Case Study

Inspection by Attributes
Inspection by Attributes Plans

• Used for parameters having binary outcomes (pass/fail, present/absent, Detect/Not-detect etc.)
  • Also applied to measurements classified as pass or fail with respect to a limit

• Typical plan described by \((n, c)\):
  • Take \(n\) samples, inspect those samples, accept batch if no more than \(c\) of them are found non-conforming
  • \(c\) is called the acceptance number for the sampling plan
  • The number of samples \(n\) and the acceptance number \(c\) are determined from specifications of the AQL (Acceptance Quality Level) and the LQL (Limiting Quality level) and their associated allowable risks of rejection and acceptance respectively.
Plan Evaluation (Purposive)

Evaluate the attributes sampling plan \((n=5, c=0)\)

- Often used in microbiology (where measurements are classified as pass fail with respect to a limit)
Design and Evaluation of Sampling Inspection Plans

Plan 1 (Purposive)
Sample size: 100
Accept number: 0

Plan 2 (Designed)
Acceptance Quality Limit (%)
Limiting Quality Level (%)
Producer's Risk (%)
Consumer's Risk (%)

Comparison of OC curves of Designed and Purposive Plans

<table>
<thead>
<tr>
<th>Plan</th>
<th>n</th>
<th>c</th>
<th>AQL</th>
<th>AQL.risk</th>
<th>LQL</th>
<th>LQL.risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purposive Plan</td>
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<td>0</td>
<td>1</td>
<td>5</td>
<td>38.3</td>
<td>10</td>
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<tr>
<td>Designed Plan</td>
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<td>0</td>
<td>1</td>
<td>5</td>
<td>38.3</td>
<td>10</td>
</tr>
</tbody>
</table>
Design and Evaluation of Sampling Inspection Plans

Type of Plan:
- Attributes
- Variables

Plan 1 (Plan Evaluation)
- Sample size: 5
- Accept number: 0

Plan 2 (Plan Design)
- Acceptance Quality Level (%): 1
- Limiting Quality Level (%): 37

Comparison of OC curves

<table>
<thead>
<tr>
<th>Plan</th>
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<th>LQL</th>
<th>LQL.risk</th>
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</thead>
<tbody>
<tr>
<td>Plan 1</td>
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<td>1.00</td>
<td>0.05</td>
<td>36.90</td>
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<tr>
<td>Plan 2</td>
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<td>0.00</td>
<td>1.00</td>
<td>0.05</td>
<td>37.00</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Notes

n is the number of samples to be taken from a lot.

c is the acceptance number, in Inspection by Attributes (Attributes) plans it is the maximum of defective samples allowed in the n samples, for the batch to be accepted.

k is the acceptability constant, in Inspection by Variables (Variables) plans it is the multiplier of the standard deviation in the acceptance criterion.

\[ \bar{x} + k \cdot s \leq USL \] for an upper specification limit, and/or \[ \bar{x} - k \cdot s \geq LSL \] for a lower limit, where \( \bar{x} \) is the average of the n results and s is their standard deviation. When the standard deviation is known, s is replaced by \( \sigma \), representing the true value of the standard deviation.
Plan Evaluation

Producer’s Risk:
• There is a 5% chance of rejection (AQL Risk) at the Acceptance Quality Limit (AQL) of 1%

Consumer’s Risk:
• There is a 10% chance of acceptance (LQL Risk) at the Limiting Quality Limit (LQL) of 37%

Notes:
• These are hypothetical e.g. If 37% of product in a lot was non-conforming then the probability of acceptance would be 10%.
How do Inspection by Variables Plans work?

Based on the Binominal Distribution:

• A lot containing items each with a probability of being non-conforming
• A sample of ‘n’ items is chosen at random
• What is the probability that more than ‘c’ of the items in the sample are found non-conforming

Typical example is coin tossing

• A coin is tossed 20 times, what is the probability of 5 heads? (0.015)

https://youtu.be/3EZbX2ftCUk
Example (n=5, c=0)

Plot of Distribution of Levels of x in a Lot

- Specification Limit
- 20% non-conforming
Plot of Distribution of Levels of x in a Lot

- Specification Limit
- Probability that a single sample taken at random from the batch is found conforming = 80%
- 20% non-conforming
Level; Frequency of Occurrence; Specification Limit; Plot of Distribution of Levels of $x$ in a Lot

- 20% non-conforming

- Plot of Distribution of Levels of $x$ in a Lot

- Specification Limit

- Probability that five samples taken at random from the batch are found conforming $= (0.8)^5 = 0.33$ (approx)

- 20% non-conforming
Plan Design - Example

• Suppose there is problem with minor blemishes on the skins of apples.

• This defect is cosmetic and does not affect the edibility of the apples either when eaten raw or following processing.

• However we do not want to accept batches (most of the time) in which the proportion of apples with blemishes is too high.

• On the other hand, because of the nature of the defect, we do not expect growers to produce apples without blemishes.
We decide that most (say 90%) of the time we do not want to accept lots containing 20% (or more) of apples with blemishes
• Accept such lots 10% of the time
• LQL = 20%, LQL Risk=10%

We also recognise that the defect is minor and that most of the time (95%) we do not want to reject lots containing 5% (or less) of apples with blemishes
• Reject such lots 5% of the time
• AQL = 5%, AQL Risk=5%
Resulting Plan is \((n=38, c=4)\)

Take 38 apples at random, accept lot if no more than 4 apples have blemishes
Case Study 2

Issue with powder browning:

- Customer can tolerate ‘low levels’ of defects
  - Up to 20% in batch
- Producer does not want to unnecessarily reject product

A sampling plan was designed in which product is:

- Accepted most of the time when 10% or less of the lot is affected
  - say 5% chance of rejection (AQL risk)
- Rejected most of the time when 20% or more of the lot is affected
  - say 10% chance of acceptance (LQL Risk)
Resulting Plan is (n=109, c=16)
Take 109 apples at random, accept lot if no more than 16 samples show browning
Design and Evaluation of Sampling Inspection Plans

Type of Plan:
- Attributes
- Variables

Plan 1 (Plan Evaluation)
Sample size: 1
Accept number: 0

Plan 2 (Plan Design)
Acceptance Quality Level (%): 0.6
Limiting Quality Level (%): 5

Comparison of OC curves

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<th>LQL.risk</th>
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<tbody>
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<td>0.05</td>
<td>20.00</td>
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