EASYPol

On-line resource materials for policy making

ANALYTICAL TOOLS

Module 050

Policy Impacts on Inequality

Welfare Based Measures of Inequality

The Atkinson Index

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The Atkinson Index

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About EASYPol
EASYPol is a an on-line, interactive multilingual repository of downloadable resource materials for capacity development in policy making for food, agriculture and rural development. The EASYPol home page is available at: *www.fao.org/tc/easypol*.
EASYPol has been developped and is maintained by the Agricultural Policy Support Service, Policy Assistance Division, FAO.

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1 SUMMARY

This tool illustrates one of the most popular welfare-based measures of inequality, the Atkinson Index[1]. In particular, it discusses the foundations of this Index, in terms of social welfare specifications, and the concept of equally distributed equivalent income on which the measure is based. The use of this measure is then exemplified in a step-by-step procedure and in a numerical example.

2 INTRODUCTION

This tool will deal with the passage from the descriptive approach to income distribution to the normative approach, i.e. from inequality to welfare.

Objective

The objective of the tool is to explain the most popular **welfare-based measure of inequality**, the Atkinson Index.

Use: This tool can be used in an operational context to derive welfare implications of alternative policy options. The use of the welfare-based measures allows the analyst to give normative content to his/her analysis. The use of these measures should always be compared with the possibility of using distributional dominance methodologies[2].

Target audience

This module targets current or future policy analysts who want to increase their capacities in measuring impacts of development policies on welfare. On these grounds, economists and practitioners working in public administrations, in NGOs, professional organisations or consulting firms will find this helpful reference material.

Required background

Users should be familiar with basic notions of mathematics and statistics.

Links to relevant EASYPol modules, further readings and references are included both in the footnotes and in section 7.2 of this module[3].

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[1] See **Atkinson's Theory** in the **EASYPol Glossary**.

[2] See EASYPol Modules 009 and 035 respectively: ***Impacts of Policies on Poverty: Distributional Poverty Measures*** and ***Poverty Analysis: Poverty and Dominance***.

[3] EASYPol hyperlinks are shown in blue, as follows:
a) training paths are shown in **underlined bold font;**
b) other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
c) links to the glossary are in **bold**; and
d) external links are in *italics*.

3 CONCEPTUAL BACKGROUND

Welfare-based measures of inequality are best used when the ordering through different possibilities of distributional dominance cannot provide a definite ranking[4]. It is however worth noting that welfare-based measures can be used in any case where a welfare analysis is needed. Welfare-based measures, however, are generally less powerful than distributional dominance methodologies. Let's see why.

Distributional dominance is a «partial ordering», as there are cases where the welfare of two income distributions cannot be ranked. Distributional dominance is also an «ordinal ranking», i.e. it says that one is preferred to the other but not by how much.

If the distributional dominance fails, or if we are interested in synthetic numbers representing the whole income distribution, welfare-based measures may provide for a «complete» ranking among alternative income distributions. However, this comes at the price of more stringent assumptions as to how to represent social welfare.

Before proceeding any further, it is therefore worth restating the main conceptual differences between distributional dominance and welfare-based measures of inequality. See Figure 1.

Figure 1: Welfare based measures and distributional dominance

	Welfare-based measures	Distributional dominance
Type of ordering	Complete ranking	Partial ranking
Characteristics	Single numbers (cardinal ranking)	Ordinal ranking
Assumptions on SWF [1]	Exact specification of the SWF	Wide classes of SWF
Robustness of outcome	Weak, need checking with other SWF	Strong

[1] **Social Welfare Function**

Welfare-based measures may provide for a **complete ordering** by reducing income distributions to a single number. In this way, they provide for a cardinal ranking that derives from the need to specify the exact functional form of the SWF. For this reason, we only get a weak robustness from the outcome of these indexes, and the results should be checked with those deriving from the use of other SWF.

[4] See EASYPol Modules 009 and 035 respectively: ***Impacts of Policies on Poverty: Distributional Poverty Measures*** and ***Poverty Analysis: Poverty and Dominance.***

Distributional dominance, instead, has the characteristic of **partial ranking**, as there are cases in which it is silent. Since it does not need the exact specification of the SWF, it is also an ordinal ranking. Income distributions are ranked, but no single number is associated to them. Since assumptions on SWF are minimal, the robustness of the outcome, is strong.

