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THE PROBABILITY DISTRIBUTION FRAMEWORK FOR ESTIMATING THE PREVALENCE OF UNDERNOURISHMENT:

Exploding the Myth of the Bivariate Distribution

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THE PROBABILITY DISTRIBUTION FRAMEWORK FOR ESTIMATING THE PREVALENCE OF UNDERNOURISHMENT:

Exploding the Myth of the Bivariate Distribution

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Page

SUMMARY

In his pioneering study carried in the early 1960's, Sukhatme had formulated the estimate of the prevalence of undernourishment in a population within a bivariate distribution framework where dietary energy consumption (DEC) and dietary energy requirement (DER) are considered as random variables. The evaluation of the formula required the specification of the joint distribution of DEC and DER. In the absence of data on the joint distribution Sukhatme had, as an approximation, formulated the estimate within a univariate distribution framework involving the distribution of DEC and a cut-off point reflecting the lower limit of the distribution of DER. FAO's methodology for estimating the prevalence of undernourishment has been traditionally based on this univariate distribution framework. However, since this approach appeared to ignore the risk of undernourishment at DEC levels overlapping the range of variation of requirement, it has been criticised as yielding an underestimate of the magnitude of the problem of undernourishment. In view of this some analysts have attempted to apply the bivariate distribution framework by modeling the joint distribution of intake and requirement. Others have applied the univariate distribution framework but used the average DER requirement rather than the lower limit of the distribution of DER as the cut-off point. All these attempts have led to very high estimates of the prevalence of undernourishment. In further studies undertaken in the 1970's Sukhatme has attempted to justify the univariate distribution framework that he proposed earlier by postulating the theory of intra-individual variation in energy requirement which implies that an individual cannot be considered to be undernourished or overnourished as long as his or her DEC is within the range of variation of DER. Since the variation in DER has been traditionally considered to be a reflection of differences between individuals, i.e. inter-individual variation, the theory of intra-individual variation has instead led to controversy and dispute rather than an understanding of the basic principle justifying the validity of the univariate distribution framework. This paper reviews the debate surrounding the issue since Sukhatme's pioneering study in the early 1960's and points out that the primary source of the controversy and debate has been the failure to realise that the distribution of DER in fact represents the realization of the joint distribution of DEC and DER with the consequence that the probability of DEC being in balance with DER is high for the DECs overlapping the range of variation of DER. Thus it is in fact the latter that explains Sukhatme's argument that an individual cannot be considered to be undernourished or overnourished if the individual's intake is within the range of variation of requirement. The failure to realise this has led to the continued belief in the myth of the bivariate distribution framework and hence the application of flawed models of the joint distribution of DEC and DER.

Key words: *Distribution of dietary energy consumption (intake); distribution of dietary energy requirement; joint distribution of dietary energy consumption(intake) and requirement; bivariate distribution framework; univariate distribution framework; inter-individual variation in dietary energy requirement; intra-individual variation in dietary energy requirement; lower and upper limits of the distribution of dietary energy requirement; correlation between consumption and requirement; probability of consumption being below requirement; probability of consumption being in balance with requirement; probability of consumption being above requirement.*

I. INTRODUCTION

FAO has been traditionally estimating the prevalence of undernourishment on the basis of food consumption data (expressed in terms of dietary energy) and dietary energy requirement for the purpose of quantifying the dimension of the food inadequacy problem particularly in the developing world. In this connection undernourishment has been defined as the state whereby dietary energy consumption (DEC)[2] is below dietary energy requirement (DER)[3] and the undernourished refers to the individuals in this state. The prevalence of undernourishment has been defined as the proportion of the undernourished in the population.

The measurement of undernourishment on the basis of food consumption data expressed in terms of dietary energy may be justified from two perspectives. Firstly, a certain amount of dietary energy is essential for the maintenance of body-weight and work performance. Secondly, an increased amount of dietary energy, if derived from normal staple foods, brings with it more protein and other nutrients as well, while raising the latter, without ensuring a certain amount of dietary energy, is unlikely to be of much benefit in terms of meeting food needs. In fact the amount of food consumed by individuals is best expressed in terms of dietary energy. Therefore the measure of undernourishment based on DEC differs from those based on anthropometric indices in the sense that while the former reflect food deprivation or hunger the latter reflect the broader concepts of undernutrition and malnutrition[4].

Another distinction between the FAO measure of undernourishment and the measures of undernutrition or malnutrition based on anthropometric indices concerns the unit of data collection. Disaggregated information pertaining to DEC is normally derived from the household food consumption data collected in national surveys and hence refer to households whereas anthropometric data pertaining to weight and height refer to individuals. Thus, while the measures of undernutrition and malnutrition are calculated on the basis of individual data, the FAO measure of undernourishment has to rely on household level data. Although the household data are normally expressed on individual basis through division by the number of household members or consumer units, the reference unit of the data remains the household and hence the intra-household distribution is not taken into account. In using the household level data for estimating the proportion of

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[2] In nutritional literature it has been common practice to refer to intake rather than consumption of food or nutrients (energy, protein, fats, etc.) This is a reflection of the fact that nutritional status is determined by the food eaten or ingested by an individual (biological consumption) and nutritional requirements reflect the intakes of well nourished individuals. However, the term adopted in connection with the data collected in national household surveys and used for estimating the prevalence of undernourishment is actually food consumption In view of this the term "consumption" rather than "intake" is used in this paper except when quoting from the relevant nutritional literature.

[3] Dietary energy requirement refers to the human biological needs after taking into account age, sex, bodyweight and physical activity.

[4] It should be noted that in the past FAO had referred to its measure based on food consumption as "undernutrition". However, beginning with the *Sixth World Food Survey*, this practice has been discontinued in order to distinguish the FAO measure from the measures based on anthropometry that reflect not only food insufficiency but also adverse health and environmental factors.

individuals having DECs that are below their respective DERs it is assumed that food is distributed according to the needs of the individuals within the households so that if household DEC is equal to household DER, the requirements of all the individuals in the household would be met. In any case, the use of data pertaining to household access to food and attempting to capture the individuals in the households whose access to food are below their needs, the measure has the merit of referring to a basic aspect of poverty.

However, the food consumption data from national surveys refer to a probability sample of households rather than the totality of the households in a population. Moreover, the DER recommended by the international expert groups on nutritional requirements refers to an average for individuals classified by sex and age which means that the actual DER of an individual is not known. Thus the estimation of the undernourished in a population cannot be viewed as a simple accounting exercise involving the comparison of the observed household DECs with calculated household DERs and counting the individuals in the households that have been found to have DECs that are below DERs.[5] Instead it has to be viewed within a probability distribution framework where the estimate is actually the proportion of the population undernourished. The number of undernourished is subsequently derived by applying the estimated proportion to the total population. Work in this direction was initiated in FAO in the early 1960's through the pioneering study of Dr. P. V. Sukhatme, who was then Director of the FAO Statistics Division. This study, which was presented at the joint meeting of the *Royal Statistical Society* and the *Nutrition Societies of London* (Sukhatme, 1961), in fact laid the foundation to FAO's use of distribution analysis in preparing estimates of the prevalence of undernourishment that began with the *Third World Food Survey* (FAO, 1963).

Sukhatme had originally formulated the estimation of the proportion of a population undernourished within a bivariate distribution framework, where DEC and DER are considered as random variables. However, in the absence of data on the joint distribution of DEC and DER, he had formulated the estimate within a univariate distribution framework that involves the distribution of DEC and a cut-off point reflecting the lower limit of the distribution of DER. This univariate distribution framework has been used by FAO in connection with its periodic assessments of the prevalence of undernourishment. Changes or improvements have taken place over the years but these have mainly concerned the specification of the distribution of DEC and the calculation of the cut-off point.

However, Sukhatme's derivation of the univariate distribution framework has proved to be elusive or not convincing to many researchers. In view of this, some have attempted to apply the bivariate distribution framework by modeling the joint distribution of DEC and DER or the conditional distribution of DER given DEC. Others have applied the univariate distribution framework but used the mean rather than the lower limit of the range of variation of DER as the cut-off point. However, all these attempts have invariably yielded estimates that are too high to be realistic. Sukhatme later attempted to justify the univariate distribution framework by invoking the theory of intra-individual variation in DER which implies that an individual with DEC intake falling within the range of variation of DER

[5] Such an approach has been applied in a recent study by IFPRI researchers (Smith, Alderman and Aduayom, 2006)

cannot be considered to be either undernourished or overnourished. Therefore only the individuals with DEC falling below the lower limit of the distribution of DER can be considered as being undernourished.

