abundant a species becomes the worse the conditions for the individuals will be, due to, for example, food competition and cannibalism. The results of the model are expressed in units of yield per recruit (grams per recruit). In the example above the yield per recruit becomes:

$$\frac{10\ 000\ 000\ 000}{100\ 000\ 000} = 100 \text{ grams per recruit}$$

The model allows us to calculate Y/R with varying inputs of the different parameters, such as F and Tc and then assess which effect the various input values have on the yield per recruit of the species under investigation. It is important to note here that the two parameters F and Tc are those which can be controlled by fishery managers, because:

1) F is proportional to effort (cf. Eq. 4.3.0.7)

2) Tc is a function of gear selectivity

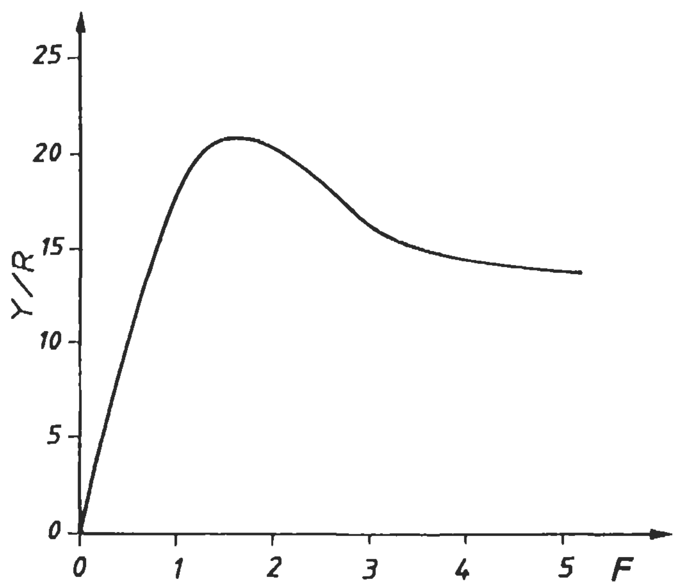

Fig. 8.2.1 Result of a yield assessment with the yield per recruit model

Therefore, Y/R is considered a function of F and Tc. Most often you will see Y/R plotted against F (or effort).

Fig. 8.2.1 shows the result of a yield assessment with the yield per recruit model. The age-at-entry to the exploited phase, Tc, is kept constant. The independent variable is the effort as expressed by the coefficient F of fishing mortality. The dependent variable is the annual yield in grams per recruit. When the total annual yield is known in a steady state situation, for a given value of F, then the number of recruits can be calculated by dividing the total yield by the yield in grams per recruit.

The "*yield per recruit curve*" often has a maximum: the "*maximum sustainable yield (MSY)*". The position of the maximum depends on the age-at-first-capture, Tc, which in turn depends on the mesh size used in the fishery.

A change of mesh size, Tc, leads to a different MSY. Fig. 8.2.2 shows three curves with different Tc's. The highest MSY is reached for the highest value of Tc, at a slightly higher level of effort, F. By combining a range of values of Tc with a range of values of F, the highest maximum sustainable yield, valid for a certain combination of Tc and F, can be determined. The term sustainable means that that yield can be maintained "forever" as long as the conditions do not change. Higher yields may be obtained by a sudden increase in effort, but cannot be maintained, and they will have to be followed by a period of much lower yields. This is always on the assumption that nothing else has changed.

The Y/R model is originally an age-based model, but age is easily converted into length, again using the principles of the conversion of the catch curve (see Sections 4.4.2 and 8.5).

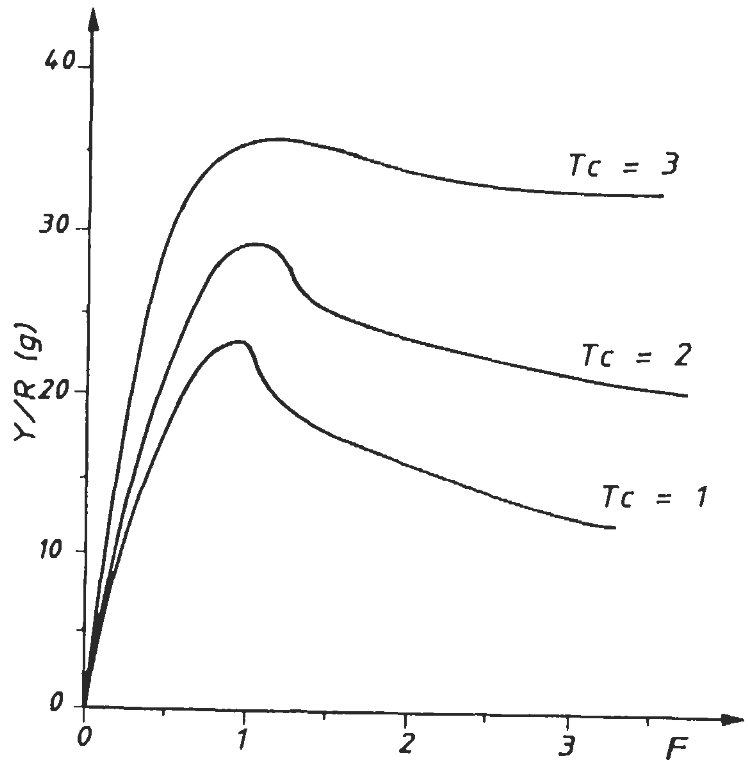

Fig. 8.2.2 **Yield per recruit curves with different ages of first capture (Tc)**

Example 28: Y/R as a function of F, for a tropical species

As an example we calculate the Y/R for *Nemipterus marginatus* as a function of F, using the following parameters:

$K = 0.37$ per year $\quad Tc = 1.0$ year $\quad t_o = -0.2$ year
$M = 1.1$ per year $\quad Tr = 0.4$ year $\quad W_\infty = 286$ grams

We start by calculating the terms in Eq. 8.2.4 which are independent of F:

$$S = \exp[-K*(Tc-t_o)] = \exp[-0.37*(1.0+0.2)] = 0.6415$$

$$3S = 1.9244 \; , \; 3S^2 = 1.2344 \; , \; S^3 = 0.2639$$

$$M+K = 1.47 \; , \; M+2K = 1.84 \; , \; M+3K = 2.21$$

$$\exp[-M*(Tc-Tr)]*W_\infty = \exp[-1.1*(1.0-0.4)]*286 = 147.8$$

Inserting these values into Eq. 8.2.4 we get:

$$Y/R = F*147.8*\left[\frac{1}{F+1.1} - \frac{1.9244}{F+1.47} + \frac{1.2344}{F+1.84} - \frac{0.2639}{F+2.21}\right]$$

To produce a graph corresponding to Fig. 8.2.1 this expression must be evaluated for a range of F-values, giving a sufficiently large number of points to fit the curve by eye.

For example, for F = 0.5:

$$Y/R = 0.5*147.8*\left[\frac{1}{0.5+1.1} - \frac{1.9244}{0.5+1.47} + \frac{1.2344}{0.5+1.84} - \frac{0.2639}{0.5+2.21}\right]$$

$$= 0.5*147.8*0.0785 = 5.8 \text{ grams per recruit}$$

Table 8.2.1 Yield per recruit and average biomass per recruit of *Nemipterus marginatus* as a function of F. Parameters as indicated in the legend of Fig. 8.2.3

F	Y/R	B/R	B/R as % of Bv	F	Y/R	B/R	B/R as % of Bv
0.0	0	22.4 = Bv	100	1.3	7.66	5.9	26
0.1	1.92	19.2	86	1.5	7.79	5.2	23
0.2	3.33	16.7	75	1.7	7.86	4.6	21
0.3	4.38	14.6	65	1.9	7.90	4.2	19
0.4	5.18	13.0	58	2.1	7.92	3.8	17
0.5	5.79	11.6	52	2.3 F(MSY)	7.93 MSY/R	3.5 MSB/R	15 MSB/Bv
0.6	6.26	10.4	46	2.5	7.92	3.2	14
0.7	6.62	9.5	42	3.0	7.88	2.6	12
0.8	6.91	8.6	38	4.0	7.77	1.9	8
0.9	7.14	7.9	35	5.0	7.66	1.5	7
1.0	7.32	7.3	33	6.0	7.57	1.3	6
1.1	7.46	6.8	30				

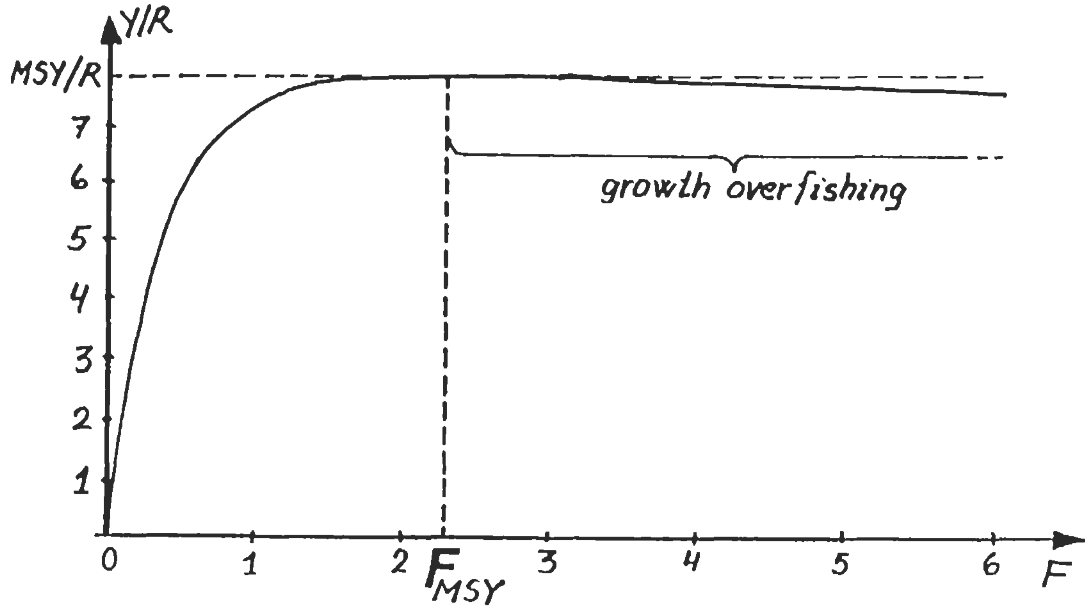

Fig. 8.2.3 Yield per recruit curve of *Nemipterus marginatus* as a function of F for the parameters:

K = 0.37 per year	Tc = 1.0 year	t_0 = -0.2 year
M = 1.1 per year	Tr = 0.4 year	W_∞ = 286 grams

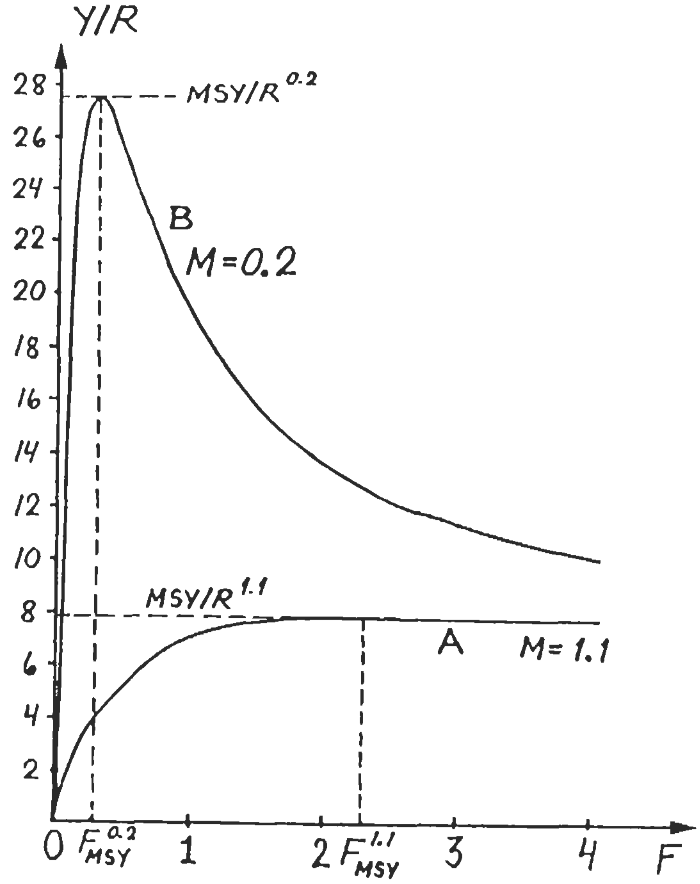

Fig. 8.2.4 Yield per recruit curve as a function of F for the parameters:

A: M = 1.1 per year	B: M = 0.2 per year	
K = 0.37 per year	Tc = 1.0 year	t_0 = -0.2 year
Tr = 0.4 year	W_∞ = 286 grams	

Repeating this calculation for F values ranging from F = 0 to F = 6.0 produces the results given in the first and fourth columns of Table 8.2.1, which have been used to produce the graph shown in Fig. 8.2.3.

