Poverty Analysis

Poverty and Dominance
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Table of Contents

1 Summary ................................................................................................. 1

2 Introduction.............................................................................................. 1

3 Conceptual background ......................................................................... 2
  3.1 Dominance criteria for the headcount ratio ...................................... 2
  3.2 Dominance criteria for poverty gaps ............................................... 3
  3.3 Dominance criteria for the FGT poverty measure ............................ 4
  3.4 The TIP curves .................................................................................. 5

4 A step-by-step procedure for dominance criteria ................................. 7
  4.1 A step-by-step procedure for dominance criteria for ....................
      headcount ratio ................................................................................. 7
  4.2 A step-by-step procedure for dominance criteria for ....................
      poverty gaps .................................................................................... 8
  4.3 A step-by-step procedure for TIP curves ....................................... 10

5 A numerical example of how to calculate dominance conditions ....... 11
  5.1 An example of dominance with the headcount ratio ..................... 11
  5.2 An example of dominance with the poverty gap ..........................
      and FGT measures ................................................................. 13
  5.3 An example of dominance with the TIP curve ............................... 16

6 Readers’ notes ....................................................................................... 18
  6.1 Time requirements .......................................................................... 18
  6.2 Frequently asked questions .......................................................... 18
  6.3 Complementary capacity building materials .................................. 19

7 References and further readings ............................................................ 19

8 Module metadata .................................................................................. 20
1 SUMMARY

This module illustrates how some simple poverty measures may be linked with dominance conditions between particular types of curves. This strongly resembles the dominance conditions already set out in the case of Lorenz curves. In particular, dominance conditions will be derived for the headcount ratio and for the Foster-Greer-Thorbecke (FGT) measures showing that, under certain conditions, the poverty line specification is not necessary. This module also introduces the concept of the Three I’s of Poverty (TIP) curve. As a way to analyse poverty, this module is based on a different approach to poverty measurement. Nor does it recourse to a poverty line or to an exact poverty measure. Rather, it relies on dominance of appropriate curves.

2 INTRODUCTION

Objectives

The aim of this module is to illustrate how poverty analysis corresponding to some common poverty indices may be carried out on the basis of particular types of curves. It illustrates how these curves can be used for policy analysis.

Target audience

The module targets policy analysts who want to have a wide range of information in order to properly advise policy-makers.

Required background

As this module completes a set of module based on poverty measurement, it is strongly recommended to read across all other EASYPol Modules on poverty measurement before going through this text. This method of investigating poverty is particularly important if we want to have a broad and visual inspection of how the income distribution may change if a given policy (e.g. investment support, public subsidies, etc.) is implemented. Of particular importance is the fact that this method, under certain conditions, provides results that are robust to the choice of the poverty line.

The trainer is strongly recommended to verify how adequate the trainees’ background is, notably their understanding of the concepts of income distribution, social welfare and poverty measurement, especially ad hoc poverty measures and generalised poverty gap measures. If their background is weak or missing, the trainer may consider delivering other EASYPol Modules beforehand, as highlighted in the

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1 EASYPol Module 000: Charting Income Inequality: The Lorenz Curve.
2 EASYPol Module 007: Poverty Analysis: Basic Poverty Measures.
introduction. Other technicalities present in this module should be understood by all people with an elementary knowledge of basic mathematics and statistics.

A complete set links of other related EASYPol modules are included at the end of this module. However, users will also find links to related material throughout the text where relevant\(^4\).

In addition, preparation and running exercises slightly more complex than the examples provided in the module with real data must be considered.

### 3 CONCEPTUAL BACKGROUND

All poverty measures so far analysed\(^5\) need a specification of the poverty line. However, there may be disagreement both about the right location of the poverty line and about the proper poverty measure. A number of dominance criteria may therefore be developed which enable poverty comparisons while at the same time allow for different ways of identifying the poor (i.e. for different poverty lines) and for different ways of measuring poverty (i.e. for different poverty measures)\(^6\).