Sukhatme's theory of intra-individual variation in DER had however aroused a major controversy and debate among nutritionists and economists and as a consequence proved to be more confusing than helpful in understanding the validity of the univariate distribution framework. Therefore the whole matter was reviewed in FAO in the course of the preparatory work for the *Sixth World Food Survey* (FAO, 1996). The review, which confirmed the validity of the univariate distribution framework, was discussed in a FAO staff article (Naiken, 1998). However, Svedberg (2003), claiming that the estimate formulated within the bivariate distribution framework reflects an "unbiased" estimate, had applied it by modeling the joint distribution in order to demonstrate that the FAO methodology and as well as data used for estimating the prevalence of undernourishment were flawed. In view of this, in an appendix of a subsequent paper presented by Naiken (2004) at the *International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition* (ISSFDU) held in Rome in 2002, it was pointed out that the flaw was rather in the joint distribution models used by Svedberg and the others who have resorted to the bivariate distribution framework.

However, as indicated by Svedberg's comments in another paper following the Symposium (Svedberg, 2002), the argument was apparently still not convincing. This was partly because the paper presented at the Symposium failed to pinpoint the primary source of the flaw, which lies in the bivariate distribution framework itself. The aim of this paper is to highlight this point while discussing the history of the debate on the subject since Sukhatme's pioneering study in the early 1960's[6].

Thus, section **II** presents the probability distribution framework for the estimation of the prevalence of undernourishment and thus introducing the bivariate and univariate distribution frameworks as conceived by Sukhatme in the early 1960's. In section **III** the attempts made by Lörstad in the early 1970's to apply the bivariate distribution framework by modeling the joint distribution of DEC and DER are described. The FAO approach, which emerged following a methodological review in the early 1970's, is discussed in section **IV**. Section **V** discusses the approach of linking the measure of undernourishment with the measure of poverty taken by Reutlinger and Selowsky in a World Bank study. Section **VI** deals with the developments following the Reutlinger and Selowsky study and the *Fourth World Food Survey* including the theory of intra-individual variation in energy requirement postulated by Sukhatme. Section **VII** discusses the developments and debate in the literature outside FAO following the *Fifth World Food Survey*. In Section **VIII** the methodological review undertaken in connection with the preparatory work for the *Sixth World Food Survey*, which confirmed the appropriateness of the univariate distribution framework, are discussed in detail. Section **IX** discusses the developments following the

[6] The paper focuses on the probability distribution framework for estimation and the issues involved in the use of dietary energy requirement in the estimation of the prevalence of undernourishment and therefore do not discuss the practical problems associated with the data and procedures used for specifying the distribution of dietary energy intake or its parameters.

Sixth World Food Survey, in particular, the Svedberg paper criticising the FAO methodology and data and the FAO paper presented at the *International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition*. In addition the irrelevance of the bivariate distribution framework originally considered by Sukhatme is highlighted. Finally, some concluding remarks are made in section **X**.

II. THE PROBABILITY DISTRIBUTION FRAMEWORK AS CONCEIVED BY SUKHATME IN THE EARLY 1960's

As indicated earlier, the DEC data refer to households rather than individuals so that the intra-household variation is excluded in the distribution analysis of DEC. As the intra-household variation in fact encapsulates the variation between individuals due to sex and age, it follows that the variation considered in the distribution analysis refers to the inter-individual variation after the effect of sex and age has been removed or taken into account. In other words the distribution is free of the effect variation between individuals due to sex and age. This means that the total variance between individuals in the population can be expressed in terms of the between (inter) household and within (intra) household variations as follows:

$$\sigma^2_I = \sigma^2_B + \sigma^2_W$$

where σ^2_I refers to the variance between individuals, σ^2_B the variance between households and σ^2_W the within household variance.

Thus in estimating the proportion of the population undernourished on the basis of household data, only σ^2_B is taken into account and σ^2_W is taken care of by assuming that, within households, food is distributed according to the household members' sex and age. However, σ^2_B includes the effect of differences is household size. Therefore, this effect also is removed by converting the household data into per capita (or per consumer) units through division by the number of household members (or consumer units). As a consequence of this conversion, the resulting distribution of DEC can be considered to refer to a population of individuals who are equivalent in so far as sex and age are concerned, i.e. the average individual implied by the expression of population aggregates on per capita or per consumer unit basis.

However the fact that there is a variation in the DER of individuals defined as above implies that there is a corresponding distribution of DER. Thus if the DEC and DER of the individuals in the population are represented by random variables X and R respectively, the probability $P(X<R)$ represents the estimate of the proportion of the population undernourished.

(a) The Bivariate Distribution Framework

Sukhatme (1961) had formulated the estimate of the proportion of the population whose DECs are below their respective DERs within a bivariate probability distribution framework as follows:

$$P(X<R) = \iint_{x<r} f_{XR}(x,r)\, dx\, dr \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where, x and r represent the individual values of X and R respectively and $f_{XR}(x,r)$, refers to the joint density function of X and R.

However, as the available information referred to the separate but not the joint distribution of X and R, Sukhatme had derived a univariate distribution formula for evaluating $P(X<R)$ involving of the distribution of X and a cut-off point representing the lower limit of the distribution of R as discussed below.

b) The Univariate Distribution Framework

For the purpose of deriving the univariate distribution formula, Sukhatme had expressed $P(X<R)$ as $P[(X/R)<1]$ so that (1) could be expressed as follows:

$$P[(X/R)<1] = \int_{(x/r)<1} f_{X/R}(x/r)\, d(x/r) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where $f_{X/R}(x/r)$ refers to the distribution of the ratio of consumption to requirement.

The distribution of the ratio X/R in fact refers to a distribution of X that is independent of R. Thus he considered that if the individual values of R corresponding to the individual values of X were known (so that the distribution of X/R also would be known), the estimation of U would be given by (2). However, while the individual values of R corresponding to the individual values of X were unknown, information pertaining to the mean and the standard deviation of R was available. Therefore, assuming independence between X and R, Sukhatme for practical evaluation replaced (2) by

$$P(X< R) = \int_{x<(\mu_R - 3\sigma_R)} f_X(x)\, d(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

where $f_X(x)$ refers to the distribution of X and μ_R and σ_R are the mean and standard deviation respectively of R.

The cut-off point, μ_R - $3\sigma_R$, in fact refers to an estimate of the lower limit of the distribution of R under the assumption that the distribution is normal with mean, μ_R and variance, σ^2_R. However, in a later paper Sukhatme had indicated that, since the distribution of R was likely to be truncated on the lower tail, $\mu_R - 2\sigma_R$ would be more appropriate than $\mu_R - 3\sigma_R$ as the cut-off point (Sukhatme, 1973). In any case, as it is clear that the cut-off point refers to an estimate of the lower limit of the distribution of R, we may for the sake of simplicity express (3) as

$$P(X< R) = \int_{x<r_L} f_X(x)\, d(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

where r_L represents the lower limit of the distribution of R.

The logic underlying Sukhatme's replacement of (2) by (3), under the assumption that X is independent of R, was in fact not clear. However it appears that Sukhatme has interpreted independence of X from of R as reflecting the distribution of X after excluding the region overlapping the range of variation of R. In other words the distribution of X that is independent of R has been taken to refer to the two regions of X lying outside the limits of the distribution of R. As $x<r$ in the region of X below the lower limit of the distribution of R and conversely, $x>r$ in the region above the upper limit, $P(X<R)$ would be given by the part of the observed distribution of X below the lower limit of the distribution R and $P(X>R)$ by the part above the upper limit.

It follows from the above that r_L in effect represents a point below which X is in all probability below R. However, Sukhatme's formulation of r_L required estimates of μ_R and σ_R. In this connection a problem was encountered since the guidelines issued by the expert groups on nutritional requirements provided for the estimation of μ_R but not σ_R. Therefore, for the purpose of the preparation of FAO's estimates of the prevalence of undernourishment in the *Third World Food Survey* (FAO, 1963), Sukhatme derived a rough figure for σ_R on the basis of data from studies relating to the normal range of variation in energy expenditure in certain population groups.