By testing various F-values it is found that F = 2.3 gives the maximum value of Y/R, the "*Maximum Sustainable Yield per Recruit*" (MSY/R):

MSY/R = 7.9 grams per recruit

which corresponds to the biologically optimum fishing mortality:

F_{MSY} = 2.3 per year (see Table 8.2.1 and Fig. 8.2.3).

Because the Y/R-model assumes a constant parameter system (cf. Section 4.4.1) the results to be read from the curve only apply after the system has had constant parameters for a while. When F is changed it takes some time before the Y/R becomes the one predicted by the curve. How long this transient period is depends on the longevity of the species in question. From Table 8.2.1 and Fig. 8.2.3 it appears that the F_{MSY}-level is not determined with any great precision. Actually, for F > 1.5 the Y/R remains the same for a wide range of effort.

Curve B in Fig. 8.2.4 is an example of a Y/R-curve which differs in shape from the one in Fig. 8.2.3 (which is reproduced as curve A). Curve B has a pronounced maximum, it has a lower value of F_{MSY} and a higher value of MSY/R compared to curve A. The only difference in the input values of the two curves is the value of the natural mortality rate, M, viz. M = 0.2 per year for curve B and M = 1.1 per year for curve A. The conclusions that can be be drawn from the differences between these two curves are:

1. A lower M produces a lower F_{MSY} and a higher MSY/R, while fishing effort levels above F_{MSY} lead to a severe reduction of the total yield

2. If M is high it is difficult to estimate F_{MSY} by the Y/R curve

These conclusions are the logical consequences of the effect of the level of natural mortality, M, on the production of biomass.

If M is high, the fish will soon reach the age where losses due to natural mortality exceed the gain in biomass due to growth. Therefore, F has to be high to catch the fish before they die of natural causes.

If M is low the gain in biomass due to growth will exceed the losses caused by natural mortality for a large part of the cohort's life span. In that case, it pays to let the fish grow to a large size and that means that for a biologically optimum exploitation F should be low.

In some cases (cf. Exercise 8.3) the Y/R-curve does not even have a maximum and an inexperienced management might come to the wrong conclusion that effort should be increased indefinitely. In such cases, which are common in tropical fisheries, it is recommended to look also at the biomass per recruit (B/R) curve, which is introduced in the next section. The two curves provide different information and it is therefore advisable to always plot them together.

The Y/R as a function of Tc, age-at-first-capture, is closely related to the estimation of the optimum mesh size (cf. Fig. 8.2.2, see Exercises 8.3 and 8.4).

8.3 BEVERTON AND HOLT'S BIOMASS PER RECRUIT MODEL

Beverton and Holt's biomass per recruit model expresses the annual average biomass of survivors as a function of fishing mortality (or effort). The average biomass is related to the catch per unit of effort (cf. Section 4.3). Eq. 4.3.0.2 expresses the relationship between CPUE and numbers caught, CPUE(t) = q*N(t), which multiplied by the body weight on both sides gives:

$$CPUE(t)*w(t) = q*N(t)*w(t)$$

or if

N(t)*w(t) is replaced by B(t), the symbol for biomass:

$$CPUEw(t) = q*B(t) \qquad (8.3.1)$$

where CPUEw is the "weight of the catch per unit of effort". Thus, the biomass is expected to show the same decline with increasing fishing effort as the CPUEw curve shown in Fig. 8.3.1.

The catch in numbers per year can be expressed as

$$C = F*\bar{N}$$

(cf. Eq. 4.2.8 with t2-t1 = 1 year). By a similar argument it can be shown that the yield per year is

$$Y = F*\bar{B}$$

where $\bar{B}$ is the average biomass in the sea during a year. It follows that:

$$\frac{\bar{B}}{R} = \frac{Y}{R}*\frac{1}{F}$$

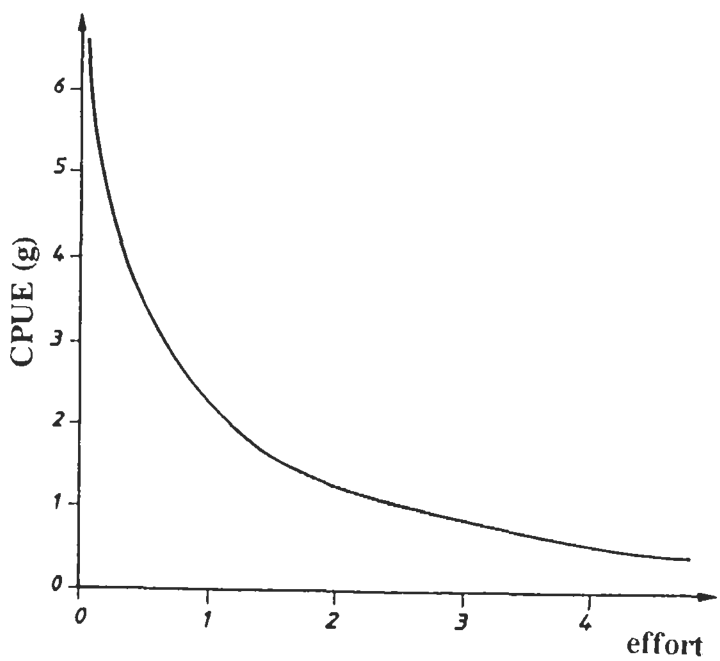

Fig. 8.3.1 Curve of CPUE (in weight) as a function of effort

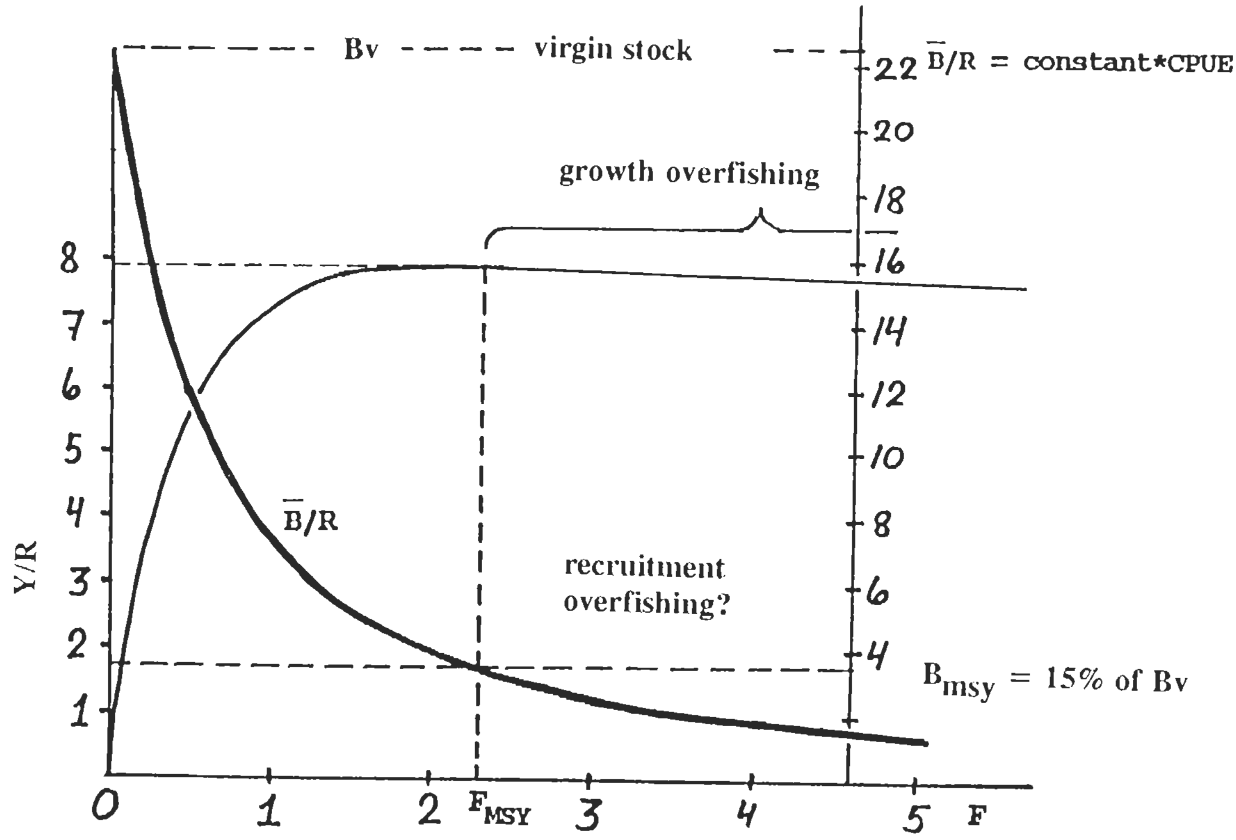

Fig. 8.3.2 **Biomass per recruit curve of *Nemipterus marginatus* corresponding to the Y/R-curve of Fig. 8.2.3 which is repeated here**

Because of the assumption of a constant parameter system (see Section 4.4.1) the yield from a stock during one year is equal to the yield from a single cohort during its life span.

Therefore we have the following simple relationship between Y/R, (Eq. 8.2.4) and average biomass per recruit, $\bar{B}/R$:

$$Y/R = F*\bar{B}/R \qquad (8.3.2)$$

The formula used to calculate $\bar{B}/R$ is the same as Eq. 8.2.4, divided by F:

$$\bar{B}/R = \exp[-M*(Tc-Tr)]*W_\infty*\left[\frac{1}{Z} - \frac{3S}{Z+K} + \frac{3S^2}{Z+2K} - \frac{S^3}{Z+3K}\right] \qquad (8.3.3)$$

It is recommended to always calculate Y/R and $\bar{B}/R$ together. The easiest way to do so is to start by calculating $\bar{B}/R$ by Eq. 8.3.3 and then use Eq. 8.3.2 to calculate Y/R. In the case of F = 0, the value of $\bar{B}/R$ is the so-called virgin biomass per recruit, Bv/R, the biomass of the unexploited stock.

The average biomass per recruit as defined by Eq. 8.3.2 or Eq. 8.3.3 is the average biomass of the **exploited** part of the cohort, i.e. the biomass of fish of age Tc or older.

The $\bar{B}/R$ values related to the Y/R values calculated in Section 8.2 are presented in Table 8.2.1, where also $\bar{B}/R$ is given as a percentage of the virgin biomass, Bv. It shows that for *Nemipterus marginatus* the biomass corresponding to the biologically optimum F level, F_{MSY}, is only 15% of the virgin biomass, Bv. Fig. 8.3.2 shows the "*biomass per recruit curve*" which is always decreasing with increasing effort. The curve is proportional to the catch per unit of effort on the assumption underlying the model (see Eq. 8.3.1). This means that in any fishery one should expect a decrease in the catch per unit effort and the biomass when effort (e.g., the number of boats) increases. A decrease in the catch per unit effort is, therefore not *per se* an indication that a stock is overfished. Overfishing occurs when the fishing effort becomes so high that the growth cannot balance the death process.

Another, and sometimes more appropriate, way of using the $\bar{B}/R$-curve is to interpret it as a CPUEw-curve. When managing a fishery, considerations on the possible income per boat are essential and this quantity of course is closely related to CPUEw (see Sparre and Willmann, 1992).

Mean age and size in the yield

Mean age, mean length and mean weight in the annual yield are easily estimated under the assumption of Beverton and Holt that Z is constant from a certain age Tc (Tr when Tr > Tc). The mean age in the annual yield is

$$\overline{Ty} = \frac{1}{Z} + Tc \qquad (8.3.4)$$

The formula is also applicable in a situation when Z varies in the exploited phase, but is constant for the oldest fish. The mean age of the old fish is then 1/Z plus the age at which Z becomes constant.

The mean length in the annual yield is

$$\overline{Ly} = L_\infty \left[1 - \frac{Z*S}{Z+K}\right] \qquad (8.3.5)$$

$$S = \exp(-K*(Tc-t_o)) = 1 - Lc/L_\infty$$

Again Tc or Lc can be replaced by any age from which the fish have a constant mortality rate, to give the mean length in that part of the population.

Similarly, the mean weight in the annual yield is

$$\overline{Wy} = Z*W_\infty*\left[\frac{1}{Z} - \frac{3*S}{Z+K} + \frac{3*S^2}{Z+2*K} - \frac{S^3}{Z+3*K}\right] \qquad (8.3.6)$$

with S the same as in Eq. 8.3.5. The expression may be applied to the plus group in VPA etc. by replacing Tc or Lc by the age or length at entry to the plus group, see Eq. 5.3.16.

