

APPENDIX TABLES

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1 JANUARY

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TABLE A1 LIST OF IMPORTANT FORMULAS

Length-weight relationship

$$W(i) = a \cdot L(i)^b \quad (2.6.1)$$

Von Bertalanffy growth equation

$$L(t) = L_{\infty} \cdot [1 - \exp(-K \cdot (t - t_0))] \quad (3.1.0.1)$$

Growth rate, von Bertalanffy growth equation

$$\frac{\Delta L}{\Delta t} = K \cdot (L_{\infty} - L(t)) \quad \text{cm/year} \quad (3.1.0.4)$$

Weight-based von Bertalanffy growth equation

$$W(t) = W_{\infty} \cdot [1 - \exp(-K \cdot (t - t_0))]^3 \quad (3.1.2.1)$$

Inverse von Bertalanffy growth equation

$$t(L) = t_0 - \frac{1}{K} \cdot \ln(1 - L/L_{\infty}) \quad (3.3.3.2)$$

Exponential decay model (1)

$$\frac{\Delta N(t)}{\Delta t} = -Z \cdot N(t) \quad (4.2.1)$$

Exponential decay model (2)

$$N(t) = N(Tr) \cdot \exp[-Z \cdot (t - Tr)] \quad (4.2.2)$$

Baranov's equation or catch equation (1)

$$C(t_1, t_2) = \frac{F}{Z} \cdot [N(t_1) - N(t_2)] \quad (4.2.4)$$

Catch equation (2)

$$C(t_1, t_2) = N(t_1) \cdot \frac{F}{Z} \cdot [1 - \exp(-Z \cdot (t_2 - t_1))] \quad (4.2.7)$$

Average number of survivors (from t1 to t2)

$$\bar{N}(t_1, t_2) = N(t_1) * \frac{1 - \exp(-Z*(t_2-t_1))}{Z*(t_2-t_1)} \quad (4.2.9)$$

Total mortality from CPUE data

$$z = \frac{1}{t_2-t_1} * \ln \frac{CPUE(t_1)}{CPUE(t_2)} \quad (4.3.0.3)$$

Linearized length-converted catch curve

$$\ln \frac{C(L_1, L_2)}{\Delta t(L_1, L_2)} = c - z * t \left(\frac{L_1 + L_2}{2} \right) \quad (4.4.5.3)$$

Pauly's formula for M (ln-based)

$$\ln M = -0.0152 - 0.279 * \ln L_{\infty} + 0.6543 * \ln K + 0.463 * \ln T \quad (4.7.2.1)$$

VPA equations (age-based)

$$\frac{C(y, t, t+1)}{N(y+1, t+1)} = \frac{F(y, t, t+1)}{M+F(y, t, t+1)} * \left[\exp[F(y, t, t+1)+M] - 1 \right] \quad (5.1.3)$$

$$N(y, t) = N(y+1, t+1) * \exp[F(y, t, t+1)+M] \quad (5.1.4)$$

Pope's age-based cohort analysis (numbers)

$$N(t) = \left[N(t+\Delta t) * \exp(M*\Delta t/2) + C(t, t+\Delta t) \right] * \exp(M*\Delta t/2) \quad (5.2.3)$$

Pope's age-based cohort analysis (fishing mortalities)

$$F(t, t+\Delta t) = \frac{1}{\Delta t} * \ln \left[\frac{N(t)}{N(t+\Delta t)} \right] - M \quad (5.2.4)$$

Jones' length-based cohort analysis (numbers)

$$N(L_1) = \left[N(L_2) * H(L_1, L_2) + C(L_1, L_2) \right] * H(L_1, L_2) \quad (5.3.4)$$

Length-based catch equation

$$C(L_1, L_2) = N(L_1) * \frac{F}{Z} * \left[1 - \exp(-Z*\Delta t) \right] \quad (5.3.5)$$

Jones' length-based cohort analysis (fishing mortalities)

$$F(L1, L2) = M \frac{F(L1, L2)/Z(L1, L2)}{1 - [F(L1, L2)/Z(L1, L2)]} \quad (5.3.6)$$

where the exploitation rate F/Z is derived from:

$$F(L1, L2)/Z(L1, L2) = \frac{C(L1, L2)}{N(L1) - N(L2)} \quad (5.3.7)$$

Logistic curve for gear selectivity (length-based)