III. LÖRSTAD's STUDIES IN THE EARLY 1970's

It was indicated in the previous section that Sukhatme had proposed and applied the univariate formula given by (3) because of the absence of data on the joint distribution of X and R. In view of this Lörstad, a statistician working in the Food and Nutrition Division of FAO in the early 1970's, had attempted to tackle the problem of absence of data by instead modeling the joint distribution of X and R and or the conditional distribution of R given $X = x$. In his 1971 study (Lörstad, 1970) the joint distribution model was applied in the context of determining the average intake of a given nutrient, N, that would allow for requirements to be met by all but a small proportion of individuals in the population. In his 1974 study (Lörstad, 1974) he had used the conditional distribution of R given $X=x$ in the context of arriving at an "unbiased" estimate of the proportion of the population undernourished - the deficiency risk estimate. Both of these studies are described below.

a) The 1970 Study

It was a usual practice for the international expert groups on nutritional requirements to provide guidelines regarding the recommended or "safe" consumption level for a population. This level was assumed to ensure that the requirements of only a small proportion of the individuals in the population would not be met. In this context, under the assumption that the distribution of requirement is normal, the "safe" level was fixed at the level corresponding to the mean plus two standard deviation of requirement. However, because DEC and DER are believed to be correlated, this "safe" level was not applied to energy. Therefore, in order illustrate the use of the bivariate probability distribution framework for determining the recommended consumption level for a given nutrient when a correlation exist between consumption and requirement, Lörstad had modeled the joint distribution of consumption and requirement .

Thus, expressing consumption and requirement of nutrient, *N*, as *X* and *R* respectively and assuming the joint distribution to be normal, the parameters of the bivariate normal distribution were expressed as follows:

μ_X, the mean of *X*;
σ^2_X, the variance of *X*;
μ_R, the mean of *R*;
σ^2_R, the variance of *R*; and
σ_{XR} , the covariance of *X* and *R*.

The covariance signifies the presence of correlation between *X* and *R*. However, as the square of coefficient of correlation is given by the ratio, $\sigma_{XR}/\sigma_X\sigma_R$, Lörstad had expressed the covariance as

$$\sigma_{XR} = \rho^2/\sigma_X\sigma_R$$

where ρ is the coefficient of correlation.

Moreover, Lörstad had actually considered the distribution of the difference *(X-R)* which implies that *P(X<R)* was expressed as *P[(X-R) <0]*. Under the assumption that the joint distribution of *X* and *R* is bivariate normal, the distribution of *(X-R)* also would be normal so that

$$P[(X-R) <0]= \Phi\ [(0-\mu_{(X-R)}) /(\sigma_{(X-R)})] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

where $\mu_{(X-R)}$ and $\sigma_{(X-R)}$ refer to the mean and standard deviation respectively of *(X-R)* and the right hand side refers to the area under the standard normal curve to the left of the point $Z = [(0-\mu_{(X-R)}) / \sigma_{(X-R)}]$.

As

$$\mu_{(X-R)} = \mu_X - \mu_R$$

and

$$\sigma_{(X-R)} =\sqrt{\sigma^2_X -2\rho\sigma_X\sigma_R + \sigma^2_R}$$

the formula for the probability of consumption being below requirement at the population level was expressed in terms of the parameters of bivariate normal distribution as follows:

$$P[(X-R) <0]= \Phi\ [\{0-(\mu_X - \mu_R)\}/ \sqrt{\sigma^2_X -2\rho\sigma_X\sigma_R + \sigma^2_R}\ ]$$

$$= \Phi\ [(\mu_R - \mu_X)/ \sqrt{\sigma^2_X -2\rho\sigma_X\sigma_R + \sigma^2_R}\ ] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

It follows from the above formula that, given μ_X, μ_R, σ^2_X, σ^2_R and ρ, the proportion of the population with consumption of N below requirement can be derived from the properties of the normal distribution.

However, since the objective was to determine the mean consumption level that would ensure that $P[(X-R) <0]$ is not higher than a fixed level, the exercise consisted of first specifying the standard normal deviate corresponding to the fixed level, i.e. $Z_{P[(X-R) <0]}$, and then equating it to the right hand side of (6) as follows:

$$Z_{P[(X-R) <0]} = [\{0-(\mu_X - \mu_R)\} / \sqrt{(\sigma^2_X - 2\rho\sigma_X\sigma_R + \sigma^2_R)}\ ] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7).$$

Thus given the standard deviate corresponding to the fixed value for $P[(X-R) <0$, μ_R, σ^2_X, σ^2_R and ρ, the recommended mean consumption level could be obtained by solving equation (7) for μ_X.

Assuming 0.025 and 0.05 as the alternative minimum levels for $P[(X-R) <0]$, Lörstad had calculated the recommended mean consumption level for various combinations of values for the parameters μ_R, σ^2_X, σ^2_R and ρ. He had noted that, “in all but few cases a recommended mean intake, meant to allow for requirements to be met for all but a small proportion of the population, becomes unrealistically high. It might even for some nutrients come close to the limits for excessive intakes hazardous to the health and well being of some individuals.”

The unrealistically high level of the recommended mean consumption level, which was insensitive to the effect of correlation, was considered by Lörstad to be an indication that the “safe” level concept was not useful. He proposed instead that, for policy purposes, attention should focus on the part of the population having intakes that are below the mean requirement, μ_R.

b) The 1974 Study

In this study Lörstad had focused his attention on Sukhatme’s univariate formula (3) for estimating the prevalence of undernourishment on the basis of DEC. Since the cut-off point was defined as corresponding to μ_R - $3\sigma_R$, he interpreted the formula as reflecting an approach that considered the probability (risk) of undernourishment only for DECs that are below the level corresponding to μ_R - $3\sigma_R$, and thus ignoring the probability of undernourishment corresponding to the DECs overlapping the range of R. He therefore argued that the approach was bound to lead to an underestimation of the prevalence of undernourishment. In view of this, given data pertaining to $f_X(x)$, he had proposed a “deficiency risk” estimate by considering the probability of undernourishment over the whole range of X. He had illustrated the methodology on the basis of the tabulated data pertaining to $f_X(x)$ for Burma and pre-partition India presented in Sukhatme’s original paper (Sukhatme, 1961).

As the data pertaining to $f_X(x)$ referred to the frequency of distribution of consumer units according to classes of household DEC per consumer unit, the approach taken was to calculate the probability of undernourishment corresponding to each of the DEC classes and aggregating them, using the proportion of consumer units in the respective classes as weights, to arrive at the "deficiency risk" estimate for the population. The DEC class specific probabilities of undernourishment were obtained by calculating the probability of undernourishment corresponding to each DEC class on the basis of the within-class distribution of R which was assumed to be normal.

Thus, given the midpoint of each *DEC* class, the methodology required the estimation of the mean and standard deviation of the distribution of R within DEC classes. These were estimated on the basis of the population mean and standard deviation, i.e. μ_R and σ_R, and the coefficient of correlation between X and R, i.e. ρ, as follows:

$$\mu_{R|X=x} = \mu_R + \rho\, \sigma_R\, (x - \mu_X)/\sigma_X \quad \text{...}(8)$$

$$\sigma_{R|X=x} = \sqrt{\sigma^2{}_R\, (1 - \rho^2)} \quad \text{...}(9)$$

where $\mu_{R|X=x}$ and $\sigma_{R|X=x}$ refer to the mean and standard deviation of the conditional distribution R given X=x, i.e. $f_{R|X}(r|X=x)$.

Evidently the conditional mean, $\mu_{R|X=x}$, varies according to the DEC class, while the standard deviation, $\sigma_{R|X}$, is constant. Therefore, given μ_R, σ_R, and ρ, the mean DER corresponding to each DEC class, i, was obtained by substituting x in equation (8) by the mid-point of the DEC class, i.e. x_i.

The probability of undernourishment corresponding to x_i, was obtained by calculating, from the conditional distribution of R given X= x_i, the probability of x_i being below the unknown DER corresponding to x_i, i.e. r_i. Thus, using the properties of the normal distribution, the probability of undernourishment corresponding to x_i was derived as follows:

$$P(x_i<r_i)=\Phi[(x_i - \mu_{R|X=x})/\sigma_{R|X=x}] \quad \text{..} \quad (10)$$

where $\Phi[(x_i - \mu_{R|X=x})/\sigma_{R|X=x}]$ refers to the proportion below the deviate $Z=[(x_i - \mu_{R|X=x})/\sigma_{R|X=x}]$ in the standard normal distribution.

Having calculated the probability of undernourishment corresponding to each DEC class, the estimate for the population was derived by calculating the weighted mean over the DEC classes as follows:

$$P(X<R)= \sum_i f(x_i)\, P(x_i <r_i) \quad \text{...}(11)$$

where $f(x_i)$ refers to the relative frequency of consumer units in the ith DEC class.