$\overline{Ty}$, $\overline{Ly}$ and $\overline{Wy}$ as well as the exploited biomass and the catch per unit effort always decrease with increasing Z, i.e., with effort, see Fig. 8.3.3. The decrease is faster for low values of F (i.e., effort) as found in the first stages of a new fishery. Tc (determined by the mesh size) is a parameter in all three equations. A larger mesh gives a higher mean age and larger size in the catch.

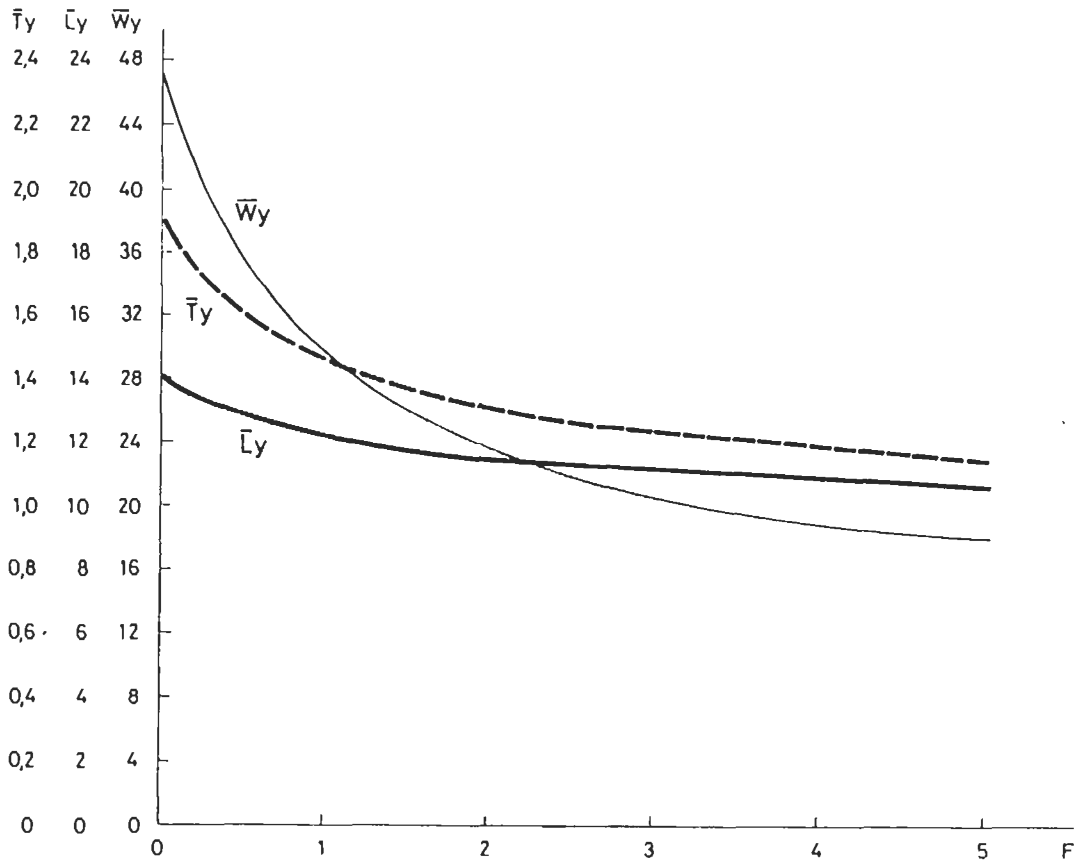

Fig. 8.3.3 Mean age ($\bar{T}y$), length ($\bar{L}y$) and weight ($\bar{W}y$) in the annual yield of *Nemipterus marginatus* corresponding to the biomass (CPUE) curve of Fig. 8.3.2

Formerly, a decreasing catch per unit effort and a decreasing size of the landed fish were often assumed to indicate overfishing, but, as is clearly shown in Figs. 8.3.2 and 8.3.3, these decreases are a logical consequence of increasing the fishing effort. An important aspect, however, is that the decreases are more pronounced at the lowest effort levels. This means that a newly developed fishery on a virgin stock will almost immediately lead to lower catch rates and smaller sizes of fish. These factors have often been overlooked in feasibility studies for developing fisheries.

(See **Exercise(s)** in Part 2.)

8.4 BEVERTON AND HOLT'S RELATIVE YIELD PER RECRUIT MODEL

For fisheries management purposes, it is important to be able to determine changes in the Y/R for different values of F, for example if F is increased by 20% then the yield will decrease by 15%. The absolute values of Y/R expressed in grams per recruit are not important for this purpose. Therefore, Beverton and Holt (1966) also developed a "*relative yield per recruit model*" which can provide the kind of information needed for management. This model has the great advantage of requiring fewer parameters, while it is especially suitable for assessing the effect of mesh size regulations. It belongs to the category of length-based models, because it is based on lengths rather than ages.

The "*relative Beverton and Holt yield per recruit model*" is defined by:

$$(Y/R)' = E*U^{M/K}*\left[1 - \frac{3U}{1+m} + \frac{3U^2}{1+2m} - \frac{U^3}{1+3m}\right] \qquad (8.4.1)$$

where

$$m = \frac{1-E}{M/K} = K/Z$$

$U = 1 - Lc/L_{\infty}$ the fraction of growth to be completed after entry into the exploited phase

$E = F/Z$ the exploitation rate or the fraction of deaths caused by fishing (cf. Section 4.2).

(Y/R)' is considered a function of U and E and the only parameter is M/K. The equation gives a quantity which is proportional to Y/R as defined by Eq. 8.2.4 as can be shown by a number of algebraic manipulations. It can be shown that

$$(Y/R)' = (Y/R)*\exp[-M*(Tr-t_0)]/W_{\infty}$$

Note that a separate estimate of K is not required as input and that Eq. 8.4.1 is based on lengths (L_{∞} and L_c in U) rather than ages.

(Y/R)' can be calculated for given input values of M/K, L_{∞} and Lc for values of E ranging from 0 to 1, corresponding to F values ranging from 0 to ∞.

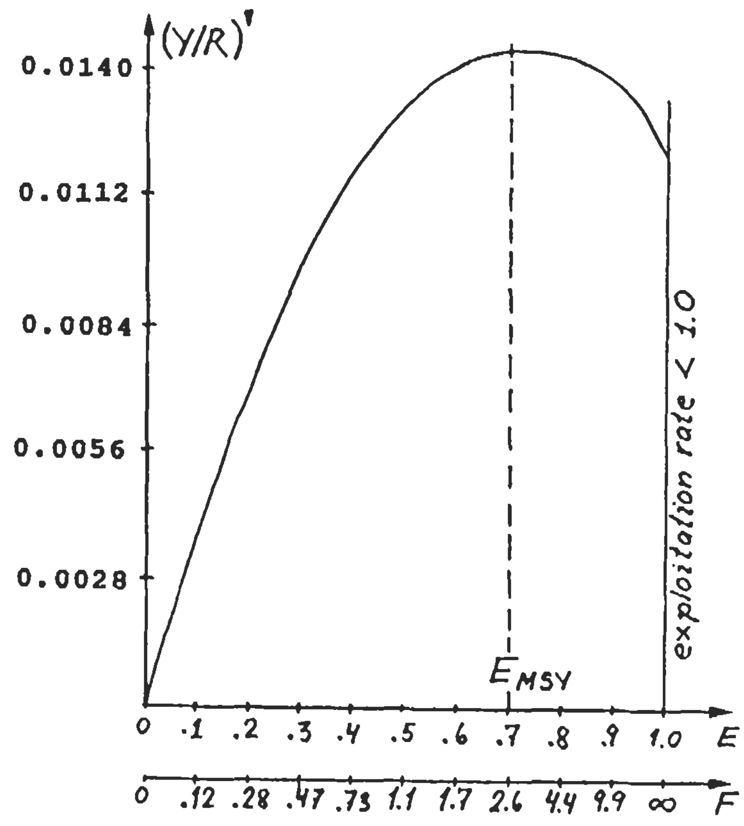

Fig. 8.4.1 **Beverton and Holt's relative yield per recruit (Y/R)' curve corresponding to the Y/R-curve of Fig. 8.2.3 (Lc = 10.2 cm)**

The plot of $(Y/R)'$ against E gives a curve with a maximum value, E_{MSY}, for a given value of Lc. Thus, when Lc, F and M/K are known for a certain fishery the actual exploitation rate can be compared with the E_{MSY} level and management measures be proposed as necessary.

Fig. 8.4.1 shows the $(Y/R)'$-curve corresponding to the Y/R-curve of *Nemipterus marginatus* (Fig. 8.2.3) in the case of:

$$Lc = L(Tc) = L(1.0) = 28.4*[1 - \exp(-0.37*(1+0.2))] = 10.2 \text{ cm}$$

where L_∞ = 28.4 cm (see Section 3.1.2 and Fig. 3.1.2.1).

$$U = 1 - Lc/L_\infty = 1 - 10.2/28.4 = 0.641$$

As an example we calculate $(Y/R)'$ for E = 0.5:

$$m = \frac{1-0.5}{1.1/0.37} = 0.168$$

$$(Y/R)' = 0.5*0.641^{2.973}*\left[1 - \frac{3*0.641}{1+0.168} + \frac{3*0.641^2}{1+2*0.168} - \frac{0.641^3}{1+3*0.168}\right] = 0.0135$$

(See **Exercise(s)** in Part 2.)

8.5 YIELD PER RECRUIT FROM LENGTH DATA

Almost the same algebra as referred to in Section 8.4 transforms the equation for Y/R (Eq. 8.2.4) into a length-based model. The original parameters and variables are F, M, W_∞, K, t_0, Tr and Tc. In the length-transformed model we have L_∞, Lr and Lc instead of t_0, Tr and Tc. The new equation is

$$Y/R = F*A*W_\infty*\left[\frac{1}{Z} - \frac{3U}{Z+K} + \frac{3U^2}{Z+2K} - \frac{U^3}{Z+3K}\right] \qquad (8.5.1)$$

where

$$U = 1 - Lc/L_\infty \text{ as in Eq. 8.4.1 and}$$

$$A = \left[\frac{L\infty-Lc}{L\infty-Lr}\right]^{M/K}$$

Recalling that several methods of parameter estimation described in the preceding chapters give Z/K or M/K it may be of interest also to formulate Eq. 8.5.1 in such terms. Division by K outside and multiplication by K inside the brackets, and substituting z for Z/K gives:

$$Y/R = \frac{F}{K}*A*W_\infty*\left[\frac{1}{z} - \frac{3U}{z+1} + \frac{3U^2}{z+2} - \frac{U^3}{z+3}\right] \qquad (8.5.2)$$

This equation contains F/K, M/K (in A) and Z/K (in z), has no reference to age and does not require a separate estimate of K.

Marten (1978), using linear growth instead of the von Bertalanffy model presents a similar length-based Y/R model.

8.6 AGE-BASED THOMPSON AND BELL MODEL

As stated in the introduction to this chapter the first predictive model was developed much earlier than the Beverton and Holt model by Thompson and Bell (1934). The Thompson and Bell model is the exact opposite of the models discussed in Chapter 5, VPA and cohort analysis. It is used to **predict** the effects of changes in the fishing effort on future yields, while VPA and cohort analysis are used to determine the numbers of fish that must have been present in the sea, to account for a known sustained catch, and the fishing effort that must have been expended on each age or length group to obtain the numbers caught (see Sections 5.1 and 5.2). Therefore, VPA and cohort analysis are called **historic** or **retrospective** models, while the Thompson and Bell model is **predictive**.

The Thompson and Bell method consists of two main stages: 1) Provision of essential and optional inputs and 2) the calculation of outputs in the form of predictions of future yields, biomass levels and even the value of the future yields.

1) **Provision of inputs**: The main input is a so-called "*reference F-at-age-array*", an array of F-values per age group. In principle any F-array could be used as input, but, of course, not just any F-array will produce results which are related to the real situation of a fishery. Therefore, it is customary to use an F-array that has been obtained from an analysis of historical data, in other words from a VPA or a cohort analysis. However, the reference F-array may also originate from other sources as is actually the case in Example 29, given below.

   Another important input parameter is the number of recruits, which may also be obtained from VPA or cohort analysis. This input is needed to obtain predictions of yields etc. in absolute quantities. However, if this input is not available the Thompson and Bell model can still be used to provide relative figures as output, for example, in the form of units "per 1000 recruits" (see Example 29).

   The model further requires a "*weight-at-age-array*", the weights of individual fish per age group. For economic analyses the model also requires inputs of value, usually in the form of the price per kg by age group. (For the length-based Thompson and Bell model the same type of input is required per length group.)

2) **Outputs**: The output of the model is in the form of predictions of the catch in numbers, the total number of deaths, the yield, the mean biomass and the value, all per age group, related to values of F for each age group. New values of F can be obtained by multiplying the reference F-array as a whole by a certain factor, usually called X, or by applying such factors only to a part of the reference F-array. The latter is applied, for example, in the case of a change in the minimum mesh size, or to separate the effect of fleets with different characteristics (e.g. artisanal and industrial) on a particular stock. By carrying out a whole series of calculations with different values for X (F-factors), graphs can be drawn that illustrate clearly the effects of changes in F on the yield, the average biomass and the value of the catch.