Let us now turn to the use of welfare-based measures. A prominent role in welfare-based measures of inequality is played by the Atkinson's Index of inequality. The Atkinson Index is directly related to the class of additive SWF:

$$W = \frac{1}{N}\sum_{i=1}^{n} U(y_i) \qquad [1]$$

Expression [1] says that social welfare is represented by average utility. The form of the function U, according to Atkinson, is the following:

$$U(y_i) = \frac{1}{1-\varepsilon} y_i^{1-\varepsilon} \quad \varepsilon \neq 1$$
$$U(y_i) = \log y_i \qquad \varepsilon = 1 \qquad [2]$$

where ε is the parameter of inequality aversion. The way [2] works is relatively simple:

If ε=0, U(yi)=yi and [2] collapses to mean income. In this case, the higher the mean income, the higher the social welfare. ε=0, therefore, makes [2] a utilitarian SWF. This form of the SWF has the only characteristic of having W'>0. This condition is easily demonstrated, as $\frac{\partial W}{\partial y_i} = \frac{1}{n}\frac{1-\varepsilon}{1-\varepsilon} y_i^{1-\varepsilon-1} = \frac{y^{-\varepsilon}}{n} > 0$.

As ε increases, increases in lower incomes are given relatively more weight in producing social welfare. This means that the SWF must have W"<0, i.e. it must be concave. The Atkinson specification respects this property: Deriving the first derivative, we get $-\varepsilon\frac{y^{-\varepsilon-1}}{n} < 0$.

As ε reflects a value judgement, the exact specification of [1] (through [2]) depends on the value of ε.

Now, the cornerstone of the Atkinson's inequality measure is the concept of Equally Distributed Equivalent (EDE) income. EDE is that level of income that, if obtained by every individual in the income distribution, would enable the society to reach the same level of welfare as actual incomes.

Figure 2 illustrates the concept of EDE. The graph reports the SWF built on the space of individual incomes. The y-axis reports the income of individual 1, while the x-axis reports the income of individual 2. Let us assume that the distribution of income is such

that point A prevails, where y2 > y1. With no inequality aversion (ε=0), the utilitarian SWF would prevail, i.e. the straight line. With this SWF, the only way to have equal incomes at the same level of welfare, is therefore to give mean income to the two individuals (point B). Since inequality aversion is zero, we are not disposed to reduce the size of the cake to have more equal slices.

With inequality aversion, the convex SWF would prevail. Now, starting from A, we can find a point where incomes are equally distributed at the same level of welfare. Since the SWF is convex, this point (point. C) must be less than the mean income. Point C is the point in the 45 degree line that has the same social welfare as A. Even though total income (the sum of the two individual incomes) is lower than in A, it is compensated by the gain in the equality of the distribution. The reason being that, as inequality aversion is positive, we are now disposed to pay the price of a smaller cake so as to have more equal slices[5].

The EDE income is graphically equal to the income level corresponding to point C. Equality is measured by the ratio OC/OB. This is equal to 1 when each individual has the same level of income or if the SWF is utilitarian (there is no perceived inequality).

The Atkinson Inequality Index[6] may therefore be expressed as follows:

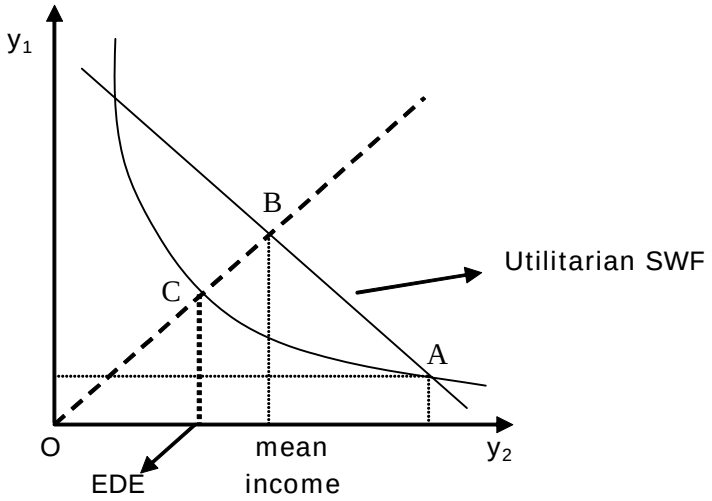
$$A(\varepsilon) = 1 - \frac{OC}{OB} = 1 - \frac{y_{EDE} * \sqrt{2}}{\bar{y} * \sqrt{2}} = 1 - \frac{y_{EDE}}{\bar{y}} \quad [3]$$

Intuitively, this Index tells us how much income we are disposed to give up in order to have equal incomes.