Thus, given the tabulated data referring to $f_X(x)$, the application of the Lörstad approach required knowledge of not only μ_R and σ_R, as in the case of the Sukhatme approach, but also of ρ. The latter parameter was not known but it was expected to be positive. Therefore in order to assess its effect on $P(X<R)$, Lörstad undertook a sensitivity analysis on the basis of the data for Burma by varying the values of ρ in (8) and (9) while keeping the other parameters unchanged. For the purpose of comparison Lörstad also derived the estimate based on Sukhatme's formula (3) but using μ_R - $2\sigma_R$ as the cut-off point (as suggested in 1973 by Sukhatme himself).

The results showed that the "deficiency risk" estimate, while being practically insensitive to ρ, was consistently very large as compared to that obtained through the Sukhatme univariate formula. The estimate varied between 66% and 72% for ρ ranging from 0 to 0.8 while the estimate obtained from the application of Sukhatme's univariate formula was only 33 %.

Lörstad interpreted the relative insensitivity to correlation as an indication that the magnitude of the correlation parameter was of no major significance. In other words one might as well assume that the two variables are not correlated, i.e. they are independent. As regards the very large prevalence of undernourishment implied by the deficiency risk estimate, he acknowledged that it was too high to be realistic. However, he claimed that this was not due to the methodology that he proposed but to the fact that undernourishment or DEC deficiency is defined as the state whereby DEC is simply below DER (or DER is above DEC). He felt that, in order to arrive at results that are similar to those of undernutrition based on anthropometric and clinical indices, a criterion reflecting the severity of undernourishment should be applied by introducing a coefficient or factor less than *1* which, if multiplied by requirement, would yield a lower limit below which an observed DEC would, in almost all probability, result in undernourishment "as a pathological state". In other words he suggested that requirement should be reduced by a factor so that the consumption class specific probabilities of energy deficiency would be expressed as follows:

$$P(x_i<qr_i)$$

where q is the reduction factor.

The introduction of the factor q meant that equations (8) and (9) would be modified as follows:

$$\mu_{R|X=x} = q\mu_R + \rho q\, \sigma_R\, (x - \mu_X)/\sigma_X \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\sigma_{R|X=x} = \sqrt{q^2\, \sigma^2{}_R\, (1-\rho^2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13).$$

Thus, by assuming the value of *0.8* for q, as suggested by Tremoliers (1957), and using the modified values of $\mu_{R|X=x}$ and $\sigma_{R|X=x}$, Lörstad noted with apparent satisfaction that the deficiency risk estimate was reduced to a figure (31%) that was in fact very close to that obtained through the Sukhatme univariate formula.

It is useful to note that the deficiency risk estimate given by (11) can in fact be expressed in within a continuous probability distribution framework as follows:

$$P(X< R) = \int_{-\infty}^{\infty} f_X(x)\ \Phi[(x - \mu_{R|X=x})/\sigma_{R|X=x}]\ dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14).$$

As $\Phi[(x - \mu_{R|X})/\sigma_{R|X}]$, which refers to the probability that x is below r is an expression for $[1- F_{R|X}(x)]$ under the assumption that the conditional distribution of R given $X=x$ is normal, it follows that (11) can in general be written as

$$P(X< R) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} [1- F_{R|X=x}(x)]\ drdx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

which is an expression of bivariate probability distribution formula given by (1) in terms of the conditional distribution of R given $X=x$.

IV. THE FAO APPROACH SINCE THE 1974 WORLD FOOD CONFERENCE

As indicated at the end of section **II**, Sukhatme's univariate distribution formula given by (3) was applied in the preparation of the FAO's estimates of the prevalence of undernourishment presented in the *Third World Food Survey*. The subsequent edition of FAO's estimates was prepared for a paper presented at the 1974 World Food Conference (FAO, 1974.a). In this connection a methodological review was undertaken particularly in the light of Lörstad's studies. The methodological review and the subsequent changes introduced vis-à-vis the Sukhatme approach, which mainly concerned the unit of the distribution and the derivation of the cut-off point, are discussed below.

(a) Methodological Review in the Early 1970's

The fact that the Sukhatme's univariate distribution formula appeared to consider only those subject to a high probability of dietary energy inadequacy, i.e. those with consumption below the lower limit of the distribution of R, as the undernourished tended to support Lörstad's claim that it was bound to underestimate the prevalence of undernourishment. However, the approach based on the conditional distribution model that he proposed instead was not convincing either. The very weak sensitivity to the effect of correlation and the very high estimate of the prevalence of undernourishment, that he himself considered to be unrealistic, were believed be due to a flaw in the model that he adopted rather than, as he claimed, the implied definition of undernourishment. In fact, according to probability theory, if the distribution of X is identical to the distribution of R, i.e. $f_X(x)= f_R(r)$,

$$P(X=R) =1.$$

This means that

$$P(X<R) = P(X>R) =0.$$

But, according to Lörstad's bivariate normal model the same condition leads to

$$P(X<R) = P(X>R) = 0.5$$

which implies that 50% of the population is undernourished and 50% overnourished. This gross anomaly built into the model was considered to be the main reason behind the lack of sensitivity to correlation and the unrealistically high estimate. Lörstad's suggestion that R in the model be reduced by factor in order to arrive at estimates that are closer to estimates of the prevalence of undernutrition derived on the basis of anthropometric or clinical data was not considered to be a valid option since it appeared to be like "putting the cart before the horse" and hence lacking objectivity.

The idea of using μ_R rather than r_L as the cut-off point in the univariate distribution formula was also investigated. It was noted that μ_R would indeed be the appropriate cut-off point if the variation of R were random as discussed below.

If the variation is random, R and σ^2_R can be written as follows:

$$R = \mu_R + \varepsilon$$

and

$$\sigma^2_R = \sigma^2_\varepsilon$$

where ε is a random variable normally distributed around mean 0 and variance σ^2_ε .

Consequently $P(X<R)$ can be written as

$$P(X<R) = P[(X- R) <0)] = P [\{X - (\mu_R + \varepsilon)\}<0] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16).$$

Being a random variable symmetrically distributed around 0, ε is expected to cancel out at the population level so that (16) reduces to the following:

$$P(X<R) = P(X< \mu_R)$$

$$= \int_{x<\mu_R} f_X(x)\, dx = F_X(\mu_R) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

which is the same as Sukhatme's univariate formula (3) but with μ_R rather than μ_R - $3\sigma_R$ (or μ_R - $2\sigma_R$) as the cut-off point.

However, as the variation in R considered here refers to the true variation and not that arising from random sources such as measurement error, the idea of using μ_R as the cut-off point in the univariate distribution framework was also rejected.

As indicated in an annex of the report entitled *Population, Food Supply and Agricultural Development* (FAO, 1974.b), other possible approaches were considered but these also did not prove to be better than Sukhatme's univariate distribution formula. Therefore it was decided to continue with this approach. In doing so FAO considered that, as the distribution of R reflects the distribution of X in a reference population composed of perfectly nourished individuals, it is quite reasonable to use the lower limit of this distribution as the cut-off point for measuring undernourishment.

In fact, following the above methodological review, Sukhatme's univariate distribution formula had emerged as the basic methodological framework underlying the different editions of FAO's estimates of the prevalence undernourishment as presented at the 1974 World Food Conference, in the *World Food Survey* reports (FAO, 1977; FAO, 1987, FAO, 1996) and more recently in the reports on the *State of Food Insecurity in the World* (FAO, 1999; FAO, 2000; FAO, 2001; FAO, 2002; FAO, 2003; FAO, 2004; FAO, 2005). However, while essentially retaining Sukhatme's univariate distribution formula, modifications were introduced with respect to the unit of the distribution and the derivation of the cut-off point, r_L. These are discussed below.

(b) The Per Capita Versus Per Consumer Unit Conversion of Household Data

As indicated in the introductory section, the food consumption data collected in national surveys and used for the estimation of $f_X(x)$ normally refer to households rather than individuals. For the purpose of an analysis on individual basis, the household data are usually divided by household size and thus expressed in terms of per capita units. However, in Sukhatme's and Lörstad's analyses, the individuals in the households were converted into equivalent consumer units and consequently the household DEC has been expressed on per consumer unit basis by dividing by the number of consumer units in the household.[7]

The expression of household data on per consumer unit basis has been considered as an improvement over the simple expression on per capita basis because of the belief that in the former case the effect of differences in the sex-age composition of households is taken into account. The conversion on per consumer unit basis is useful in the context of inter-household comparisons of DEC since it leads to a change in the ranking of the households, but in the context of the estimation of the prevalence of undernourishment, it is an unnecessary exercise as discussed below.