The Thompson and Bell model is a very important tool for the fishery scientist to demonstrate the effect that certain management measures, such as changes in the minimum mesh size, decreases or increases of fishing effort, or closed seasons will have on the yield, the biomass and the value of the catch. Since a large number of calculations is required, it is essential to use computers.

An important aspect of the Thompson and Bell model is that it allows for the incorporation of the value of the catch. Therefore, the model has become the basis for the development of so-called bio-economic models, which are extremely useful for the provision of predictions needed for management decisions.

Computer programs

The LFSA package contains programs to carry out relatively simple Thompson and Bell analyses, both length-based and age-based. Similar programs have been incorporated in the FiSAT package. A series of computer programs for bio-economic analysis of fisheries has been developed and published by FAO, the so-called BEAM (Bio-Economic Analytical Model) programs (BEAM 1 and 2, Coppola *et al.*, 1992, BEAM 3, Cochet and Gilly, 1990 and BEAM 4, Sparre and Willmann, 1992).

Example 29: Age-based Thompson and Bell analysis, tropical shrimp

Input

To illustrate the model we use data from the Kuwait shrimp fishery (from Garcia and van Zalinge, 1982). Columns A to E in Table 8.6.1 contain the input data. In this case the fishing mortalities, the F's, were estimated from catch data and estimates of the biomass obtained by the swept area method (cf. Chapter 13). However, the F-array could also have been estimated by cohort analysis or VPA.

The life span of the shrimp *Penaeus semisulcatus* is not much over one year so the age groups in Column A of Table 8.6.1 are given in months. The species is recruited to the fishery at the age of one month (Tr = 1). Column B gives the average weight per age group. Column C contains a relative value, proportioned to the price per kg of unpeeled tails per age group.

Table 8.6.1 Age-based Thompson and Bell model illustrated by data from the Kuwait shrimp fishery (from Garcia and van Zalinge, 1982). M = 3.0 per year for all ages

INPUT					OUTPUT					
A	B	C	D	E	F	G	H	I	J	K
age *) t months	mean wght. $\bar{w}(t)$ g	value per g $\bar{v}(t)$ money unit	fish. mort. F(t) per year	total mort. Z(t) per year	popu-lation *) N(t) number	deaths N(t)-N(t+Δt) number	catch C(t) number	yield Y(t) g	mean biom. $\bar{B}(t)$ g	value $Y*\bar{v}$ money unit
1=Tr	5.7	0.73	1.20	4.20	1000.0	295.3	84.4	481	4809	351
2	9.3	0.93	1.32	4.32	704.7	213.0	65.1	605	5504	563
3	13.0	1.20	1.32	4.32	491.6	148.6	45.4	590	5367	708
4	17.6	1.45	1.44	4.44	343.0	106.1	34.4	606	5046	878
5	22.0	1.70	1.92	4.92	236.9	79.7	31.1	684	4276	1163
6	26.1	1.90	1.20	4.20	157.2	46.4	13.3	346	3463	658
7	30.3	2.08	1.56	4.56	110.8	35.0	12.0	363	2793	755
8	33.8	2.14	1.20	4.20	75.8	22.4	6.4	216	2161	462
9	37.0	2.18	1.20	4.20	53.4	15.8	4.5	167	1667	363
10	40.3	2.23	1.80	4.80	37.6	12.4	4.7	187	1250	418
11	43.1	2.24	2.76	5.76	25.2	9.6	4.6	199	863	445
12	44.7	2.27	2.52	5.52	15.6	5.8	2.6	117	559	267
13	-	-	-	-	9.9	-	-	-	-	-
							Totals	4561	37758	7031
						Mean biomass:		37758/12 = 3146.5		
*) At beginning of period										

Column D contains the fishing mortalities, the "reference F-at-age-array", and Column E the total mortality per year per age group.

In Column F we start with 1000 recruits, which have an age of 1 month at the beginning of the period. In other words, the population or stock number of age group 1 is 1000. All subsequent calculations are relative to 1000 recruits. In case a cohort analysis had been carried out and an estimate of the actual number of recruits obtained, the values obtained per 1000 recruits could be converted into actual yields and stock size (see Section 5.2).

Output based on reference F-at-age-array

On the basis of the input figures presented in Columns A to E and the number of recruits at age 1 month (= 1000), the population per age group, expressed in numbers present at the beginning of each month can be calculated (Column F). Also the following can be calculated: the number of deaths per month (Column G), the catch in numbers, equivalent to the number of deaths due to fishing (Column H), the yield in grams (Column I), the mean biomass in grams (Column J), and the value expressed in money units (Column K).

The computation procedures will now be presented step-by-step, using as numerical examples the calculations for the first three age groups.

Step 1: Calculate the population number at the beginning of each period (month):
N(1) = 1000, use
N(t+Δt) = N(t)*exp(-Z*Δt), where
Δt = 1 month = 0.08333 year, to calculate subsequent numbers
N(2) = 1000*exp(-4.20*0.08333) = 704.7
N(3) = 704.7*exp(-4.32*0.08333) = 491.6

Step 2: Calculate the total number of deaths in each period:
Total number of deaths D(t) = N(t) - N(t+Δt)
D(1) = 1000-704.7 = 295.3
D(2) = 704.7-491.6 = 213.1
D(3) = 491.6-343.0 = 148.6

Step 3: Calculate the numbers caught in each period:
C(t) = [N(t) - N(t+Δt)]*F(t)/Z(t) = D(t)*F(t)/Z(t)
C(1) = 295.3*1.20/4.20 = 84.4
C(2) = 213.1*1.32/4.32 = 65.1
C(3) = 148.6*1.32/4.32 = 45.4

Step 4: Calculate the yield (= catch in weight) in each period:
Y(t) = C(t)*$\bar{w}$(t)
Y(1) = 84.4*5.7 = 481
Y(2) = 65.1*9.3 = 605
Y(3) = 45.4*13.0 = 590

Step 5: Calculate the mean biomass in each period:
$\underline{B}$(t) = Y(t)/[F(t)*Δt]
$\underline{B}$(1) = 481/(1.20*0.08333) = 4810
$\underline{B}$(2) = 605/(1.32*0.08333) = 5500
B(3) = 591/(1.32*0.08333) = 5373

Note: This calculation of biomass is derived from Eq. 4.2.8, $C = F*\Delta t*\bar{N}$, which by multiplication by $\bar{w}$ on both sides becomes

$$Y = F*\Delta t*\bar{B} \text{ and } \bar{B} = Y/(F*\Delta t)$$

Step 6: Calculate the value of yield in each period:
$V(t) = Y(t)*\bar{v}(t)$
$V(1) = 481*0.73 = 351$
$V(2) = 605*0.93 = 563$
$V(3) = 590*1.20 = 708$

Step 7: Calculate the total yield, the mean biomass over the whole period and the total value (see last row of Table 8.6.1):

Total yield is the sum of all monthly yields.
Total value is the sum of all monthly values.

The approximate average biomass is (cf. Fig. 4.2.3):

$$\bar{B} = \sum_{t=1}^{12} (\bar{B}(t)*\Delta t) \Big/ \sum_{t=1}^{12} \Delta t$$

As $\Delta t = 1/12$ and the total period 12 months, in this case

$$\bar{B} = \frac{\Sigma\bar{B}(t)}{12} = \frac{37758}{12} = 3146.5$$

The mean biomass concept in the more complicated case where Δt does not remain constant is discussed in Section 5.3.

The following block of equations summarizes the formulas for the age-based Thompson and Bell model in a general form, including X (F-factor). The index i refers to the age interval $(t_i, t_i+\Delta t)$. The index t_i refers to the start of the interval, while the index $t_i+\Delta t$ refers to the end of it.

age interval: $i = (t_i, t_i+\Delta t)$

$Z_i = M + X*F_i$

$N(t_i+\Delta t) = N(t_i)*\exp(-Z_i*\Delta t)$

$C_i = [N(t_i)-N(t_i+\Delta t)]*X*F_i/Z_i$

$\bar{w}_i = w(t_i+\Delta t/2)$ (8.6.1)

$Y_i = C_i*\bar{w}_i$

$\bar{B}_i = Y_i/[F_i*\Delta t*X]$

$V_i = Y_i*\bar{v}_i$

Thompson and Bell using a plus-group

The last age group line in Table 8.6.1, age 13, contains only the number of survivors and none of the other entries. That is because in Example 29 the number of survivors older than 12 months has been considered to be an insignificant number and has therefore been ignored.

In cases where the number is significant there is a way to account for it, even when taking only 12 age groups into account. This is done by treating the age group 12 as a plus group, i.e., replacing the number of deaths between ages 12 and 13, N(12)-N(13), by the total number of deaths after age 12. Since all specimens will eventually die, this number is in Example 29 N(12) = N(12+) = 15.6.

Assuming further that the older age groups have the same mortalities as age group 12, the number of shrimp caught from the plus group becomes

$$C(12+) = \frac{F(12)}{Z(12)}*N(12)$$

$$C(12+) = (2.52/5.52)*15.6 = 7.1$$

Thus, by leaving out the plus groups in Table 8.6.1 a catch of 7.1-2.6 = 4.5 has been ignored.

If growth has stopped at age 12 then $\bar{w}(12)$ is the maximum body weight and the yield corresponding to C(12+) becomes 44.7*4.5 = 201 g. This yield corresponds to a value of 201*2.27 = 456 money units, which constitutes some 6% of the total a significant amount. Therefore, to be on the safe side with regard to ignoring significant catches, it is better to always treat the last group as a plus-group.

Prediction, output based on different F-arrays

With the output based on the reference F-at-age-array calculated above all the basic data are available to predict the effect of increases and decreases in fishing effort or fishing mortality. New figures for total yield, total mean biomass and total value can be obtained by raising the fishing mortalities in Column D of Table 8.6.1 by a certain percentage. The F-array presented in Table 8.6.1 called the reference F-array is then replaced by a new one, by multiplying the reference F-array, or a part of it with the factor X = (new F)/(reference F).

If, for example, the effort is increased by 20 percent, the new fishing mortalities in Column D would become

$$1.20*1.20 = 1.44,\ 1.32*1.20 = 1.58,\ \text{etc.}$$

By then going again through the whole procedure using the new F's the related total yield, total mean biomass and total value are obtained.

An example of results of such a series of calculations with X = F-factors ranging from 0 to 3.0 is presented in Table 8.6.2. The reference F-array, where X = 1.0, gives a total yield of 4560, a total mean biomass of 3146 and a total value of 7029. (These amounts were obtained with the same input data as used in Table 8.6.1, however there are slight differences with the results presented in that table due to the fact that the computer program used to calculate Table 8.6.2 used the maximum number of digits for all calculations. Such small

Table 8.6.2 **Yield, value of yield and biomass for various F-levels. The reference F-array is given in column D of Table 8.6.1 (compare Fig. 8.6.1)**

F-factor X	total yield	total mean biomass	total value
0	0	5382	0
0.4	2549	4271	4209
0.8	4055	3466	6396
1.0	4561*)	3146*)	7031*)
1.2	4954	2870	7465
1.5	5383	2522	7842
2.0	5814	2075	8025
3.0	6138	1497	7683
*) cf. Table 8.6.1			

and, from the stock assessment point of view, insignificant differences can also be found in some other calculations presented in this manual.)

In Fig. 8.6.1 the total yield, total mean biomass and total value figures corresponding to Table 8.6.2 have been plotted against X (F-factor) and the respective curves were drawn. Note that the value curve has a maximum, whereas the yield curve has no maximum in the range of F-Factors (X) considered. When the price per kg varies with the size of the shrimps the two curves will have their maximum at different levels of F.

Recall that biomass is proportional to catch per unit of effort (Sections 4.3 and 8.3). Fig. 8.6.1 illustrates the important conflict between the desire to maximize the total yield from a fishery, by weight or by value, and the need to give the fishermen and boat owners the necessary income. The catch per boat decreases steadily as the effort increases and may in practice become too small to make the fishery profitable, even at effort levels smaller than those corresponding to the maximum on the curve for total value of the yield.