In other EASYPol modules, the link between social welfare and dominance criteria has also been discussed. In particular, it has been found that the dominance of Lorenz curves has a correspondence with social welfare rankings. As poverty may be thought of as a focus on a part of the income distribution, we can expect that corresponding dominance criteria may be developed also for poverty analysis.

All dominance criteria below are developed assuming a common poverty line. Dominance criteria when using different poverty lines can also be accommodated, but the discussion is left for more advanced tools\(^7\).

#### 3.1 Dominance criteria for the headcount ratio

The first dominance criteria that is worth developing is related to the most common poverty measure, the headcount ratio \(HC\). The headcount ratio may be directly expressed by the Cumulative Distribution Function (CDF) measured up to the poverty

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\(^4\) EASYPol hyperlinks are shown in blue, as follows:

- a) training paths are shown in [underlined bold font](#)
- b) other EASYPol modules or complementary EASYPol materials are in [bold underlined italics](#);
- c) links to the glossary are in [bold](#); and
- d) external links are in [italics](#).


\(^6\) This literature has developed since the contribution of Atkinson, 1987. A recent review is from Zheng, 2000.

\(^7\) We can refer to Lambert, 2001, Chapter 6, for details on this issue.
line. The cumulative distribution function $F(x)$, for any given income level $x$, gives the proportion of people who have incomes below that level. Therefore, if the income level is taken to be the poverty line $z$, the cumulative distribution function $F(z)$ gives the proportion of people who have incomes below $z$, i.e. the proportion of poor people.

Suppose there are two different income distributions, $A$ and $B$, (relating to, say, different years or countries, etc.) having the same poverty line $z$. Therefore, $F_A(z)$ measures the proportion of people in poverty in income distribution $A$, while $F_B(z)$ measures the same proportion in income distribution $B$. If the following condition holds:

$$F_A(z) > F_B(z)$$

for all $x < z$ (i.e., if the CDF of income distribution $A$ is everywhere above the CDF of income distribution $B$ up to the income level $z$) the headcount ratio will always be higher in $A$ than in $B$ for all poverty lines up to $z$. If there is disagreement about the right location of $z$, we can test the dominance criteria up to a maximum conceivable poverty line, say $z_{\text{max}}$. If dominance occurs up to that point, the headcount ratio of the dominating distribution will be higher for all poverty lines up to that point. If the dominance of distribution $A$ over $B$ extends over the whole CDF, the result will hold for any arbitrary poverty line.

### 3.2 Dominance criteria for poverty gaps

Dominance criteria can also be established for another popular measure of poverty, the poverty gap. Quite interestingly, the dominance criteria, in this case, is related to the Generalised Lorenz (GL) curve.

Given two income distributions, $A$ and $B$, if the GL curve of income distribution $A$ is «everywhere» above the GL curve of income distribution $B$, the poverty gap of income distribution $B$ will always be higher than the poverty gap of income distribution $A$. In other words, GL dominance implies less poverty as measured by the poverty gap. In this case, unlike welfare analysis, it is necessary that dominance occurs for a subset of the income distribution, corresponding to the income level just above any conceivable poverty line.

It is also worth briefly discussing an alternative way of expressing this result, as it appears in the specialized literature about poverty measurement. To this purpose, let us define a **poverty deficit (PD) curve** as the cumulated sum of normalised poverty gaps:

\[ \text{PD} = \sum_{x} \frac{ \text{poverty gap at income } x }{ \text{poverty gap at } z_{\text{max}} } \]

Just recall that for welfare analysis, dominance must occur over the whole income distribution. For poverty analysis, it is sufficient that it occurs until the maximum conceivable poverty line. Obviously, dominance over the whole income distribution implies dominance for any poverty line.

See Deaton, 1997.
where $y$ is the individual income, $z$ is the poverty line, $p$ is the number of poor. Note that ranking income distributions by ascending incomes is equivalent to rank income distributions by descending poverty gaps. The individual with the lowest income indeed has the highest poverty gap. The dominance criterion can therefore be stated as follows: if the PD curve of income distribution $A$ is everywhere below the PD curve of income distribution $B$, poverty as measured by the poverty gap will always be lower in $A$ than in $B$.