As indicated earlier, the methodology implies an inference regarding whether the DEC of each unit in the population is below the unit's DER. Therefore, it is evident that this inference will not be affected by the scalar change in the unit implied by the conversion of household DEC on per consumer unit rather than per capita unit basis. In other words the inference on food inadequacy would be same irrespective of the scalar change in the unit. In view of this and the fact that the consumer unit conversion of the individuals in all households is a tedious process, FAO had, beginning with the *Fourth World Food Survey (FAO, 1977)*, abandoned Sukhatme's consumer unit approach and instead adopted the approach based on the per capita unit.

[7] The "reference" man concept then used in the derivation of energy requirements was taken to be the consumer unit.

(c) The Derivation of the Cut-off Point

Sukhatme's approach for deriving the cut-off point (r_L) may be considered as being "parametric" in the sense that it is based on the parameters of the distribution *R*, i.e. μ_R and σ_R, and the assumption that the distribution is normal However, as indicated later by Sukhatme, the distribution of *R* is not likely to be normal. Moreover, the focus of the guidelines set by the international expert groups on nutritional requirements has been on the estimation of μ_R but not σ_R. In fact the estimate of σ_R used by Sukhatme was a rough one based on fragmented information. In view of this, FAO has adopted a procedure that attempts to derive r_L by considering the components of the variance of *R* separately.

As previously indicated, the distribution of *R* in the present context refers to units that are free of the effect of sex and age. Hence the components of variance are considered to be those due to the factors that determine the level of DER for an individual of given sex and age, i.e. body-weight and physical activity, and a residual component reflecting the contribution of unknown factors. Thus *R* can be written as

$$R = BW + PA + \varepsilon \quad \text{...(18)}$$

where *BW* refers to the contribution of body-weight, *PA* the contribution of physical activity and ε the contribution of the unknown factors.

Consequently, assuming that *BW*, *PA* and ε are independent, the variance of *R* can be been written in terms of its components as follows:

$$\sigma^2_R = \sigma^2_{BW} + \sigma^2_{PA} + \sigma^2_\varepsilon \quad \text{..(19)}$$

where σ^2_{BW} refers to the component of variation due to body-weight, σ^2_{PA} the component due to physical activity and σ^2_ε to the component due to unknown factors.

The component of variance due to unknown factors, i.e. σ^2_ε, was assumed to be a random variation associated with estimation or measurement error and therefore ignored in defining the variance of the distribution of *R*.

The procedure of deriving r_L by considering the two non-random components of σ^2_R separately has been facilitated by the expression of *R* in terms of the energy expenditure for maintaining body-weight, expressed as the *Basal Metabolic Rate* (*BMR*), and a multiplying factor to take into account the needs for physical activity. This component of energy expenditure approach in estimating DER was first introduced by the 1971 *FAO/WHO Ad Hoc Expert Committee on Energy and Protein Requirements* (FAO/WHO, 1973) and formalized by the *1981 FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements* (FAO/WHO/UNU, 1985) as discussed below.

In the past, the sex-age specific average DER recommended by the international expert groups on nutritional requirements were based on the average DER of the "reference" man and woman. The "reference" man and woman were defined as adults in ages *20 – 29* living in a climate with mean annual temperature of 10^0 *C* with fixed body-weight (*65 kg.* for males and *55 kg.* for females) and performing moderate physical activity. The "reference" man's and woman's average DERs were based on the measurements of the energy intakes of groups of healthy and active individuals of the "reference" woman and woman types. .Adjustments were made to the "reference" man and woman average DERs to account for different states and situations such as growth, pregnancy, lactation, age, climate etc. Thus the "reference" man and woman average DERs constituted the base line for the derivation of the average DER by sex-age groups (i.e. the energy requirement scale). In country applications the latter were adjusted to reflect the average body-weight of the adult males and females in the "reference" age group (20-29) and the mean temperature in the country. The sex-age specific average DERs were then weighted by the proportion of the population in the corresponding age groups and averaged to arrive at the per capita average DER or the average DER for the population. This concept of DER, which represents the average for individuals maintaining actual body-weight and performing moderate physical activity, was considered as μ_R.

However, while continuing the above approach, the 1971 *FAO/WHO Ad Hoc Expert Committee on Energy and Protein Requirements* (FAO/WHO, 1973) had introduced the concept of *maintenance energy requirement* defined as corresponding to the *Basal Metabolic Rate (BMR)* and a multiplying factor to account for the performance of a minimum level of physical activity. The BMR multiplier was referred to as the physical activity level (PAL) index.

The *FAO/WHO/UNU Expert Consultation on Energy and Protein* that met in 1981 (FAO/WHO/UNU, 1985) in fact discontinued the practice of deriving DER on the basis of the energy intakes of the "reference" man and woman. Instead it defined DER as the "energy intake level that will balance energy expenditure when individuals have a body-size and physical activity level that are consistent with good health and that will allow for the maintenance of economically necessary and socially desirable physical activity". In other words DER was defined as the energy expenditure corresponding to normatively specified rather than actual body-weight and physical activity of individuals.

In line with the above expenditure approach, the 1981 Expert Consultation formalized the expression of DER in terms of BMR and the PAL index. For the purpose of practical application, the Expert Consultation provided a set of regression equations for the estimation of the average BMR by sex-age groups on the basis of a liner equation linking BMR with body-weight (expressed in kg.) and three PAL indices reflecting light, moderate and heavy physical activity levels or lifestyles were given.[8] The BMR calculated on the basis of the regression equations is considered to be an average over the residual variation due

[8] The 1981 FAO/WHO/UNU Expert Consultation was not able to recommend the component energy expenditure approach for children below age *10*. For this segment of the population it provided a set of sex-age specific energy requirements per kg of body-weight that were based on the intakes of reference groups composed of healthy and well-nourished children in developed countries. However, this was remedied by the 2001 FAO/WHO/UNU Expert Consultation on Human Energy Requirements (FAO, 2004) which extended the expenditure approach to infants and children as well. But according to the new recommendations the approach based on the *BMR* and the *PAL* index has been limited to adults age 18 and above. For the infants, children and adolescents below age 18 the approach is to estimate the total energy expenditure (TEE) on the basis of body-weight.

to the effect of the unknown factors, i.e. σ^2_ε. This new approach of estimating DER was in recognition of the fact that there is a range of body-weight norms that are consistent with good health and a range of physical activity norms that are consistent with the performance of the necessary and socially desirable physical activity. Consequently the variance of *R* reflects the composite effect of these ranges of body-weights and physical activity norms.

The above approach enabled the direct derivation of r_L on the basis of the lower limit of the range of variation of body-weight (for the calculation of the average BMR) and the lower limit of the range of variation of the PAL index. In this connection the range of weight for given height in the relevant WHO reference populations was taken as the range of body-weight norms that are consistent with good health and the PAL indices corresponding to light and heavy physical activities were taken to reflect the range of physical activity norms that are consistent with the performance of the necessary and socially desirable physical activity. Consequently the body-weight corresponding to the lower limit of the relevant WHO reference weight for given height distribution was used to calculate the average BMR and the PAL index corresponding to light activity was applied to arrive at the lower limit of the range of variation of *R*. This lower limit has been referred to as the minimum dietary energy requirement (MDER).

However, as the distribution of *R* in the present context refers to units that are free of the effect of differences due to age and sex, the MDER has to be calculated by sex-age groups and then averaged over the sex-age groups using the population sex-age structure as weight, in order to arrive at the estimate of r_L. The procedure for calculating the sex-age specific MDERs is formulated below.

The first step in the procedure is to calculate the average BMR on the basis of the lower limit of the range of variation of body-weight as follows:

$$\overline{BMR} = (a + b \times BW_L)$$

where *a* and *b* are the constants of the linear equations for BMR and BW_L is the lower limit of the range of variation of body-weight in the reference weight for given height distribution.

The next step is the calculation of the MDER by applying the PAL index corresponding to light activity to the estimated average BMR as follows:

$$MDER = PAL_L \times \overline{BMR} \quad \text{...(20)}$$

where PAL_L refers to the PAL index for light physical activity.

It may be noted that the lower limit of the range of weight for attained height in the WHO reference population distribution, i.e. BW_L, which is used to derive the BMR component of MDER, is precisely what is adopted as cut-off point in the anthropometric measure of undernutrition. The latter measure is conceptually different from the food-based measure of undernourishment in the sense that it reflects not only food inadequacy but adverse health and environmental conditions. Nevertheless, as the two concepts are expected to overlap due to the common food inadequacy factor, there is a need to ensure a consistency between the two measures. Thus by using the same anthropometric criterion, i.e. BW_L, in determining BMR, the consistency between the two measures is ensured.

The country sex-age specific height figures, needed to specify the range of weight for attained height in the WHO reference population distribution, are obtained from the anthropometric data collected in nutrition surveys or demographic and health surveys.