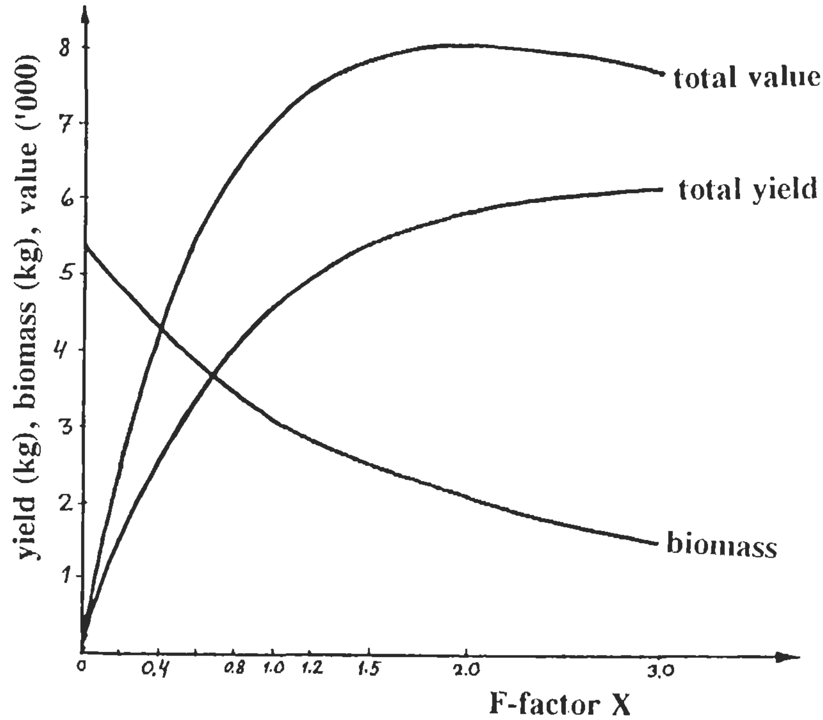

Fig. 8.6.1 **Yield, biomass and value of yield per 1000 shrimps calculated by the age-based Thompson and Bell model. Based on data in Table 8.6.2**

Prediction by fleet

The shrimp fishery in Kuwait waters is composed of an "artisanal fishery" and an "industrial fishery". Table 8.6.3 shows the results of a division of the total fishing mortality given in Table 8.6.1 into an artisanal component and an industrial component (from Garcia and van Zalinge, 1982). Such a division of fishing mortalities caused by different fleets is usually based on the proportions of the numbers of shrimps (or fish) caught by each fleet.

The fishing mortality exerted by one fleet, say, fleet no. i, F(i), is

$$F(i) = F_{total} * C(i) / C_{total} \qquad (8.6.2)$$

where C(i) is the number of shrimps (or fish) caught by fleet no. i, and F(total) and C(total) are the fishing mortality and the numbers caught by all fleets. F(total) may be derived from cohort analysis. The split of the catch (column H in Table 8.6.1) into fleet components is obtained by:

$$C(i) = C_{total} * F(i) / F_{total} \qquad (8.6.3)$$

Thus, yield and value of yield are easily separated into fleet components. Table 8.6.4 shows the split of total yield and value of yield given in Table 8.6.2 between the two fleets.

In this case the same factors, $X_A = X_I$ are applied to the F-values of both fleets, i.e. in the exercise demonstrated in Table 8.6.4 it has been assumed that the effort of the artisanal fleet is always the same proportion of the total effort. Fig. 8.6.2 shows the graphs corresponding to Table 8.6.4.

Table 8.6.5 and Fig. 8.6.3 show an example where the factor, X_A, for the artisanal fleet is kept constant whereas the factor X_I, for the industrial fleet is varied. This corresponds to a situation where the industrial fishery is changing, whereas the artisanal fishery is assumed to remain at the same level. Note that the artisanal fleet gets a smaller share of the total catch the higher the effort level of the industrial fleet. This is what may be expected as an increase in the effort of the industrial fleet reduces the stock so that a smaller share is left for the artisanal fleet.

It is possible to assess in a similar manner the effect of any regulatory measure for each fleet component as long as one can convert the effort regulation into the proper fishing mortalities. For example, Garcia and van Zalinge (1982) used the Thompson and Bell model to assess the effect of a closed season.

The Thompson and Bell model may also be used to assess the effect of a change in mesh size. In this case the selection curve for the current fishery should be estimated using one of the methods described in Chapter 6. The method will be discussed in Section 8.8.

The application of the Thompson and Bell model described above (including the mesh assessment described below) is essentially the method applied today to predict catches and to set catch quotas in the ICES area (Northeast Atlantic) and in many other places.

(See **Exercise(s)** in Part 2.)

Table 8.6.3 **Division of total fishing mortality from the Kuwait shrimp fishery into an artisanal and an industrial component (from Garcia and van Zalinge, 1982)**

	Fishing mortality (F)		
age months	F_A per year	F_I per year	F_{total} per year
1	0.720	0.480	1.20
2	0.960	0.360	1.32
3	0.840	0.480	1.32
4	0.480	0.960	1.44
5	0.600	1.320	1.92
6	0.480	0.720	1.20
7	1.080	0.480	1.56
8	0.480	0.720	1.20
9	0.084	1.116	1.20
10	0.120	1.680	1.80
11	0.240	2.520	2.76
12	0.240	2.280	2.52

Table 8.6.4 **Division of yields and values of yield from Table 8.6.1 into an artisanal and an industrial component (from Garcia and van Zalinge, 1982). The X-factors are as in Table 8.6.2 ($X_A = X_I$) (see Fig. 8.6.2)**

		artisanal fleet			industrial fleet		
total yield g	total value unit	F-factor X_A per year	yield g	value unit	F-factor X_I per year	yield g	value g
0	0	0	0	0	0	0	0
2549	4209	0.4	1048	1531	0.4	1501	2678
4055	6396	0.8	1773	2486	0.8	2284	3910
4560	7029	1.0	2048	2815	1.0	2512	4216
4954	7465	1.2	2281	3073	1.2	2673	4392
5383	7842	1.5	2563	3354	1.5	2819	4488
5814	8025	2.0	2903	3627	2.0	2910	4398
6138	7683	3.0	3291	3783	3.0	2847	3900

Table 8.6.5 **Assessment of the effect of varying the industrial effort (X_I) while the artisanal effort is kept constant (X_A = 1.0) (see Fig. 8.6.3)**

		artisanal fleet			industrial fleet		
total yield	total value	F-factor X_A	yield	value	F-factor X_I	yield	value
2479	3603	1.0	2479	3603	0	0	0
3522	5403	1.0	2289	3250	0.4	1234	2154
4270	6598	1.0	2124	2950	0.8	2146	3648
4560	7029	1.0	2048	2815	1.0	2512	4216
4811	7383	1.0	1979	2691	1.2	2832	4692
5120	7783	1.0	1883	2530	1.5	3237	5263
5501	8203	1.0	1740	2271	2.0	3761	5932
5951	8499	1.0	1510	1880	3.0	4441	6619

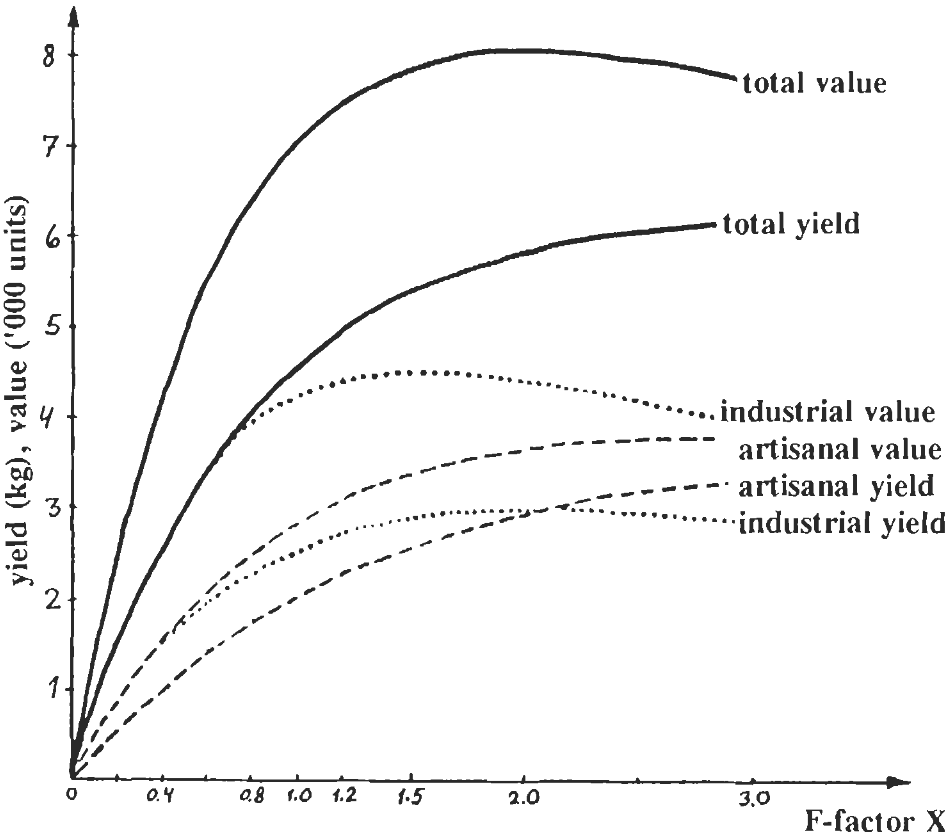

Fig. 8.6.2 Total yield and total value of yield from Fig. 8.6.1 separated into an artisanal and an industrial component (cf. Table 8.6.4)

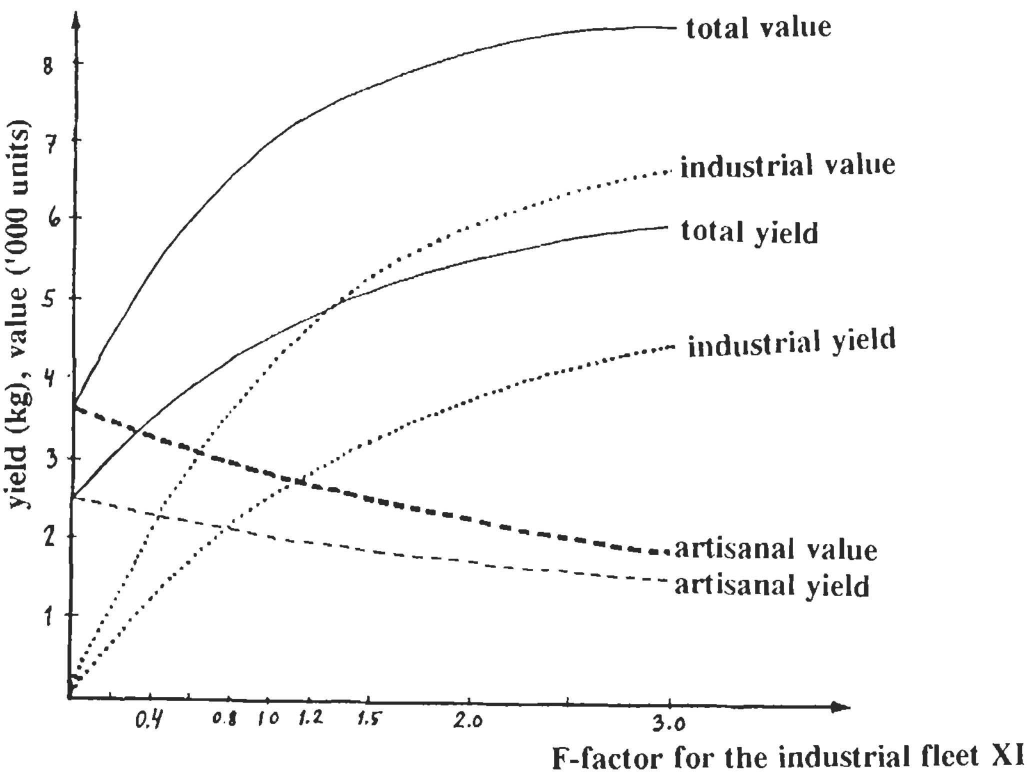

Fig. 8.6.3 Assessment of the effect of changes in the industrial fishery while the artisanal fishery is kept at a constant level (cf. Table 8.6.5)

Example 29a: Short-term and long-term predictions

Immediately following a change of effort or mesh size there occurs a change in catches which is not reflected in the steady state situation reached when all age groups in the population have been exposed to the new effort or new mesh size during all their life. Example 29 dealt with the estimation of the new steady state only.

In order to calculate the catches year by year after a change in effort (i.e., F) we continue the cohort analysis (or VPA) into future years using the estimated numbers in each age group at the end of the period with the old effort level, and the new values which F take on after the change. The calculations begin with the youngest age group and proceed downwards in the tables in contrast to VPA and cohort analysis. The recruitment cannot be predicted. Constant recruitment is therefore assumed.

Returning to the whiting data of the age-based cohort analysis, Section 5.2, Example 19a, we use the average recruitment of 2849 million for the years 1974-79, see Table 5.2.2. We shall investigate the effect of doubling the effort. Thus, the factor X of Example 29 takes on the value 2:

```
F(new,t,t+1) = F(1980,t,t+1)*2
```

However, the values of F at disposal for 1980 are "terminal F" values, i.e., they are guessed. Actually, the estimates of F for the 1974 year class were used in Example 19a. Inspection of Table 5.2.2 shows that there is little, if any, indication of an effort change over the years of sampling. We therefore replace the original final F values by averages of the estimates for the sampling years 1974-79 as given in the last column of Table 5.2.2. These we adopt as F(1980,t,t+1).