$$s_L = \frac{1}{1 + \exp(s1 - s2*L)} \quad (6.1.1)$$

Set of formulas for age-based Thompson and Bell analysis

age interval: $i = (t_i, t_i + \Delta t)$

$$Z_i = M + X * F_i$$

$$N(t_i + \Delta t) = N(t_i) * \exp(-Z_i * \Delta t)$$

$$C_i = [N(t_i) - N(t_i + \Delta t)] * X * F_i / Z_i$$

$$\bar{w}_i = w(t_i + \Delta t / 2) \quad (8.6.1)$$

$$Y_i = C_i * \bar{w}_i$$

$$\bar{B}_i = Y_i / [F_i * \Delta t * X]$$

$$V_i = Y_i * \bar{v}_i$$

Set of formulas for length-based Thompson and Bell analysis

$$\begin{aligned}
 &\text{length interval: } i = (L_i, L_{i+1}) \\
 &Z_i = M + X \cdot F_i \\
 &N(L_{i+1}) = N(L_i) \cdot \frac{1/H_i - X \cdot F_i / Z_i}{H_i - X \cdot F_i / Z_i} \quad \text{where} \\
 &H_i = \left[\frac{L_\infty - L_i}{L_\infty - L_{i+1}} \right]^{M/2K} \\
 &C_i = [N(L_i) - N(L_{i+1})] \cdot X \cdot F_i / Z_i \quad (8.7.7) \\
 &\bar{w}_i = q \cdot [(L_i + L_{i+1}) / 2]^b \\
 &Y_i = C_i \cdot \bar{w}_i \\
 &V_i = Y_i \cdot \bar{v}_i \\
 &\bar{N}_i \cdot \Delta t_i = [N(L_i) - N(L_{i+1})] / Z_i \\
 &\bar{B}_i \cdot \Delta t_i = \bar{N}_i \cdot \Delta t_i \cdot \bar{w}_i
 \end{aligned}$$

Schaefer model

$$Y(i)/f(i) = a + b \cdot f(i) \quad \text{if } f(i) \leq -a/b \quad (9.1.2)$$

Fox model

$$\ln (Y(i)/f(i)) = c + d \cdot f(i) \quad (9.1.3)$$

TABLE A2
METHODS BASED ON LINEAR TRANSFORMATIONS AND ORDINARY LINEAR REGRESSION ANALYSIS: $y = a + b \cdot x$

Method	Assumption	Independent (x)	Dependent (y)	Slope (b)	Intercept (a)	Eq.
Length-weight relationship	$W_i = q \cdot L_i^p$	$\ln L_i$	$\ln W_i$	b	$\ln q$	2.6.1
Bhattacharya plot	lengths-at-age are $N(x, s^2)$	$x + dL/2$	$\Delta \ln N(x+dL/2)$	$-dL/s^2$	$\bar{x} \cdot dL/s^2$	2.6.5
GROWTH						
Gulland and Holt plot	Δt small but not constant	$\bar{L}(t)$	$\Delta L/\Delta t$	-K	$K \cdot L_\infty$	3.3.1.1
Ford-Walford plot	Δt constant	$L(t)$	$L(t+\Delta t)$	$\exp(-K \cdot \Delta t)$	$[1 - \exp(-K \cdot \Delta t)] \cdot L_\infty$	3.3.2.1
Chapman's method	Δt constant	$L(t)$	$L(t+\Delta t) - L(t)$	$\exp(-K \cdot \Delta t) - 1$	$[1 - \exp(-K \cdot \Delta t)] \cdot L_\infty$	3.3.2.2
Von Bertalanffy plot	L_∞ known	t	$-\ln(1 - L(t)/L_\infty)$	K	$-K \cdot t_0$	3.3.3.1
MORTALITY						
Age-based	Steady state $t \geq T'$					
Catch curve	Δt small and variable	$t + \Delta t/2$	$\ln[C(t, t+\Delta t)/\Delta t]$	-Z	not used	4.4.4.2
Catch curve	Δt constant	t	$\ln C(t, t+\Delta t)$	-Z	not used	4.4.3.1
Cumulated catch curve	$\Delta t = \infty$	t	$\ln C(t, \infty)$	-Z	not used	4.4.4.1
Length-based	$L \geq L'$					
Catch curve		$t \left(\frac{L1+L2}{2} \right)$	$\ln \frac{C(L1, L2)}{\Delta t(L1, L2)}$	-Z	not used	4.4.5.3
Jones-van Zalinge	L_∞ known	$\ln(L_\infty - L1)$	$\ln C(L1, L_\infty)$	Z/K	not used	4.4.6.1
Powell-Wetherall	$L \geq L1 = L'$	L1	$\bar{L} - L1$	$-1/(1+Z/K)$	$L_\infty/(1+Z/K)$	4.5.4.1