Finally the sex-age specific MDERs are averaged over the sex-age groups to arrive at the MDER corresponding to the population represented by r_L as follows:

$$r_L = \sum_{ij} p_{ij}\, MDER_{ij} \quad \text{...(21)}$$

where $MDER_{ij}$ and p_{ij} refer to the *MDER* and the proportion of the population respectively in age group i and sex j.

It has to be pointed out that the above approach of deriving r_L reflects an attempt to circumvent the problem of absence of an estimate for σ^2_R and in essence does not depart from the idea behind consideration of r_L as the lower limit of the distribution of R.

V. THE REUTLINGER/SELOWSKY STUDY AND LINKAGE OF MEASURE OF UNDERNOURISHMENT WITH MEASURE OF POVERTY

The second edition of FAO's estimates of the prevalence of undernourishment was presented at the *1973 World Food Conference*. This had aroused the interest of some researchers outside FAO in assessing the size of the food inadequacy problem in the developing world. A major example in this connection was the study undertaken in the World Bank by Reutlinger and Selowsky (1976). This study in fact represented an attempt to use income distribution data, which are more readily available than food consumption distribution data, to estimate the prevalence of undernourishment in Africa, Asia, Latin America and the Middle East.

Using 1965 as the base year, the authors started by stratifying the population in the four developing regions according to eight per capita (per person) income classes (expressed in U.S. dollars). Then the per capita average DEC corresponding to each income class was derived though use of the following function:

$$x_i = \alpha + \beta \log_e v_i \quad \text{... (22)}$$

where x_i represents per capita average DEC of income class i and v_i the corresponding per capita average income and α and β are parameters.

Thus the use of the above function to derive the DEC corresponding to each income class requires the estimation of α and β. The appropriate data for estimating these parameters for each region were not available. Therefore *ad hoc* procedures were used to arrive at approximations. As the ratio of β to x represents the elasticity of DEC with respect to income, β was estimated on the basis of assumptions regarding the elasticity taken at DEC = μ_R. Thus β was derived as follows:

$$\beta = \zeta \, \mu_R$$

where ζ represents the elasticity of DEC with respect to income.

The other parameter, α, was taken to be the value that would ensure that the weighted mean of the income class specific DECs derived through function (22)is equal to the mean DEC for the population of the region using the following relationship:

$$\mu_X = \alpha + \beta \sum p_i \, log_e \, v_i$$

where p_i refers to the proportion of the population in income class i and μ_X to the per capita average DEC for the region.

Thus, having derived the DEC corresponding to each income class through the use of function (22), the weighted average of the proportions of population in the income classes with DEC below μ_R was taken as the proportion of undernourished. As this estimate was interpreted as reflecting the proportion of individuals having DEC deficits greater than zero, a worse off category, defined as those with deficits greater *250* kcal/day, was also derived by using ($\mu_R - 250$) as the cut-off point.

The regional per capita averages, μ_X and μ_R, were obtained by aggregating the national per capita averages which were by then regularly derived by FAO for practically all countries. The national per capita averages pertaining to DEC were those derived through the food balance sheets while those pertaining to DER were derived on the basis of sex-age composition of the population and the moderate activity energy requirement scale provided in the report of the 1971 *FAO/WHO Expert Group on Energy and Protein Requirement*.

The Reulinger and Selowsky approach was in fact practically the same as that used by Dandekar and Rath (1971) in estimating the prevalence (incidence) of poverty in India. Dandekar and Rath had used the Indian National Sample Survey (NSS) data referring to the average DEC of households grouped according per capita expenditure (taken as proxy of income) classes and considered the proportion of the households/individuals in the income groups having average DEC below μ_R as the incidence of poverty. Thus the only difference between the approaches is that while Dandekar and Rath had used actual data referring to the average DEC of households grouped according to income classes, Reutlinger and Selowsky have derived the data through the use of function (22).

The above implies that Reutlinger and Selowsky had practically equated their measure of undernourishment to Dandekar's and Rath's measure of poverty. Dandekar and Rath had referred to poverty presumably because the measure is actually based on the distribution of income rather than DEC. In fact, as function (22) is normally estimated on the basis of the data set used by Dandekar and Rath, both measures may be considered as being based on the distribution of income and the application of a poverty line derived as follows:

$$v_{pov} = exp[(\mu_R - \alpha)/\beta]$$

where v_{pov} represents the poverty line and $exp[(\mu_R - \alpha)/\beta]$ is the income level corresponding the DEC level equal to the μ_R.

Thus, given the poverty line as defined above, the two estimates can be formulated in terms of a continuous distribution function, as follows:

$$\int_{v < v_{pov}} f_V(v)\, dv = F_V(v_{pov}) \qquad (23)$$

where $f_V(v)$ refers to the density function of income.

It is evident from the above that, with all other factors remaining the same, the size of the resulting estimate will depend on the dietary energy adequacy norm used. Both studies had taken this to be μ_R.

On the basis of alternative assumptions regarding economic growth Reutlinger and Selowsky have projected the distribution of the population by the eight income classes and the average DEC from 1965 to 1975 and 1990 and thus derived estimates of the prevalence of undernourishment for the developing regions corresponding to these periods also.

The results of the estimate of the prevalence of undernourishment for 1975 according to the most optimistic assumptions regarding economic growth and the elasticity of DEC with respect to income was 55% for the developing regions as a whole. The estimate referring to worse off category (i.e. those having energy deficits greater than 250 kcal/person/day) was 19%.

VI. THE DEVELOPMENTS FOLLOWING THE REUTLINGER/SELOWSKY STUDY AND THE FOURTH WORLD FOOD SURVEY

The *Fourth World Food Survey* (FAO, 1977), which presented the third edition of FAO's estimates of the prevalence of undernourishment, was issued just a year after the Reutlinger and Selowsky study. FAO's estimates covered the developing regions (excluding the Asian Centrally Planned Economies) for 1969-71 and 1972-74. The estimate for the period that was closer to that in the Reultlinger and Selowsky study, i.e. 1972-74, was about 25%. The large difference in the assessment of the extent of undernourishment in the developing world (55% as compared to 25%) by these two studies attracted the attention of many critics. Meanwhile, Sukhatme, who has continued to study the problem of estimating the prevalence of undernourishment following his retirement from FAO in 1972, had come up with the theory intra-individual variation in requirement in order to justify the use of r_L as the cut-off point. Since Reutlinger and Selowsky as well as Dandekar and Rath have used μ_R in their studies, this had led to an intense dispute mainly between Sukhatme and Dandekar. These events are discussed below.

(a) The Large Difference Between the Reutlinger/Selowky and FAO Estimates

Following the publication of the *Fourth World Food Survey*, the world was confronted by the large difference between the Reutlinger and Selowsky and the FAO estimates of the size of the undernourishment problem in the developing world. As the Reutlinger and Selowsky study covered all the developing regions while FAO's excluded China and the countries of the Asian Centrally Planned Economies, the comparison was made in terms of the percentage undernourished for the two roughly comparable time periods, i.e. 1975 in the case of the Reutlinger and Selowsky estimates and 1972-74 in the case of the FAO estimates. Although the estimates had been found to useful in the sense of indicating that at least a quarter of the population in the developing world were undernourished in the mid-1970's, critics were divided regarding which was the more reliable or realistic assessment. It was in fact not easy to disentangle the real issue(s) because of the different assumptions and data manipulations involved in the application of the two approaches. This problem coupled with the very large discrepancy between the two assessments – 55% in the case of Reutlinger and Selowsky and 25% in the case of FAO - have led Poleman (1978) to cast serious doubts on the validity of the methodology and the reliability of the data used by both the FAO and the World Bank analysts. Poleman's view was that both had overestimated the prevalence of undernourishment.

However, the key factor explaining the large difference between the two estimates is the difference between the DER level used as dietary energy adequacy norm in the Reulinger and Selowsky approach and that represented by the cut-off point in the FAO approach. In other words it is to a large extent due to the fact that Reutlinger and Selowsky had used μ_R rather r_L.

As one of the criticisms made regarding the Reutlinger and Selowsky approach was that it had not taken into account the joint distribution of DEC and DER within the income groups, Reutlinger and Alderman (1980) had attempted to rectify this by using the Lörstad's bivariate normal distribution model discussed in section **III(*a*)** to estimate the proportion of undernourished within each of the 8 income groups and averaging the results over the income groups to arrive at the prevalence of undernourishment in the population.