Changing the final F values for 1980 requires also a recalculation of the numbers in the population at the beginning of 1980. We use again the expression for the "final N" of Example 19:

$$N(1980,t) = \frac{C(1980,t,t+1)}{(F/Z)*(1-\exp(-Z))}$$

where F = F(1980,t,t+1), Z = F+M, with M = 0.2 as before. The results are given in the first column of Table 8.6.6.

The doubling of effort is assumed to occur on 1 January 1981. In order to predict the effect of this the numbers N(1981,t) must be calculated from the old F values and used as a starting point. They are

```
N(1981,t+1) = N(1980,t)*exp[-(F(1980,t,t+1) + M)]
```

as for instance

```
N(1981,4) = 412*exp[-(1.02 + 0.2)] = 122
```

see Table 8.6.6. For N(1981,0) is inserted the average recruitment in 1974-79, 2849 million, from Table 5.2.2. With an X-factor of 2 we insert the new numbers into

```
N(1982,t+1) = N(1981,t)*exp[-(X*F(1980,t,t+1) + M)]
```

such that

```
N(1982,5) = 122*exp[-(2*0.91 + 0.2)] = 16
```

and similarly for the following years. Keeping the X-factor and the recruitment constant the population gradually stabilizes as the year classes present in 1980 are dying out. In 1987 the numbers in age groups 0-6 years are all stabilized. The 1987 column represents the long-term prediction of the effects of doubling the effort.

Similar calculations, Table 8.6.7, are made for the catches using Eq. 4.2.7:

```
C(y,t,t+1) = N(y,t)*[1 - exp(-Z)]*X*F(1980,t,t+1)/Z
```

where

```
Z = X*F(1980,t,t+1) + M
```

Thus, for age group 4 in 1981:

```
Z = 2*0.91 + 0.2 = 2.02

C(1981,4,5) = 122*[1 - exp(-2.02)]*2*0.91/2.02 = 95.
```

The weight of the annual yield is found by multiplying numbers by the weight of the individual fish in each age group is shown in Fig. 8.6.5 as a drawn line. The catches in the first year after the increase of effort are large because the stock is still adjusted to the previous situation with lower effort. In 1987 and later when a steady state has been reached the catches of fish over 4 years old are almost negligible as seen in Table 8.6.7.

The results are influenced by the situation at the date of the change, 1st January 1981. The same change effectuated in another year would give different short-term results depending on the age distribution in the stock at that time whereas the long-term prediction would be the same. To generalize the effects of an increase in effort a situation must be examined in which the change is from one steady state to another. Fig. 8.6.4 shows such effects on the North Sea stock of herring. In this case the effort was increased and the mesh reduced at the same time. Changes of this kind usually occur when a fishery develops unregulated and is already at an effort level beyond MSY.

The effect was an immediate increase in the numbers caught and in the total yield by weight (Fig. 8.6.4, C and A) followed by a rapid decrease to a new level. The biomass and the mean size of the fish decrease all the time until the new steady state is reached (Fig. 8.6.4, B and D). The temptation to increase the effort, for instance by improving the efficiency of the gear and by reducing the mesh size is obvious because the immediately observable effect is a higher yield.

We return to the whiting example and examine the effect of reducing the fishing mortalities (Table 8.6.6, column 3) to half their present value, i.e., X = 0.5 (Table 8.6.8, column 2). A considerable increase in the number of old fish in the catch is anticipated such that it is not advisable to neglect the fish of age group 7 and older. Age group 6 is therefore replaced by a plus-group, 6+. This does not affect the calculation of numbers N(t) because the number of 6-year-olds on the first day of the year is still the quantity needed. The catch in the plus-group is defined by Eq. 5.1.7:

```
C(6+) = N(6) * F(6+)/Z(6+)
```

where the mortalities are defined by the "final F" of VPA multiplied by the X-factor which is now 0.5.

Table 8.6.6 Short-term and long-term predictions of population numbers (millions) after an increase in effort ($F_{new} = 2*F_{old}$). North Sea whiting, from Tables 4.4.3.1 and 5.2.2. Constant recruitment. M = 0.2. Fish over 7 years old are neglected. Catches are in Table 8.6.7

					numbers at beginning of successive years				
age group t	1 year before change N(1980,t)	during last year before change F(1980,t,t+1)	1st Jan. of year of change N(1981,t)	new F F(t,t+1)	N(1982,t)	N(1983,t)	N(1984,t)	....	long-term change N(1987,t)
0	1700	0.24	2849	0.48	2849	2849	2849		2849
1	801	0.50	1095	1.00	1443	1443	1443		1443
2	614	0.85	398	1.70	330	435	435		435
3	412	1.02	215	2.04	60	49	65		65
4	146	0.91	122	1.82	23	6	5		7
5	75	0.60	48	1.20	16	3	1		1
6	22	0.50	34	1.00	12	4	1		0

Table 8.6.7 Short-term and long-term predictions of catches (millions) after an increase in effort. North Sea Whiting. Population numbers are in Table 8.6.6

		after change of effort				
age group t	last year before change C(1980,t,t+1)	1st year C(1981,t,t+1)	2nd year C(1982,t,t+1)	3rd year C(1983,t,t+1)	....	long-term change C(1987,t,t+1)
0	330	992	992	992		992
1	288	638	841	841		841
2	323	303	251	331		331
3	243	175	49	40		53
4	80	95	18	5		5
5	31	31	10	2		1
6	9	20	7	2		0

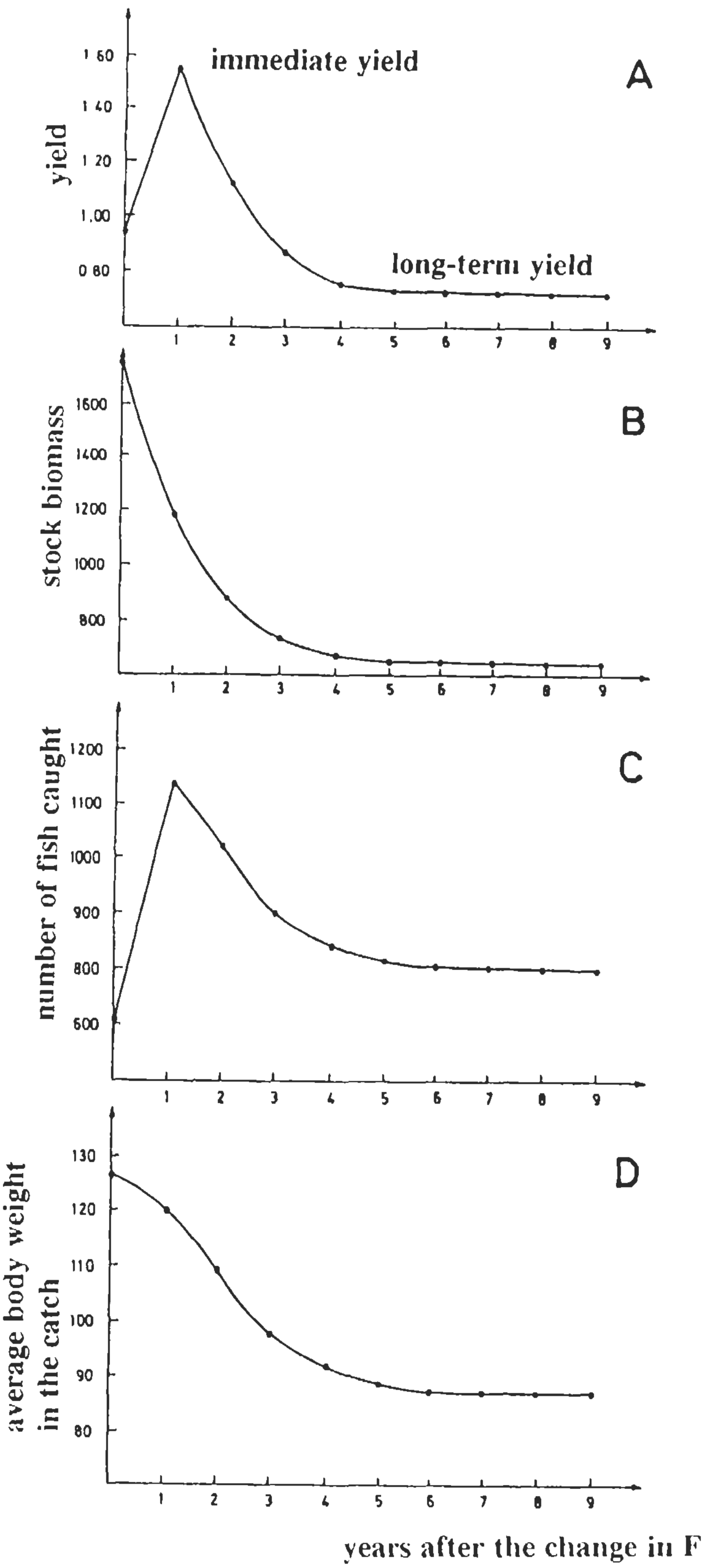

Fig. 8.6.4 **Immediate and long-term effects of simultaneously increasing the effort and reducing the mesh size. Transition from one steady state to another over nine years. North Sea herring (*Clupea harengus*). Redrawn from Beyer and Sparre (1983)**

If the mean number of all age groups in the plus-group is wanted it is given by:

```
N(6+) = N(6)/Z(6+)
```

The number of all ages in the plus-group on the first day in the year may be found as the sum of a geometric progression with first term = N(6) and a quotient of exp(-Z(6+)):

```
N(6+) = N(6)/[1-exp(-Z(6+))]                                    (8.6.3)
```

Any treatment of a plus-group is approximative. The above expressions assume that all year-classes in the plus-group have had an identical history: that they had the same recruitment and the same mortality in the years before they entered the plus-group. An effect of this is that it seems as if a new steady state is reached when the last year-class born before the change of effort joins the plus-group. If the plus-group were defined as 7+ instead of 6+ this would happen one year later, in 1988. Adopting an 8+ group it would be in 1989, etc. Using 6+ as in this example means that we recognize the need to consider fish over 7 years old, but deem it satisfactory to treat them cursorily.

Table 8.6.8 gives the numbers in the stock on January 1st. and the annual catches after reduction of the fishing mortality to half its original value. After an immediate drop the catches of large fish increase markedly. The broken line in Fig. 8.6.5 shows the development of the yield (catch by weight) as obtained by multiplying the numbers caught by the mean weight of the individual fish in each age group. The yield drops to half in the first year and then increases beyond the original value. At first the fishing industry suffers an immediate lack of material which may influence the price level. How the change would affect an

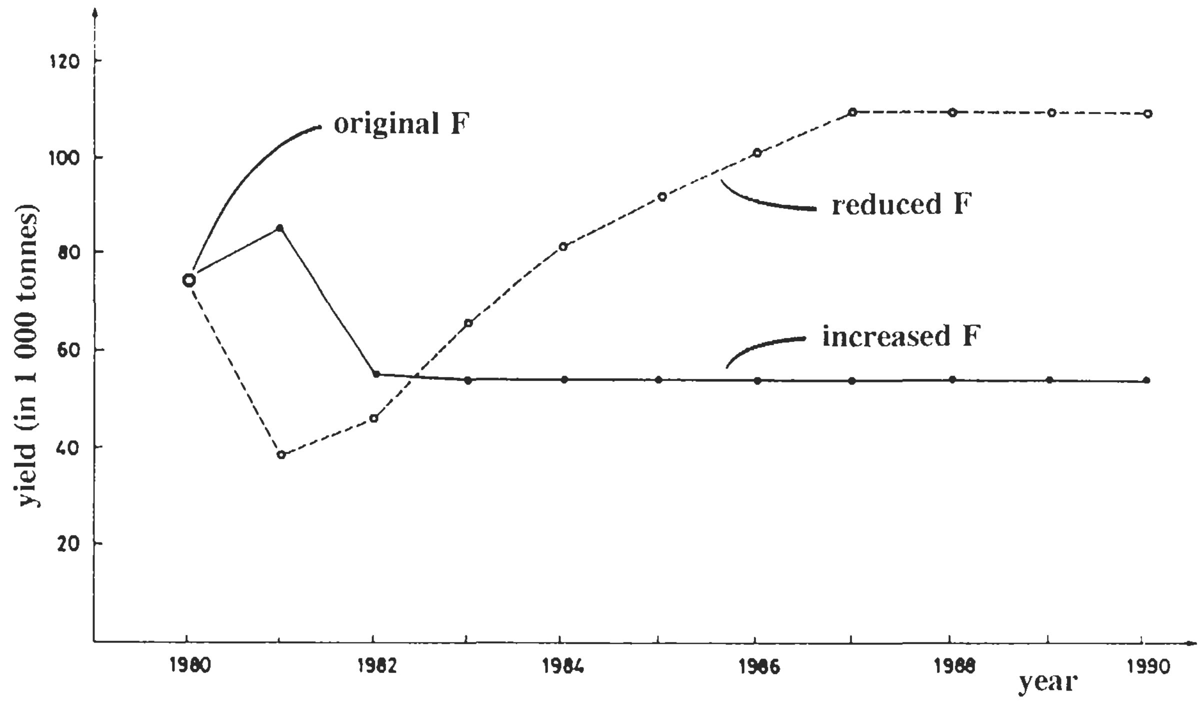

Fig. 8.6.5 **Transition phase after a change in effort. North Sea whiting. Yield in thousands of tonnes**