TABLE A2 (CONTINUED)
METHODS BASED ON LINEAR TRANSFORMATIONS AND ORDINARY LINEAR REGRESSION ANALYSIS: $y = a + b*x$

Method	Assumption	Independent (x)	Dependent (y)	Slope (b)	Intercept (a)	Eq.
Plot of Z on effort (f)	constant catchability (q)	f_i	Z_i	q	M	4.6.1
GEAR SELECTIVITY						
Covered codend	logistic curve	L	$\ln(1/S_L - 1)$	-S2	S1	6.1.2
Resultant curve (from catch curve)	steady state $M_L = Z - Fm * S_L$	t	$\ln(1/S_L - 1)$	-T2	T1	6.4.3.3
Gill net selection (two net experiment)	bell-shaped curve (Im, s^2): $Im = SF * m$	$(L1+L2)/2$	$\ln \left[\frac{Cb(L1, L2)}{Ca(L1, L2)} \right]$	$SF * (m_b - m_a) / s^2$ (not given in Chapter 6)	$-(SF)^2 * (m_b^2 - m_a^2) / (2s)^2$	6.2.1.2
SURPLUS PRODUCTION MODELS						
Schaefer model	$f < -a/b$	f_i	Y_i / f_i	$-MSY / f_{MSY}^2$	$2 * MSY / f_{MSY}$	9.1.2
Fox model	constant catchability (q)	f_i	$\ln(Y_i / f_i)$	$-1 / f_{MSY}$	$1 + \ln(MSY / f_{MSY})$	9.1.3
STATISTICS						
Mean	n data: (x_i, y_i)	$\bar{x} = \frac{1}{n} \sum x_i$ sx	$\bar{y} = \frac{1}{n} \sum y_i$ sy	$b = sxy / sx^2$	$a = \bar{y} - \bar{x} * b$ sa	2.1.1 2.4.5/6
Standard deviation						
Variance	the y's are $N(ax+b, s^2)$ with constant s	$sx^2 = \frac{\sum [x(i) - \bar{x}]^2}{n-1}$	$sy^2 = \frac{\sum [y(i) - \bar{y}]^2}{n-1}$	$sb^2 = \frac{sb \left[\frac{sy}{sx} \right]^2 - b^2}{n-2}$	$sa^2 = sb^2 * \left[\bar{x}^2 + \frac{n-1}{n} * sx^2 \right]$	2.1.2 2.4.11/12
Covariance		$sxy = \frac{1}{n-1} (\sum x(i)y(i) - \frac{1}{n} \sum x(i) \sum y(i))$				
Confidence limits	t-values in Table 2.3.1			$b \pm sb * t_{n-2}$	$a \pm sa * t_{n-2}$	2.4.9

TABLE A3
IMPORTANT DATES EXPRESSED AS FRACTIONS OF A YEAR FROM 1 JANUARY

date	no. of days (cumulated)	fraction of year	date	no. of days (cumulated)	fraction of year
1 Jan	0	0.00	15 Jan	14	0.04
1 Febr	31	0.08	15 Febr	45	0.12
1 March	59	0.16	15 March	74	0.20
1 April	90	0.25	15 April	104	0.28
1 May	120	0.33	15 May	135	0.37
1 June	151	0.41	15 June	165	0.45
1 July	181	0.50	15 July	196	0.54
1 Aug	212	0.58	15 Aug	227	0.62
1 Sept	243	0.67	15 Sept	257	0.70
1 Oct	273	0.75	15 Oct	288	0.79
1 Nov	304	0.83	15 Nov	318	0.87
1 Dec	334	0.92	15 Dec	349	0.96
1 Jan	365	1.00	15 Jan	365	1.00
1 Febr	396	1.08	15 Febr	379	1.04
1 March	424	1.16	15 March	407	1.12
etc.	etc.	etc.	etc.	etc.	etc.

TABLE A4 FRACTILES OF THE t-DISTRIBUTION (STUDENT'S DISTRIBUTION)

degrees of freedom f	fractiles			degrees of freedom f	fractiles		
	90% t_f	95% t_f	99% t_f		90% t_f	95% t_f	99% t_f
1	6.31	12.71	63.66	15	1.75	2.13	2.95
2	2.92	4.30	9.93	16	1.75	2.12	2.92
3	2.35	3.18	5.84	17	1.74	2.11	2.90
4	2.13	2.78	4.60	18	1.73	2.10	2.88
5	2.02	2.57	4.03	19	1.73	2.09	2.86
6	1.94	2.45	3.71	20	1.73	2.09	2.85
7	1.90	2.37	3.50	25	1.71	2.06	2.79
8	1.86	2.31	3.36	30	1.70	2.04	2.75
9	1.83	2.26	3.25	40	1.68	2.02	2.70
10	1.81	2.23	3.17	50	1.67	2.01	2.68
11	1.80	2.20	3.11	60	1.67	2.00	2.66
12	1.78	2.18	3.06	80	1.67	1.99	2.64
13	1.77	2.16	3.01	100	1.66	1.98	2.63
14	1.76	2.15	2.98	∞	1.65	1.96	2.58

This volume describes in detail a selection of methods on fish stock assessment, with examples of calculations. Special emphasis is placed on methods based on the analysis of length frequencies. After a short introduction to statistics, the publication covers the estimation of growth parameters and mortality rates; virtual population methods, including age-based and length-based cohort analysis; gear selectivity; sampling; prediction models, including Beverton and Holt's yield-per-recruit model and Thompson and Bell's model; surplus production models; multispecies and multilient problems; the assessment of migratory stocks; stock/recruitment relationships; and demersal trawl surveys, including the sweep-area method. The manual ends with a review of stock assessment, giving an indication of methods to be applied at different levels of availability of input data. The publication also includes a review of relevant computer programs produced by or in cooperation with FAO, and a bibliography, including material for further reading. The companion volume, Part 2: Exercises, gives a number of exercises (and their solutions) directly related to the various chapters and sections of the manual.

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