The two criteria are equivalent and the equivalence may be restated by observing that the PD curve of income distribution $B$ is everywhere above the PD curve of income distribution $A$ only if the GL curve of $B$ is everywhere below the GL curve of $A$. Both criteria lead to more poverty in $B$ than in $A$ as measured by the poverty gap.\textsuperscript{10}

### 3.3 Dominance criteria for the FGT poverty measure

The Generalized Lorenz dominance criterion can also be used in relation to the FGT poverty index with $\alpha=2$. In particular, if an income distribution $A$ generalized Lorenz dominates an income distribution $B$, poverty as measured by FGT with $\alpha=2$ is always lower in income distribution $A$ than in income distribution $B$. In fact, as reported by Lambert, 2001, the generalized Lorenz dominance condition extends to a wide class of poverty indexes that are decreasing and convex with respect to income. If we define this class by $C$, analytically the generalized Lorenz dominance condition applies to all members $c$ of this class for which $\frac{\partial c}{\partial y} < 0; \frac{\partial^2 c}{\partial^2 y} > 0$. The first condition means that the poverty index should decrease if income of poor individuals increases; the second condition means that the decrease in the poverty index is higher if a given amount of income is added to relatively poorer individuals. A graphical intuition of decreasing and convexity in income may be given in Figure 1, below.
When the poverty index is decreasing and convex in income, an increase in income reduces its level, but this reduction is higher if that same income increase occurs at lower income levels. This can be seen in the graph, where the highest \( dP \) on the \( y \)-axis, following an increase of income at lower income levels, is greater than the lowest \( dP \) on the \( y \)-axis, following the same increase at higher income levels.

### 3.4 The TIP curves

For all poverty indexes expressed in terms of normalised poverty gaps, another useful dominance criterion is available. Before proceeding any further, it is worth recalling the Three I’s of Poverty (TIP) defined by the Sen index:

- **Incidence**
- **Intensity**
- **Inequality**

Following this approach, Spencer and Fisher, 1992, Jenkins and Lambert, 1997, and Shorrocks, 1998, have defined (and refined) a curve giving a synthesis of these three I’s and stated a useful dominance criterion.

The TIP curve is defined by plotting the cumulated proportion of population on the \( x \)-axis (as in Lorenz or Generalised Lorenz curve\(^{11}\)) and the cumulated per capita poverty gap \( PG \) on the \( y \)-axis from the biggest one downwards. Note the difference between the TIP curve and a standard Lorenz curve. In this latter, incomes are cumulated from the lowest to the highest; in the TIP curve, the normalised poverty gaps are cumulated from the biggest to the smallest. This make sense, if one think that the highest normalised poverty gap is equivalent to the lowest income, as the poverty gap measures the distance

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\(^{11}\) See EASYPol Modules 001 and 002 respectively: [Social Welfare Analysis of Income Distribution: Lorenz Curves](#) and [Social Welfare Analysis of Income Distribution: Generalised Lorenz Curves](#).
between each income and the poverty line. The maximum distance (the biggest poverty gap) is therefore equivalent to the lowest income.

The dominance criterion is the following: Given two income distributions A and B and a common poverty line \( z \), if the TIP curve of income distribution B dominates the TIP curve of income distribution A up to the maximum conceivable poverty line, there will always be more poverty in B than in A as measured by the class of normalised poverty gap measures.

The TIP curve has a typical configuration and may be given maximum and minimum benchmarking. When the income distribution exhibits maximum poverty, i.e. all individuals have zero income, the TIP curve is linearly increasing. When the income distribution exhibits no poverty, i.e. all individuals have incomes equal to the poverty line, the TIP curve corresponds to the horizontal axis. Figure 2 illustrates this situation, drawing a typical TIP curve (A) with corresponding extreme cases (TIP max and TIP min).

**Figure 2 - A typical TIP curve, maximum and minimum benchmarking**

As mentioned above, the TIP curve gives a synthesis of the three I’s of poverty. Figure 3, below, illustrates how this can be done, making reference to LENGTH, CURVATURE and HEIGHT.