Table 8.6.8 Short-term and long-term predictions of numbers in the stock and of catches after a decrease of effort ($F_{new} = 0.5*F_{old}$). North Sea whiting. Fish over 6 years are in a plus-group. Cf. Tables 8.6.6 and 8.6.7

		numbers at beginning of successive years					catches				
								after change of effort			
age group t	new F	at date of change N(1981)	N(1982)	N(1983)	.....	long-term change N(1987)	last year before change C(1980)	1st year C(1981)	2nd year (C1982)	.....	long-term C(1987)
0	0.120	2849	2849	2849		2849	330	293	293		293
1	0.250	1095	2069	2069		2069	288	220	417		417
2	0.425	398	698	1319		1319	323	126	221		417
3	0.510	215	213	374		706	243	79	78		258
4	0.455	122	106	105		347	80	41	35		116
5	0.300	48	64	55		180	31	11	15		42
6+	0.250	34	29	39		109	9	19	16		61

individual fishing vessel depends on how the reduction of effort was achieved. If the fleet were reduced to half its former size there would be half as many boats to share the yield and incidentally, the immediate drop was to half the yield of the year before (from 76,000 to 38,000 tonnes) such that the catch per unit effort would remain the same in the first year and then increase in the following years. If on the other hand the reduction of effort was achieved by putting a ban on particularly efficient gear types or by introducing closed seasons there would still be the same number of boats to share the reduced yield in the first year. In that case only after a few years would the catch per boat reach its former value, and beyond.

When in fish stock management a reduction of effort is considered it is paramount to look into the problems likely to turn up in the transition period. A boat owner with little capital will find no consolation in the prospect of an increased income in a few years' time if he goes broke in the first meagre year.

8.7 LENGTH-BASED THOMPSON AND BELL MODEL

The "*length-based Thompson and Bell model*" takes its inputs from a length-based cohort analysis. The inputs consist of the fishing mortalities by length group (the so-called F-at-length-array), the number of fish entering the smallest length group, and the natural mortality factor H by length group, which must be the same as the ones used in the cohort analysis. Additional inputs are the parameters of a length-weight relationship (or the average weight of a single fish or shrimp by length group) and the average price per kg by length group.

The outputs are the same as for the age-based model, *viz.*, for each length group the number at the lower limit of the length group, N(L1), the catch in numbers, the yield in weight, the biomass multiplied by Δt, i.e. the time required to grow from the lower limit to the upper limit of the length group and the value. Finally, the totals of the catch, mean biomass * Δt, yield and value are obtained. The calculations are repeated for a range of X values (F-factors) and the final results (totals) are plotted in graphs. The principle is the same as that described above for the age-based models, only the formulas are slightly different. They can be derived from those used for Jones' length-based cohort analysis, Eqs. 5.3.4 and 5.3.7 as follows:

First Eq. 5.3.7 is rearranged:

$$C(L1,L2) = [N(L1)-N(L2)]*\frac{F(L1,L2)}{Z(L1,L2)} \qquad (8.7.1)$$

then it is inserted into into Eq. 5.3.4 which gives:

$$N(L1) = \left[N(L2)*H(L1,L2) + \frac{N(L1)-N(L2)}{Z(L1,L2)}*F(L1,L2)\right]*H(L1,L2)$$

where

$$H(L1,L2) = \left[\frac{L\infty-L1}{L\infty-L2}\right]^{M/2K}$$

which is the same factor as used in Jones' length-based cohort analysis (Eq. 5.3.3).

Solving this equation with respect to N(L2) gives:

$$N(L2) = N(L1)*\frac{1/H(L1,L2) - F(L1,L2)/Z(L1,L2)}{H(L1,L2) - F(L1,L2/Z(L1,L2)} \qquad (8.7.2)$$

In order to calculate the yield (catch in weight) by length group the catch C (in numbers) has to be multiplied by the mean weight of the length group, $\bar{w}(L1,L2)$, which is obtained from Eq. 5.3.11 as follows:

$$\bar{w}(L1,L2) = q*[(L1+L2)/2]^b$$

where q and b are the parameters of the length-weight relationship.

The yield of this length group is then given by

$$Y(L1,L2) = C(L1,L2) * \bar{w}(L1,L2) \qquad (8.7.3)$$

The value of the yield is given by:

$$V(L1,L2) = Y(L1,L2) * \bar{v}(L1,L2) \qquad (8.7.4)$$

where $\bar{v}(L1,L2)$ is the average price per kg of fish between lengths L1 and L2.

During the time $\Delta t(L1,L2)$ that it takes a cohort to grow from L1 to L2, the number of survivors decreases from N(L1) to N(L2). The mean number of survi-vors of that length group is calculated as follows:

$$\bar{N}(L1,L2) * \Delta t(L1,L2) = [N(L1)-N(L2)]/Z(L1,L2) \qquad (8.7.5)$$

The corresponding mean biomass * Δt is:

$$\bar{B}(L1,L2) * \Delta t(L1,L2) = \bar{N}(L1,L2) * \Delta t(L1,L2) * \bar{w}(L1,L2) \qquad (8.7.6)$$

The annual yield is simply the sum of the yield of all length groups:

$$Y = \Sigma Y_i$$

The annual value is likewise the sum of the value of all length groups:

$$V = \Sigma V_i$$

As discussed in Section 5.3

$$\bar{B} = \Sigma \bar{B}_i*\Delta t_i$$

is an estimate of the average biomass during the life span of a cohort, or of all cohorts during a year. In the age-based method, Section 8.6, it was not necessary to multiply each biomass by Δt because this was constant and equal to 1/12 of a year, or one month, but in this case Δt is variable.

Eqs. 8.7.1 to 8.7.6 have been presented for one specific length class (L1,L2). Like in the age-based version, the following block of equations summarizes the formulas for the length-based Thompson and Bell model in a general form, including X (F-factor). The index i refers here to the length interval (L_i,L_{i+1}). The index L_i refers to the lower limit of that length interval, while the index L_{i+1} refers to the upper limit.

length interval: $i = (L_i, L_{i+1})$

$$Z_i = M + X*F_i$$

$$N(L_{i+1}) = N(L_i) * \frac{1/H_i - X*F_i/Z_i}{H_i - X*F_i/Z_i} \quad \text{where}$$

$$H_i = \left[\frac{L_\infty - L_i}{L_\infty - L_{i+1}}\right]^{M/2K}$$

$$C_i = [N(L_i) - N(L_{i+1})]*X*F_i/Z_i \qquad (8.7.7)$$

$$\bar{w}_i = q*[(L_i + L_{i+1})/2]^b$$

$$Y_i = C_i*\bar{w}_i$$

$$V_i = Y_i*\bar{v}_i$$

$$\bar{N}_i*\Delta t_i = [N(L_i) - N(L_{i+1})]/Z_i$$

$$\bar{B}_i*\Delta t_i = \bar{N}_i*\Delta t_i*\bar{w}_i$$

Basic features of the length-based Thompson and Bell analysis

Since the length-based Thompson and Bell analysis is derived from Jones' length-based cohort analysis (Section 5.3) which in turn is based on Pope's age-based cohort analysis (Section 5.2), the length-based Thompson and Bell method has the same limitations as Pope's age-based cohort analysis. The approximation to VPA in the predictive mode is valid for values of $F*\Delta t$ up to 1.2 and of $M*\Delta t$ up to 0.3 (Pope, 1972). If the F's are high, nonsensical results will come out of the analysis, such as negative stock numbers. If that is the case, smaller length groups and hence, smaller Δt values, are required.

The approximation is not necessary, however, because the forward version of the VPA does not involve the iterative solution for F (Eq. 5.1.3). Using VPA technique in Thompson and Bell requires the replacement of the second and third formula of Eq. 8.7.7 by Eq. 4.2.6 and Eq. 4.4.5.1:

$$N(L_{i+1}) = N(L_i)*\exp(-Z_i*\Delta t_i) \qquad (8.7.8)$$

where

$$\Delta t_i = \frac{1}{K} * \ln \frac{L_\infty - L_i}{L_\infty - L_{i+1}}$$

The mean body weight in the plus group is given by Eq. 5.3.16.

Example 30: Length-based Thompson and Bell analysis, hake, Senegal

As an example of a length-based Thompson and Bell analysis we use the data from Table 5.3.3 for hake (*Merluccius merluccius*) caught off Senegal. The following input parameters are used (cf. Section 5.3):

L_∞ = 130 cm, K = 0.1 per year, M = 0.28 per year, q = 0.00001 kg/cm^3,
b = 3, N (first length group) = N(6) = 98919.3

Using the F-values and the natural mortality factors, H, from Table 5.3.3 and the body weights derived from $\bar{w}_i = q*[(L_i+L_{i+1})/2]^b$, the length-weight relationship, and some (in this case arbitrarily selected) prices per kg for hake, the input may be summarized as in Table 8.7.1.

Using Eqs. 8.7.7 with X = 1 and the input data from Table 8.7.1 we can calculate the numbers in the subsequent length classes, the catch, yield, mean biomass * Δt and the value, as presented in the following example:

N(12) = N(6)*[1/H(6,12) - F(6,12)/Z(6,12)]/[H(6,12) - F(6,12)/Z(6,12)]

= 98919.3*[1/1.0719 - 0.04/0.32]/[1.0719 - 0.04/0.32]

= 84400.8

C(6,12) = [N(6)-N(12)]*X*F(6,12)/Z(6,12)

= [98919.3-84400.8]*1*0.04/0.32 = 1814.8

$\bar{w}$(6,12) = q*[(6+12)/2]^b

= 0.00001*9^3 = 0.007290

Y(6,12) = C(6,12)*$\bar{w}$(6,12)

= 1814.8*0.007290 = 13.23

$\bar{B}$(6,12)*Δt(6,12) = [(N(6)-N(12)]/Z(6,12)]*$\bar{w}$(6,12)

= [(98919.3-84400.8)/0.32]*0.007290 = 330.7

V(6,12) = Y(6,12)*$\bar{v}$(6,12) = 13.23*1.0 = 13.23

Table 8.7.1 Input data for length-based Thompson and Bell analysis, hake, Senegal

length group (L_i,L_{i+1})	F(L_i,L_{i+1})	H(L_i,L_{i+1})	$\bar{w}$(L_i,L_{i+1}) kg	$\bar{v}$(L_i,L_{i+1}) unit/kg
6-12	0.04	1.0719	0.0073	1.0
12-18	0.39	1.0758	0.0338	1.0
18-24	1.07	1.0801	0.0926	1.0
24-30	0.65	1.0850	0.196	1.5
30-36	0.49	1.0905	0.359	1.5
36-42	0.59	1.0967	0.593	2.0
42-48	0.65	1.1039	0.911	2.0
48-54	0.39	1.1122	1.33	2.5
54-60	0.29	1.1220	1.85	2.5
60-66	0.31	1.1337	2.50	2.5
66-72	0.40	1.1478	3.29	3.0
72-78	0.39	1.1652	4.22	3.0
78-84	0.11	1.1873	5.31	3.0
84-∞	0.28	-	12.25	3.0

These calculations will then continue until the last length group is reached. Since that is a so-called plus group a few additional assumptions have to be made: $N(\infty) = 0$ and $\bar{w}(84,\infty) = \bar{w}(84,90)$. The results are:

$$C(84,\infty) = [N(84)-N(\infty)]*F(84,\infty)/Z(84,\infty)$$

$$= [92-0]*0.28/0.56 = 46$$

$$\bar{w}(84,\infty) = \bar{w}(84,130) = q*[(84+130)/2]^b$$

$$= 0.00001*107^3 = 12.25$$

$$Y(84,\infty) = C(84,\infty) * \bar{w}(84,\infty)$$

$$= 46*12.25 = 563.5$$

$$\bar{B}(84,\infty)*\Delta t(84,\infty) = [(N(84)-N(\infty)]/Z(84,\infty)] * \bar{w}(84,\infty)$$

$$= [(92-0)/0.56]*12.25$$

$$= 2012.5$$

$$V(84,\infty) = Y(84,\infty)*\bar{v}(84,\infty)$$

$$= 563.5*3.0 = 1690.5$$

Following these procedures the final result will be like Table 8.7.2. However, it should be noted that there are differences between the results of calculations presented above, calculated by means of a pocket calculator, and those in Table 8.7.2 calculated by using 8 significant digits in all calculations.