The application of the Lörstad model required estimates of the means and standard deviations of DEC and DER and the correlations between the two variables within each income group. The energy average DEC derived through equation (22) was taken as mean for each income group while the mean requirement was assumed to the same for all income groups and hence equated to μ_R. The standard deviations of DEC and DER within each income group were obtained by assuming the coefficient of variation (CV) to be 15% in both cases. Thus given the means and standard deviations of DEC and DER and the coefficient of correlation corresponding to each income group, the proportion of undernourished within each group was derived using formula (6) but with the means and standard deviations referring to each income group rather than the population as a whole. However, as the CVs of DEC and DER and the coefficients of correlation were assumed to be the same across the income groups, this approach boiled down to be the same as that applied earlier by Lörstad (1970) on the basis of the population parameters, i.e. μ_X, μ_R, σ_X, σ_R and ρ.

Assuming different values for the coefficient of correlation, ρ, Reutlinger and Alderman applied the approach to each of the 36 developing countries in the Reutlinger and Selowsky (1976) study for which income distribution data were available and compared the results with those obtained through the application of formula (23). As Lörstad had already noted, the estimate based on the application of the bivariate normal distribution was not sensitive to the coefficient of correlation. Furthermore there was not much difference as compared to the estimate based on formula (23). The aggregated result for the 36 counties in 1960 was 63% according the approach based on the application of bivariate normal distribution (with ρ assumed to be 0.7) while that based on formula (23) was 65%. The conclusion reached by the authors from this exercise was that taking into account the variations in DEC and DER within the income groups did not matter much and hence the use of formula (23) was sufficiently reliable.

(b) The Sukhatme Versus Dandekar Dispute Over the Use of the Average Energy Requirement as the dietary energy adequacy norm for assessing food inadequacy

Sukhatme has contested the use of μ_R as the dietary energy adequacy norm in both the Reutlinger and Selowsky and Dandekar and Rath studies. This had led to a hotly contested debate mainly between Dandekar (1981 and 1982) and Sukhatme (1981.a; 1981.b; 1982.a) in the Indian *Economic and Political Weekly*. Since Dandekar had considered his estimate as referring to poverty he argued that poverty and undernourishment are different though overlapping concepts and in measuring poverty on the basis of the distribution of income it has been the practice has been to use μ_R as the food adequacy norm. Hence he could not see why this should be reduced by $2\sigma_R$ just because Sukhatme considers it to be appropriate in the context of estimating the prevalence of undernourishment. However Sukhatme's point was that, by using a food adequacy norm, the measure of poverty is intrinsically linked with the measure undernourishment. Therefore, in order to ensure consistency between these two measures the food adequacy norm used should be the same. This point is in fact analogous to that made in section **IV(*c*)** regarding the use of the lower limit of the reference distribution of body-weight for attained height (BW_L) in deriving the BMR (and hence MDER) so as to ensure consistency between the food-based measure of undernourishment and the measure of undernutrition based on the anthropometric indices (weight-for-height or body mass indices).

However, the problem actually concerns the use of average DER for the population, i.e. μ_R, as a food adequacy norm in assessing food inadequacy of individuals. The following statement in the report of the *1981 FAO/WHO/UNU Expert Consultation on Energy and Protein Requirement* clearly indicates that this average does not reflect a food adequacy norm:

"When the population is the unit of observation for food intake, an estimate of the mean energy requirement can be obtained from demographic, anthropometric and activity profile data. As for protein, its interpretation is doubtful. It is known that the distribution of intakes within populations is not uniformly proportionate to needs; acute malnutrition exists in populations that appear to have sufficient food to meet the estimated energy needs for the country as a whole. Even though a correlation exists between intake and requirement, it is not perfect and inferences cannot be made about the situation of individuals from a knowledge of intake and requirement at the population level.

For energy more than for protein, the aggregate requirement estimate at the population level may be a useful marker in studies of trends. It may be a meaningful way of taking into account of demographic changes and differences in comparisons across populations. It is not a useful index of satisfaction of need or a meaningful target for production." (underlining mine).

It is precisely for the above reasons that the reports of the international expert groups on nutritional requirements have indicated that μ_R was not meant to be applied at the individual level for making inference on food inadequacy or excess. As discussed in section **IV(*a*)**, the use of μ_R as the food adequacy norm in this context would imply that the variance, σ^2_R, is random when in fact it is not so. While it is true that in the case of both the Reutlinger and Selowsky and the Dandekar and Rath estimates, μ_R was compared with the average DEC corresponding to the income classes rather than DEC of the individuals, the fact of the matter is that all the individuals in the energy deficit income classes have been classified as undernourished (in the case of the Reutlinger and Selowsky study) or poor (in the case of Dandekar and Rath study). This means that the inference on individuals has been based on μ_R as the food adequacy norm.

It may be also noted here that in the light of the 1981 FAO/WHO/UNU Expert Consultation's statement regarding the fact that μ_R is not a useful index of satisfaction of need FAO had discontinued the past practice of presenting tables showing the national per capita average DEC (as estimated through the food balance sheet) as percentage of the national per capita average DER, i.e. μ_R.

(c) Sukhatme's Theory of Intra-individual Variation in Requirement

The basic question in the minds of Dandekar and others contesting Sukhatme's cut-off point was whether an individual's basic need for food can be satisfied by a level of DEC that is as low as that implied by the lower limit of the distribution of *R*? Sukhatme's response to that was the theory of intra-individual variation in requirement, which is discussed below.

As indicated in section **IV(*c*),** in the past human requirements for energy were based on special dietary studies on the intakes of individuals in reference groups composed of healthy individuals of the same age, sex and body-size and engaged in similar physical activities, i.e. the concepts of "reference" man and woman. However, because of the existence of day-to-day variation in intake, requirement was based on the concept of usual intake. In this connection daily intakes averaged over a number days (e.g. a week) was considered by Widdowson (1947) to be a fairly reliable estimate of habitual intake and hence the requirement of an individual. Accordingly, the energy requirement of an individual of given age, sex, and body-size and physical activity has been considered to be fixed and the differences observed between the habitual intakes of the individuals in the reference group attributed to the effect of unknown factors, i.e. ε. This interpretation was contested by Sukhatme (1982.b) who postulated that "a greater part of the observed variation arises from intra-individual variation which is stochastic stationary in character, thereby meaning that requirement is dynamic and self-regulated and not static as assumed in nutrition literature and further has adaptive significance".

The representation of requirement by daily intakes averaged over a week implied that the day-to-day (within week) variation is interpreted as being random. Sukhatme's argument was that if this variation was truly random it would have disappeared in the process of averaging to reflect DER and hence there would not have been any difference between the DER of the individuals in the reference group. Thus, following his retirement from FAO in 1971, Sukhatme had examined the issue by analysing the series of daily energy intakes and expenditures of a reference group composed of a number of healthy army recruits maintaining bodyweight and engaged in similar physical activities reported by Edholm *et al* (1970). Through an analysis of variance carried out on the data, he had noted that when the daily intakes and expenditures were averaged over a period of five consecutive days the differences from period to period for the same subject did not disappear, as one would expect if the day-to-day variation was random, but persisted. He took this as a suggestion that the body regulates its energy balance on a range of intakes (the range of homeostasis) by varying the efficiency of utilization of its daily intakes in the same manner as was shown in an earlier study pertaining to nitrogen by Sukhatme and Margen (1978).

In the case of the nitrogen study, the presence of regulation was shown by testing for auto-correlation in a data series relating to day-to-day changes in nitrogen balance (expressed in terms of the difference between intake and expenditure) in adult subjects maintaining body-weight. The series was found to be auto-correlated and hence adequately represented by the following auto-regressive model of order one:

$$w_t = \rho_a w_{t-1} + e_t \quad \ldots\ldots(24)$$

where w_t is the difference between intake and expenditure on the t^{th} day, ρ_a is the auto-correlation coefficient of order one between w_t and w_{t-1} and e_t is a random variable distributed with mean zero and variance σ_e^2.