**LENGTH** – A TIP curve cumulates the PG measure up to the maximum poverty line. Until the poverty line is achieved, the cumulated sum increases (at decreasing rates, however, as it adds lower and lower poverty gaps so far as incomes approach the poverty line). Once the maximum poverty line is achieved, the TIP curve becomes horizontal, as there is no addition to poverty gaps as incomes are now higher than the poverty line. A typical TIP curve is therefore concave up to the poverty line and then
flat. The length of the non-horizontal portion of the TIP curve reveals the INCIDENCE OF POVERTY. In fact, the length is equivalent to the headcount ratio, i.e. the proportion of people below the poverty line. In the example of figure 3, this proportion is about 65 per cent of population.

CURVATURE – A TIP curve is concave up to the maximum poverty line. The degree of concavity summarizes INEQUALITY AMONG POOR, as it reveals the rate at which gaps decrease as income rises. If there is a higher degree of inequality among poor with, say, very large poverty gaps for few individuals and very low poverty gaps for the others, the TIP curve becomes more concave.

HEIGHT – The TIP curve becomes horizontal after the maximum poverty line is achieved. The height of the TIP curve on the y-axis corresponding to that point reveals the INTENSITY OF POVERTY. In particular, the height of the TIP curve corresponds to the value of the PG index at that poverty line, 0.25 in the graph.

**Figure 3 - TIP curve and the three I’s of poverty**

![TIP curve and the three I's of poverty](image)

### 4 A STEP-BY-STEP PROCEDURE FOR DOMINANCE CRITERIA

In order to implement dominance condition, some steps are required that are different for each corresponding poverty measures.

**4.1 A step-by-step procedure for dominance criteria for headcount ratio**

Figure 4 illustrates the very simple steps to check for dominance conditions corresponding to the headcount ratio.
### Figure 4 - A step-by-step procedure to check for dominance for the headcount ratio

<table>
<thead>
<tr>
<th>STEP</th>
<th>Operational content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If not already sorted, sort income distributions by income level</td>
</tr>
<tr>
<td>2</td>
<td>Calculate the cumulative distribution function (CDF)</td>
</tr>
<tr>
<td>3</td>
<td>Plot the CDFs of the income distributions to be compared</td>
</tr>
<tr>
<td>4</td>
<td>Check for dominance. If dominance does not occur over the overall income distribution, identify the income level until which dominance occurs</td>
</tr>
<tr>
<td>5</td>
<td>Ask whether the income level in Step 4 is such to include any conceivable poverty line</td>
</tr>
</tbody>
</table>

**Step 1** requires, as usual, that we sort income distributions in ascending order of income. **Step 2** requires that we calculate the cumulative distribution function of the income distributions. In **Step 3** we have to plot the cumulative distribution functions of the income distributions to be compared. **Step 4** is qualitative, as it requires a visual inspection of whether a given CDF dominates over another. If one CDF dominates over another for the whole range, there is no need to define the proper poverty line, as the result holds for any poverty line. If overall dominance does not occur, it is important to identify the income level until dominance occurs. In **Step 5** we indeed need to investigate whether the income level identified in Step 4 is such that all conceivable poverty lines are below it. If yes, the poverty ranking will not depend on the exact choice of the poverty line. If not, the result may depend on the choice of the poverty line.

### 4.2 A step-by-step procedure for dominance criteria for poverty gaps

Figure 5, below, illustrates the steps required to check for dominance conditions that correspond to higher poverty gaps. After the usual requirement of ranking income...
distributions by income levels (Step 1), we are required to plot either GL curves or PD curves (either Step 2 or Step 3). How to build GL curves has already been illustrated elsewhere. Whereas, building PD curves, requires that we first calculate, for each individual, the normalised poverty gap (Step 3a). Then, normalised poverty gaps must be cumulated up to the income level below any conceivable poverty line (Step 3b). Finally, income levels must be plotted against the cumulated poverty gap (Step 3c).