[5] See EASYPol Module 041: ***Social Welfare Analysis of Income Distributions: Social Welfare, Social Welfare Functions and Inequality Aversion***.

[6] Where A(ε) recalls that the value of Atkinson's Index depends on the parameter of inequality aversion.

Figure 2: The equally distributed equivalent income



In order to have an operational approach to the Atkinson's Index of inequality, we need an expression for EDE. We can get this expression by observing that, analytically, [2] implies the following:

$$U(y_{EDE}) = \frac{1}{1-\varepsilon}(y_{EDE})^{1-\varepsilon} \quad [4]$$

According to Figure 2, social welfare as in expression [1] must be the same with [2] and [4], i.e.:

$$W = \frac{1}{n}\sum_i \frac{y_i^{1-\varepsilon}}{1-\varepsilon} = \frac{1}{n} n \frac{(y_{EDE})^{1-\varepsilon}}{1-\varepsilon} \quad [5]$$

From [5], we can directly get an expression for the EDE:

$$y_{EDE} = \left[\frac{1}{n}\sum_i y_i^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad [6]$$

Repeating the same reasoning for ε=1 gives:

$$y_{EDE} = \prod_i (y_i)^{\frac{1}{n}} \quad [7]$$

Given any income distribution, therefore, EDE can be easily calculated for different levels of inequality aversion. Different levels of inequality aversion ε give different values of yEDE. For ε=0, the equally distributed equivalent income is simply the average level of income. With ε>0, yEDE decreases (for convex SWF, its level is always below average income) and A(ε) increases. For example, if with ε=2, A(2)=0.379, the interpretation is that society is disposed to release 37.9 per cent of the size of the cake to have equal slices. If $\varepsilon \to \infty$, the Rawlsian criterion is used, i.e. the SWF becomes more and more inequality averse.

A nice feature of the Atkinson's Index is that it allows us to directly derive a social evaluation function in abbreviated terms. Solving [2] for yEDE yields:

[8] $$y_{EDE} = \bar{y}(1 - A(\varepsilon)) = W$$

For any income distribution, if we know the mean incomes and the value of A(ε), the level of welfare can be calculated and compared. In this sense, yEDE is a direct measure of welfare.

4 A STEP-BY-STEP PROCEDURE TO CALCULATE THE ATKINSON INDEX

Figure 3 illustrates the simple steps needed to calculate the Atkinson Index. Step 1, as usual, asks us to sort the income distribution by income level. Step 2 asks us to calculate the mean of the income distribution, while Step 3 asks us to choose the parameter ε to calculate yEDE. Once calculated yEDE as in [6], we can directly apply [3] (Step 4).

Figure 3: A step-by-step procedure to calculate the Atkinson Index

STEP	Operational content
1	Sort the income distribution by income level
2	Calculate the mean of the income distribution
3	Choose the value of the parameter e to calculate equally distributed equivalent income
4	Apply formula [3] to calculate Atkinson

5 AN EXAMPLE OF HOW TO CALCULATE THE ATKINSON INDEX

Table 1 exemplifies the procedure for ε=2. Step 1 and Step 2 are very easy, as they sort the income distribution by income level and calculate the average income level.

In Step 3, yEDE is calculated by applying formula [6]. Its value is 2,190. Step 4 directly applies formula [3], giving rise to an Atkinson Index of 0.270. This number means that society is ready to «give up» 27 per cent of total income in order to have equally distributed incomes.

It is worth noting that by increasing ε, the value of the Atkinson Index also increases. It means that society is prepared to give up increasing shares of total income in order to achieve equality in incomes. For example, not reported in the table but easy to do as an exercise, with ε=3, the Atkinson Index would be 0.382, i.e. 38.2 per cent of total income could be sacrificed in order to have equal incomes.