Notice that the values of $N(L_i)$ and $C(L_i,L_{i+1})$ are exactly the same as those calculated by Jones' length-based cohort analysis in Tables 5.3.3 and 5.3.4 respectively. Small differences, like in the calculations of mean biomass *Δt can easily occur due to rounding of F and $\bar{w}$.

Table 8.7.2 Output from length-based Thompson and Bell analysis, hake, Senegal, using the F-factor X = 1.0. Weights are in tonnes (cf. Table 5.3.4)

length group (L_i,L_{i+1}) cm	F (L_i,L_{i+1}) X = 1.0	$N(L_i)$ '000 number	C (L_i,L_{i+1}) '000 number	yield (L_i,L_{i+1}) tonnes	mean biomass *Δt $\bar{B}$*Δt tonnes	value (L_i,L_{i+1}) '000 units
6-12	0.04	98919.3	1823	13.3	330.7	13.3
12-18	0.39	84392.7	14463	488.1	1260.1	488.1
18-24	1.07	59475.8	25277	2336.3	2191.5	2336.3
24-30	0.65	27623.0	8143	1601.0	2475.2	2401.5
30-36	0.49	15967.8	3889	1397.6	2845.9	2096.4
36-42	0.59	9861.5	2959	1755.3	2970.1	3510.5
42-48	0.65	5500.5	1871	1704.9	2638.4	3409.9
48-54	0.39	2818.8	653	866.2	2247.1	2165.5
54-60	0.29	1691.5	322	596.3	2069.4	1490.8
60-66	0.31	1056.6	228	570.1	1853.8	1710.3
66-72	0.40	621.0	181	594.6	1481.9	1783.8
72-78	0.39	313.7	96	405.0	1040.1	1215.0
78-84	0.11	148.7	16	85.0	772.0	255.1
84-∞	0.28	92.0	46	563.5	2012.6	1690.6
Total			59908	12977.2	26189.0	24567.1

The calculations can now be repeated for different values of X.

Table 8.7.3 shows the results corresponding to Table 8.7.2 but with the F-factor X = 2.0, i.e. the prediction of catch, yield, mean biomass *Δt and value under the assumption of a doubling of fishing effort. In this case the effect of doubling the effort would be a dramatic decrease in yield and value.

Table 8.7.3 **Output from length-based Thompson and Bell analysis, hake, Senegal, using the F-factor X = 2.0. (cf. Table 8.7.2)**

length group L_i, L_{i+1} cm	F (L_i, L_{i+1}) X = 2.0	$N(L_i)$ '000	C (L_i, L_{i+1}) '000	yield (L_i, L_{i+1}) tonnes	mean biomass *Δt $\bar{B}$*Δt tonnes	value (L_i, L_{i+1}) '000 units
6-12	0.08	98919.3	3611.6	26.3	327.6	26.3
12-18	0.77	82724.1	26041.2	878.9	1134.4	878.9
18-24	2.13	47271.6	32863.1	3043.4	1427.4	3043.4
24-30	1.29	10092.9	5154.2	1014.5	784.2	1521.7
30-36	0.98	3823.1	1652.3	593.8	604.6	890.7
36-42	1.18	1699.7	881.6	523.0	442.5	1046.0
42-48	1.29	609.2	351.7	320.5	248.0	640.9
48-54	0.77	181.3	74.9	99.3	128.8	248.3
54-60	0.58	79.3	27.4	50.8	88.1	126.9
60-66	0.62	38.5	14.9	37.3	60.6	111.9
66-72	0.80	16.8	8.5	27.9	34.7	83.6
72-78	0.78	5.4	2.8	11.9	15.3	35.8
78-84	0.22	1.5	0.3	1.7	7.5	5.0
84-∞	0.56	0.8	0.5	6.7	12.0	20.2
Total			70685.0	6636.0	5315.9	8679.7

Table 8.7.4 **Results of the length-based Thompson and Bell analysis, hake, Senegal.**
MSY = Maximum Sustainable Yield.
MSE = Maximum Sustainable Economic Yield (value) (cf. Fig. 8.7.1)

A	B	C	D
F-factor X	total yield tonnes	total mean biomass *Δt tonnes	total value units
0.0	0	571297	0
0.2	18903	268193	18329
0.4	20717	135343	49701
0.6	18360	73209	40925
0.8	15474	42376	31836
1.0*)	12977*)	26189*)	24567*)
1.2	10999	17216	19168
1.4	9470	11976	15236
1.6	8287	8761	12370
1.8	7365	6697	10259
2.0**)	6636**)	5316**)	8680**)
2.2	6053	4357	7480
2.4	5580	3670	6554
2.6	5191	3163	5829
2.8	4868	2780	5253
3.0	4596	2484	4790
MSY = 20,919 t for F-factor X = 0.343 Biomass at MSY = 163,296 t			
MSE = 51,544 t for F-factor X = 0.301 Biomass at MSE = 188,207 t			
*) cf. Table 8.7.2			
**) cf. Table 8.7.3			

Table 8.7.4 shows the summary results for 16 different F-factors (X). Each row is based on calculations like those illustrated by Tables 8.7.2 and 8.7.3. The total yield, mean biomass and value given in the last row of these two tables can also be found in Table 8.7.4. The two last rows of Table 8.7.4 show the maximum sustainable yield (MSY) and the maximum sustainable economic yield (value) (MSE) together with the corresponding F-factor and stock biomass. When kg prices differ from one length group to another the F-factor giving MSY usually differs from the F-factor giving MSE. Tables 8.7.2 to 8.7.4 were calculated by the program "MIXFISH" in the LFSA package (Sparre, 1987). This program calculates MSY and MSE by using an iterative technique.

The results of Table 8.7.4 have been plotted in Fig. 8.7.1. The graphs clearly show that the present level of fishing effort is well above that giving the maximum sustainable yield and the conclusion to be drawn from this analysis is that the stock is overfished because a reduction in effort would give a higher yield.

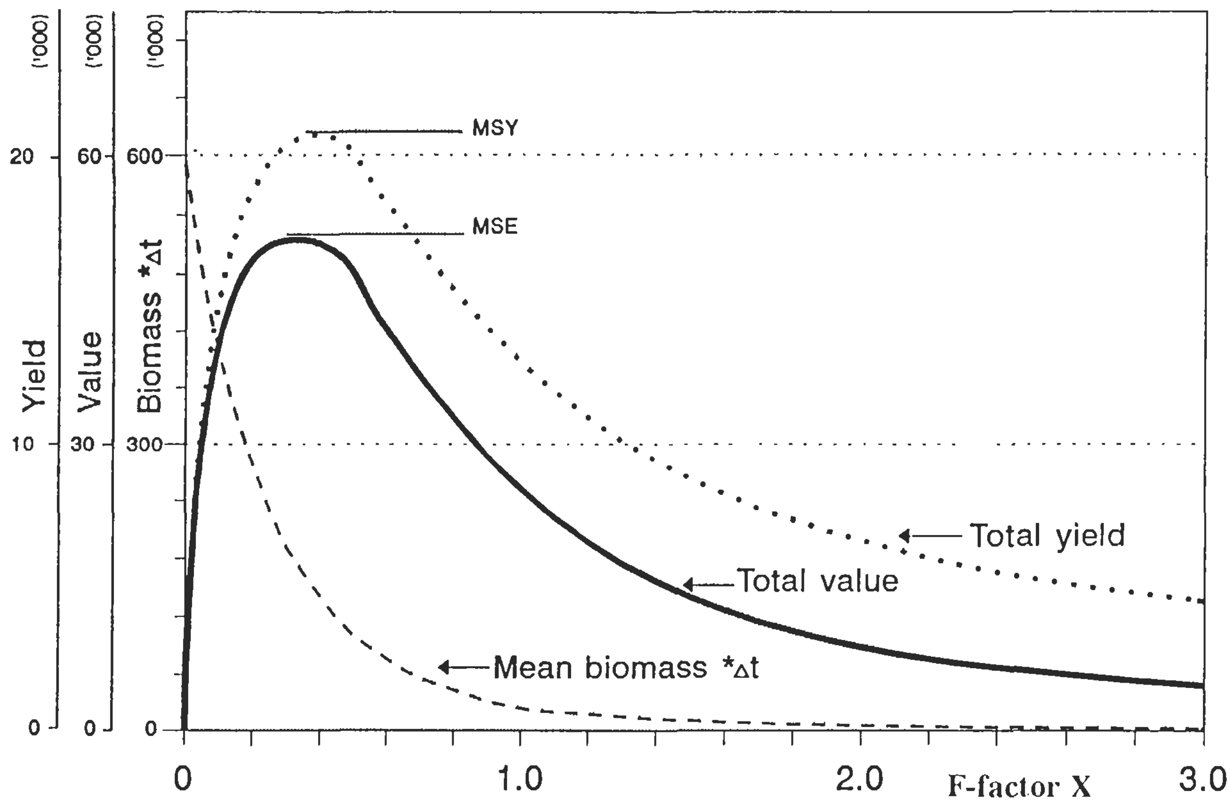

Fig. 8.7.1 **Graphic presentation of the results of the length-based Thompson and Bell analysis, hake, Senegal (cf. Table 8.7.4)**

In case of economic interaction, where several fleets are exploiting one resource the catches predicted by the length-based Thompson and Bell analysis can be partitioned in exactly the same way as shown in Section 8.6 (cf. Tables 8.6.3 to 8.6.5).

The assumption behind the length-based Thompson and Bell analysis (and behind Jones' length-based cohort analysis) is that the stock remains in a steady state, with all parameters (e.g. recruitment) remaining constant. Thus, we obtain a prediction of the "*average long term catches*". Deviations from the predicted catches are therefore to be expected in individual years.

(See **Exercise(s)** in Part 2.)

8.8 PREDICTION OF THE EFFECTS OF CHANGES OF MESH SIZES USING THE THOMPSON AND BELL METHOD

The regulation of mesh sizes is an important management tool for many fisheries. It is, therefore, important to be in a position to predict the result of a change of mesh size. Since a change of mesh size will cause a change in the fishing pattern, the array of F-values, we can use the formulas presented in Sections 6.6.1 and 6.6.2 to come to a prediction, in other words use the "current" situation to predict a "new" situation.

We may express the current fishing mortality by the age-based or the length-based model (cf. Eq. 6.6.1.1):

$$F_t current = Fm*S_t current \quad (8.8.1)$$

and

$$F_L current = Fm*S_L current \quad (8.8.2)$$

where Fm is the maximum fishing mortality and $S_t$current or $S_L$current the selection curve for the current gear, for example, if the gear has the trawl type of selection ogive:

$$S_t current = 1/[1 + exp(T1 - T2*t)] \quad (8.8.3)$$

and

$$S_L current = 1/[1 + exp(S1 - S2*L)] \quad (8.8.4)$$

The parameters T1 and T2 are defined by Eqs. 6.4.3.4 and 6.4.3.5 respectively, while S1 and S2 are defined by Eqs. 6.1.6 and 6.1.7 respectively.

The parameters t50% and t75% are the ages at which 50% and 75% of the fish are retained by the gear, respectively. Usually we know the lengths L50% and L75% which correspond to t50% and t75%.

With the known parameters L50% and L75% for the gear currently in use we are in a position to calculate a new age-based or length-based selection curve for new values of L50% and L75% (or t50% and t75%). From the new selection ogive, and the Fm of the current fishery, we can calculate a new array of fishing mortalities, using Eq. 6.6.1.1:

$$F_t new = Fm*S_t new \quad (8.8.5)$$

and

$$F_L new = Fm*S_L new \quad (8.8.6)$$

The new F's are then used as inputs to the Thompson and Bell model, and the results for the alternative F-patterns, F(current) and F(new), can be compared (Hoydal *et al.*, 1980 and 1982). This method is a generalization of the methods suggested by Gulland (1961), Jones (1961) and Kimura (1977).

Computer programs

The program "MIXFISH" in the LFSA package (Sparre, 1987) contains an option for mesh assessment corresponding to the procedure described above. It produces an output table showing the total yield for various combinations of effort and L50%, i.e. a table of the form shown below:

		Relative Effort				
		- 20%	- 10%	no change	+ 10%	+ 20%
	- 30%					
Relative value of L50%	- 15%					
	no change			YIELD		
	+ 15%					
	+ 30%					

MIXFISH assumes L75% to be proportional to L50%. MIXFISH allows you to test any combination of L50% and effort, and thereby enables you to determine the optimum combination of L50% and effort. The middle cell (marked "YIELD") corresponds to the current fishing regime.

A similar program has also been incorporated in FiSAT.