**Figure 5 - A step-by-step procedure to check for dominance for the poverty gap**

<table>
<thead>
<tr>
<th>STEP</th>
<th>Operational content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If not already sorted, sort income distributions by income level</td>
</tr>
<tr>
<td>2</td>
<td>Either calculate GL curves... 3a Calculate the normalised poverty gap for each individual</td>
</tr>
<tr>
<td>3</td>
<td>...or calculate PD curves 3b Cumulate the normalised poverty gaps up to the maximum poverty line</td>
</tr>
<tr>
<td>4</td>
<td>Check for dominance. If dominance does not occur over the overall income distribution, identify the income level until which dominance occurs 3c Plot income levels against the cumulated poverty gap</td>
</tr>
<tr>
<td>5</td>
<td>Ask whether the income level in Step 4 is such to include any conceivable poverty line</td>
</tr>
</tbody>
</table>

**Step 4** is qualitative, as it requires a visual inspection of whether a given GL curve or PD curve dominates over another. If one of them dominates over the other on the whole range, there is no need to define the proper poverty line, as the result holds for any poverty line. If overall dominance does not occur, it is important to identify the income level until dominance occurs. In **Step 5** we indeed need to investigate whether the income level identified in Step 4 is such that all conceivable poverty lines are below it. If yes, the poverty ranking will not depend on the exact choice of the poverty line. If not, the result may depend on the choice of the poverty line.
4.3 **A step-by-step procedure for TIP curves**

Figure 6, below, illustrates how to build a TIP curve. After having sorted the income distribution by ascending level of income, which corresponds to a ranking by decreasing level of poverty gaps (**Step 1**), we need to define the maximum conceivable poverty line (**Step 2**). Using this poverty line, we must then calculate the PG measure for each individual falling below the poverty line. In other words, we must apply the formula for PG measure $PG_i = \frac{1}{p} \left( \frac{z - y_i}{z} \right)$ for each individual $i$, where $p$ is the number of poor people corresponding to the maximum poverty line (**Step 3**). All these poverty gaps must then be cumulated (**Step 4**) and plotted against the cumulative proportion of population (**Step 5**).

**Figure 6 - A step-by-step procedure to check for dominance for TIP curves**

<table>
<thead>
<tr>
<th>STEP</th>
<th>Operational content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If not already sorted, sort income distributions by income level</td>
</tr>
<tr>
<td>2</td>
<td>Define the maximum conceivable poverty line</td>
</tr>
<tr>
<td>3</td>
<td>Calculate the PG measure for each poor individual falling below the max poverty line</td>
</tr>
<tr>
<td>4</td>
<td>Cumulate the PG measure across individuals up to the maximum poverty line</td>
</tr>
<tr>
<td>5</td>
<td>Plot the cumulative proportion of population against the cumulated PG measure</td>
</tr>
</tbody>
</table>
5 A NUMERICAL EXAMPLE OF HOW TO CALCULATE DOMINANCE CONDITIONS

5.1 An example of dominance with the headcount ratio

Table 1, below, reports an example of the elements needed to derive dominance conditions corresponding to the headcount ratio. In order to properly represent these conditions, recourse is made to an extended income distribution of thirty individuals.
Table 1 - An example of how to check for dominance criteria for the headcount ratio