Table 1: A numerical example of how to calculate the Atkinson Index

STEP 1
Sort income distribution by income levels

Individuals	Income distribution A
1	1,000
2	2,000
3	3,000
4	4,000
5	5,000
Total	**15,000**

STEP 2
Calculate the mean of the income distribution

Mean	**3,000**

STEP 3
Choose e and calculate y(ede)

e	3
Individuals	**y(ede)**
1	0.00000
2	0.00000
3	0.00000
4	0.00000
5	0.00000
y(ede)	**1,848**

STEP 4
Apply formula [3] in text

Atkinson	**0.384**

6 THE ATKINSON INDEX AND THE GENERALISED ENTROPY CLASS

The Atkinson Index has one nice characteristic that may prove useful in its empirical application. We can show that a transformation of Atkinson's Index is a member of the General Entropy (GE) class of inequality measures[7].

In particular, we can state this relation in two alternative ways:

$$A(\varepsilon) = 1 - \left[\varepsilon(\varepsilon - 1)GE_{1-\varepsilon} + 1\right]^{1/(1-\varepsilon)}$$
$$\frac{\left[1 - A(\varepsilon)\right]^{1-\varepsilon} - 1}{\varepsilon(\varepsilon - 1)} = GE_{1-\varepsilon} \qquad [9]$$

The first directly reveals that the Atkinson's Index is related to the (1-ε) member of the GE class. The second shows that a transformation of the Atkinson's Index is a member

[7] See EASYPol Module 051: ***Policy Impacts on Inequality: The Theil Index and the Other Entropy Class Inequality Indexes***

of the GE class of inequality measures. Using this transformation has the advantage of being perfectly decomposable in, within and between inequality (welfare)[8].

As the procedure to calculate the transformation of Atkinson's Index is based first on the calculation of Atkinson's Index and then on the application of [9], the step-by-step procedure and the example are not reported here, as the previous discussion also holds in this case.

For example, let us take the Atkinson Index of Table 1 (0.270). The corresponding GE transformation [9], for ε=2, would give the value 0.185.

7 READERS' NOTES

7.1 Time requirements

Time required to deliver the module is estimated at about two hours.

7.2 EASYPol links

Selected EASYPol modules may be used to strengthen the readers' background and to further expand their knowledge on welfare analysis.

This module belongs to a set of modules that discuss how to implement a welfare analysis comparing income distributions generated by different policy options. It is part of the modules composing a training path addressing **Analysis and monitoring of socio-economic impacts of policies**.

The following EASYPol modules form a set of materials logically preceding the current module, which can be used to strengthen users' background:

- ✓ EASYPol Module 041: ***Social Welfare Analysis of Income Distributions: Social Welfare, Social Welfare Functions and Inequality Aversion***

7.3 Frequently asked questions

- ✓ In which way can inequality measures be related to an explicit SWF?
- ✓ How do we embody different value judgements into an inequality index?
- ✓ What is the equally distributed equivalent income?

[8] See EASYPol Module 053: ***Policy Impacts on Inequality: Decomposing Inequality by Source.***

8 FURTHER READINGS

Atkinson A.B., 1970. On the Measurement of Inequality, *Journal of Economic Theory*, 2, 244-263.

Atkinson A.B., 1983. *The Economics of Inequality*, Clarendon Press, II ed. Oxford, UK.

Lambert, P., 1993. *The Distribution and Redistribution of Income – A Mathematical Analysis*, 2nd edition. Manchester University Press, Manchester, UK.

Sen A., 1997. *On Economic Inequality*, Oxford University Press, 2nd edition. Oxford, UK.

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4. Summary

This tool illustrates one of the most popular welfare-based measures of inequality, the Atkinson Index. In particular, it discusses the foundations of this index, in terms of social welfare specifications, and the concept of equally distributed equivalent income on which the measure is based. The use of this measure is then exemplified in a step-by-step procedure and in a numerical example.

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7. Module type	☐ Thematic overview ☐ Conceptual and technical materials ☒ Analytical tools ☐ Applied materials ☐ Complementary resources
8. Topic covered by the module	☒ Agriculture in the macroeconomic context ☐ Agricultural and sub-sectoral policies ☐ Agro-industry and food chain policies ☐ Environment and sustainability ☐ Institutional and organizational development ☐ Investment planning and policies ☒ Poverty and food security ☐ Regional integration and international trade ☐ Rural Development
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10. Training path	Analysis and monitoring of socio-economic impacts of policies
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