<table>
<thead>
<tr>
<th>Individual</th>
<th>Income distribution A</th>
<th>CDF A</th>
<th>Individual</th>
<th>Income distribution B</th>
<th>CDF B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,417</td>
<td>0.033</td>
<td>1</td>
<td>1,417</td>
<td>0.033</td>
</tr>
<tr>
<td>2</td>
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<td>2</td>
<td>3,392</td>
<td>0.067</td>
</tr>
<tr>
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<td>0.100</td>
<td>3</td>
<td>4,200</td>
<td>0.100</td>
</tr>
<tr>
<td>4</td>
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<td>0.133</td>
<td>4</td>
<td>4,948</td>
<td>0.133</td>
</tr>
<tr>
<td>5</td>
<td>6,500</td>
<td>0.167</td>
<td>5</td>
<td>5,500</td>
<td>0.167</td>
</tr>
<tr>
<td>6</td>
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<td>0.200</td>
<td>6</td>
<td>6,048</td>
<td>0.200</td>
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<tr>
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<td>7,280</td>
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<td>6,280</td>
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<td>6,800</td>
<td>0.267</td>
</tr>
<tr>
<td>9</td>
<td>7,800</td>
<td>0.300</td>
<td>9</td>
<td>6,800</td>
<td>0.300</td>
</tr>
<tr>
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<td>10</td>
<td>6,814</td>
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<td>8,295</td>
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<td>7,295</td>
<td>0.433</td>
</tr>
<tr>
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<td>0.467</td>
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<td>7,450</td>
<td>0.467</td>
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<td>11,906</td>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>26</td>
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<td>15,519</td>
<td>0.967</td>
</tr>
<tr>
<td>30</td>
<td>15,239</td>
<td>1.000</td>
<td>30</td>
<td>26,239</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The example is developed considering two income distributions, A and B. Steps 1 and 2 only require that we sort income distributions by income level and that we calculate the corresponding cumulative distribution functions. Steps 3 to 5 of the step-by-step procedure are best illustrated by Figure 7.
Step 3 requires that we plot CDFs of the two income distributions. This gives the result reported in Figure 7, where income distribution A is identified by the bold line, while income distribution B is identified by the solid line. Step 4 requires that we check whether dominance occurs. As can be easily seen, dominance of income distribution B occurs up to around 10,000 units of income. Up to this point, therefore, the headcount ratio of income distribution B is higher than the headcount ratio of the income distribution A, regardless of the exact specification of the poverty line. Is this income level such that all conceivable poverty lines are below it? If yes, the example in Figure 7 gives unambiguous results for poverty rankings as measured by the headcount ratio, otherwise the outcome will depend on the specific poverty line chosen (Step 5), as the cumulative distribution functions of the two income distributions cross after that income level.

5.2 An example of dominance with the poverty gap and FGT measures

Table 2, below, reports the required elements to check for dominance criteria associated with the poverty gap. Income distributions must be sorted and then GL curves and PD curves must be calculated (Step 1 and 2). Note that in Table 2, the PD curves are calculated up to income levels just below 10,000 income units, which is the maximum poverty line.
### Table 2 - An example of how to check for dominance

Sort income distributions by income level and define GL and PD

<table>
<thead>
<tr>
<th>Individual</th>
<th>Income distribution A</th>
<th>Define GL(A)</th>
<th>Define PD(A)</th>
<th>Individual</th>
<th>Income distribution B</th>
<th>Define GL(B)</th>
<th>Define PD(B)</th>
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<td></td>
<td>30</td>
<td>16,239</td>
<td>9,069</td>
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</tr>
</tbody>
</table>

Mean income | 9,069 | Mean income | 9,069 |
Total income | 272,060 | Total income | 272,060 |
Max poverty line | 10,000 | Max poverty line | 10,000 |

The elements of Table 2 can then be used to draw Figure 8.
Figure 8 - An example of how to check for dominance

GL curves

PD curves

Figure 8 reports the test for dominance using both GL and PD curves. As can be easily seen from the top graph, income distribution A, GL dominates income distribution B over the whole income range. It means that the poverty gap of income distribution A is always lower than the poverty gap of income distribution B regardless of the exact specification of the poverty line. The same result can be read from the bottom graph of Figure 8, where poverty deficit curves are depicted up to the maximum poverty line.
(10,000 income units). In this case, the poverty deficit curve of income distribution B dominates the poverty deficit curve of income distribution A, which means that the poverty gap is always higher in B than in A, regardless of the exact specification of the poverty line. Just note again that the interpretation of dominance between income distribution is reversed when passing from GL curves to PD curves. In GL curves, the dominating distribution has less poverty; in PD curves, the dominating distribution has more poverty.

The examples reported in Table 2 and Figure 8 are also useful to check for dominance criteria associated to FGT indices. The top graph of Figure 8, giving GL dominance of income distribution A, assures that the FGT index of income distribution A will always be lower than the FGT index of income distribution B, regardless of the specific poverty line.

5.3 An example of dominance with the TIP curve

An example of how to build a TIP curve is developed in Table 3, below, for two income distributions, A and B, with the same mean and the same total income. In Step 1, both distributions are ranked by ascending income levels. In Step 2, the maximum poverty line is chosen (10,000 income units). In Step 3, the PG measure is calculated for each individual. As can be easily seen, this measure is decreasing in income. As far as income approaches the poverty line, the poverty gap obviously decrease. Step 4, finally, requires that we only cumulate individual poverty gaps calculated in Step 3.
Checking for dominance requires to plot the TIP curves. This is done in Figure 9, below. What does this graph reveal? The TIP curve of income distribution B is always above the TIP curve of income distribution A up to the maximum poverty line. This means that poverty, as measured by FGT indices, is always greater in B than in A, regardless of the exact specification of the poverty line. The height of the TIP curves, as discussed above, is equal to 0.35 for income distribution B and to 0.25 for income distribution A, which are the PG measures at the maximum poverty line.
6 READERS’ NOTES

6.1 Time requirements

The delivery of this module to an audience already familiar with poverty measurement may take about three hours.

6.2 Frequently asked questions

✓ Does poverty analysis always require to specify a poverty line? Dominance conditions provide a method to derive poverty results regardless of the exact specification of the poverty line. All that is required is that we define a maximum conceivable poverty line. If the dominance extends over the whole income distribution, even this maximum poverty line may be ignored.

✓ How do I proceed if dominance conditions are not verified for any conceivable poverty line? In this case, traditional poverty measures must be used, with the aim of verifying until which poverty line results may be considered robust.

✓ Do dominance conditions encompass all poverty indexes? No, dominance conditions are strictly linked to either specific poverty measures or classes of poverty indices. However, classes are wide enough to encompass the most used poverty indexes in empirical works.
6.3 Complementary capacity building materials

The following module should be used as a complement to dominance issues:

✓ EASYPol Module 004: *Poverty Analysis: The Definition of Poverty*
✓ EASYPol Module 005: *Poverty Analysis: Absolute Poverty Lines*
✓ EASYPol Module 006: *Poverty Analysis: Relative Poverty Lines*
✓ EASYPol Module 007: *Poverty Analysis: Basic Poverty Measures*
✓ EASYPol Module 009: *Poverty Analysis: Distributional Poverty Measures*
✓ EASYPol Module 010: *Poverty Analysis: Generalised Poverty Gap Measures*

7 REFERENCES AND FURTHER READINGS

Module metadata

1. EASYPol module 035

2. Title in original language
   English: Poverty Analysis
   French: 
   Spanish: 
   Other language: 

3. Subtitle in original language
   English: Poverty and Dominance
   French: 
   Spanish: 
   Other language: 

4. Summary
   This module illustrates how some simple poverty measures may be linked with dominance conditions between particular types of curves. This strongly resembles the dominance conditions already set out in the case of Lorenz curves (link Lorenz curves). In particular, dominance conditions will be derived for the headcount ratio (link Ad hoc poverty measures) and for the FGT measures (link Generalised poverty gap measures) showing that, under certain conditions, poverty line specification is not necessary. This module also introduces the concept of TIP curve. As a way to analyse poverty, this module is based on a different approach to poverty measurement. It does not make recourse neither to a poverty line nor to an exact poverty measure. Rather, it relies on dominance of appropriate curves.

5. Date
   November 2005

6. Author(s)
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   Paolo Liberati, University of Urbino, "Carlo Bo", Institute of Economics, Urbino, Italy

7. Module type
   ☒ Thematic overview
   ☒ Conceptual and technical materials
   ☒ Analytical tools
   ☒ Applied materials
   ☒ Complementary resources

8. Topic covered by the module
   ☒ Agriculture in the macroeconomic context
   ☒ Agricultural and sub-sectoral policies
   ☒ Agro-industry and food chain policies
   ☒ Environment and sustainability
   ☒ Institutional and organizational development
   ☒ Investment planning and policies
   ☒ Poverty and food security
   ☒ Regional integration and international trade
   ☒ Rural Development

9. Subtopics covered by the module

10. Training path
    Analysis and monitoring of socio-economic impacts of policies

11. Keywords