The meaning of the above model, called the stochastic stationary Markov process, is that if it were possible to repeat the circumstances which gave rise to the observed value of the w on any day, t, then the balance will be distributed around zero within limits (limits of homeostasis) given by

$$0 \pm 2\sigma_e/\sqrt{1-\rho_a^2} \quad \ldots\ldots(25).$$

The daily series pertaining to energy intake and expenditure reported by Edholm *et al* (1970), being limited to 3 non-consecutive weeks, did not permit a direct study of auto-correlation in daily energy balances in order to verify whether energy expenditure also is self-regulated over a range of energy intake. In view of this, Sukhatme had examined the presence of autocorrelation by the indirect method of computing the variance of mean balance when the daily balance is averaged over periods of 2, 3, or more consecutive days. In doing so, he had noted that the variance of the mean did not vary inversely as the length of the period, but decreased slowly, thus indicating that the successive (daily) values were serially correlated. Therefore he concluded that the hypothesis of daily balance being distributed in a stochastic stationary manner of Markovian type with serial correlation of the first order equal to 0.3 noted earlier in the case of the nitrogen study seemed perfectly reasonable for energy also. In view of this he stated that the data from Edholm *et al* (1970) "must be interpreted to mean that although energy intake may not be equal to energy expenditure even when averaged over a week, man is in balance every day in a probabilistic sense with varying intervals of peaks and troughs and varying amplitudes in daily balance". As the coefficient of variation of the within period daily energy intakes of the individuals in the reference group was about 20%, he indicated that "man's requirement for any day or period is not fixed but dynamic, adapting itself to intake over a fixed range from 60 to 140 per cent of the average dietary allowance". Consequently an individual of the reference type could be classified as being undernourished only if his or her DEC was below the lower limit of the range of DER. Thus Sukhatme had used this argument to justify the univariate distribution formula (with μ_R - $3\sigma_R$ as cut-off point) that he had derived and applied in his 1961 study. In other words

$$\mu_R \pm 3\sigma_R$$

reflected the limits of the range of intra-individual variation of R.

However, since Sukhatme's variance analysis referred to the intakes and expenditures of adults males of the "reference" man type, i.e. individuals of the same sex and age maintaining body-weight and performing similar physical activity, the theory of intra-individual variation of R was taken by most observers to imply that the residual variance component, i.e. σ^2_ε, is not random but systematic arising from the capacity of an individual to modify his or her efficiency of energy utilisation. But, the variance of the distribution R, as discussed in section **IV(c)**, refers to the components of variation due to bodyweight and physical activity, i.e. σ^2_{BW} and σ^2_{PA}, which are clearly of inter-individual nature. In view of this, the obvious question that was raised by his critics was how the theory of intra-individual variation in requirement could be invoked to justify the reduction of μ_R by $3\sigma_R$ or $2\sigma_R$ when σ^2_R includes σ^2_{BW} and σ^2_{PA}? (See section **VII** regarding the debate over the theory of intra-individual variation).

VII. THE FIFTH WORLD FOOD SURVEY AND THE ENSUING DEBATE OUTSIDE FAO

The fourth edition of FAO's estimates of the prevalence of undernourishment was presented in the *Fifth World Food Survey* (FAO, 1987). In this edition consideration was given to Sukhatme's theory of intra-individual variation in DER. Since the theory was interpreted as referring to the variation in efficiency of energy utilization, it was assumed to be associated with the BMR component of energy requirement. Thus acceptance of Sukhatme's theory was taken to imply that the calculated average BMR in (20) had to be reduced by $2\sigma_{BMR}$.

There was much uncertainty regarding Sukhatme's theory but, there was no evidence to either prove or disprove it. In view of this FAO decided to apply, in addition to the cut-off point described in section **IV(c),** an alternative where the average BMR was adjusted downwards to account for the intra-individual variation. That is to say for the purpose of the alternative cut-off point *MDER* was reduced to its lower limit as follows:

$$MDER_L = PAL_L\,(\overline{BMR} - 2\sigma_{BMR})\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26).$$

Thus, given the average BMR, the evaluation of the above required an estimate of σ_{BMR}. Actual data referring to this variation was not available. However, the results of measurements of energy expenditure on male and female reference subjects of the same body-weight performing a specified minimum activity programme over 24 hours indicated that the coefficient of variation (CV) was about 7% for either sex . Hence this CV was used to estimate σ_{BMR}.

The publication of the *Fifth World Food Survey* in 1987 was followed by commentaries and debates in the literature involving a number of nutritionists and economists concerning Sukhatme's theory of intra-individual in energy requirement and the probability framework for estimating the prevalence of undernourishment. These are reviewed below under different headings.

(a) Reactions within the Nutrition Community Regarding Sukhatme's Theory Intra-individual Variation

Some nutritional experts had contested the theory of intra-individual variation in DER that had emerged following the studies undertaken by Sukhatme and Margen (James, Healy and Waterlow, 1989). The point contested, however, was not whether an individual has the capacity to vary his or her efficiency of energy utilization but whether the range of variation could be as large that suggested by Sukhatme (CV of 20%). In other words the existence of intra-individual variation due to efficiency in energy utilization was recognized but the range of variation was believed to be small and hence negligible (James and Schofield, 1990).

Beaton and Tarasuk (1989) had viewed Sukhatme's theory of intra-individual variation in requirement as a statistician's answer to the anomaly in Lörstad's bivariate distribution model referred to in **IV(a)**. Noting that the intra-individual variation attributed to variation in efficiency of energy utilization would imply reducing individual DER to the lower limit of the intra-individual variation, they showed, within the bivariate distribution framework, that this would indeed reduce the prevalence of undernourishment to a more acceptable level. However they indicated that more research was needed on the basis of longitudinal dietary data pertaining to reference groups of perfectly nourished individuals in order to confirm the existence of intra-individual variation in DER as large as that indicated by Sukhatme.

However, while considering that Sukhatme's theory of intra-individual variation would attenuate the dilemma posed by Lörstad's bivariate distribution model, Beaton and Tarasuk still believed in the latter model as being the appropriate probability framework for taking into account the inter-individual variation in requirement, i.e. $\sigma^2_{BW} + \sigma^2_{PA}$. But, in this context, they admitted that the insensitivity to the effect of correlation was a "very difficult question". It is probably because of this that Beaton (1991) had stated the following:

"A simple assumption of the probability assessment is that intakes and requirements are not correlated when examined within strata of the population (e.g. young children) and when factors potentially affecting both are controlled (e.g. when thiamine is examined as mg per kcal rather than mg per day). In the case of energy, there is strong reason to believe that over moderate time periods energy intake and energy expenditure ("requirement") are strongly correlated as part of regulated energy balance. This violates the core assumption of the probability approach". Hence he concluded that, "there is at present no satisfactory way of estimating the prevalence of inadequate energy intakes". In other words he had considered the insensivity of the estimate based on the bivariate distribution formula to correlation as an indication that the prevalence of undernourishment cannot be satisfactorily estimated.

b) Articles in the Study by the World Institute for Development Economics Research (WIDER)

The study entitled *Nutrition and Poverty* (ed. Osmani, 1992) included a number of articles by different authors relating to the problem of estimating the prevalence of undernourishment based on food consumption data as well as the prevalence of undernutrition based on anthropometric data. In so far as the subject being discussed in this paper is concerned, the relevant articles are those authored by Osmani (1992), Anand and Harris (1992), Kakwani (1992), Srinivasan (1992) and Payne (1992). The main points made in all these papers are briefly described below.

1) Osmani's article

Osmani attempted to explain Sukhatme's theory of intra-individual variation in DER and discussed the ensuing controversy in the nutrition field by referring to the many opponents as well as the few supporters of the theory. While discussing the controversy, he highlighted the fact in some instances Sukhatme's theory has been misinterpreted and consequently criticised for the wrong reasons. The misinterpretations were partly due to the confusion arising from the fact that although his theory was referring to the variation within individuals he had later (Sukhatme, 1982.b) used it to explain the derivation of cut-off point μ_R - $3\sigma_R$ in his original study (Sukhatme, 1961) where σ_R referred to the inter-individual variation.

In explaining the theory of intra-individual variation, Osmani pointed out that Sukhatme's theory referred to the variation in the DER of an individual due to variable efficiency of energy utilization Viewed within this perspective it would be logical to consider an individual to be undernourished only if his or her DEC is below the lower limit of the intra-individual range of variation (or range of homeostasis).

However, Osmani considered the above logic to be flawed since, according to him, it is based on the presumption that *every one* with DEC within the range of intra-individual variation in DER is well nourished when, in fact some of such individuals may not actually be so because of some extraneous constraints on DEC rather than variable efficiency in energy utilization. Thus he explained the flaw as being a failure to realize that one's DEC being within the range of homeostasis is only a *necessary* condition and by no means a *sufficient* condition for being well nourished. Consequently he considered the use of the lower limit of the range of homeostasis as the cut-off point would in general lead to an underestimation of the prevalence of undernourishment.

Osmani noted that Sukhatme had also interpreted intra-individual variation in requirement as a process of no risk adaptation to DEC constraints. In this context varying efficiency of energy utilization is regarded as a process of biological adjustment in response to changes in DEC. This interpretation would seem to satisfy the *sufficient* condition referred to above. In view of this and the fact that the interpretation was made at a later stage in the development of the theory of intra-individual in requirement, Osmani had considered it to be a "mutation" largely in response to earlier criticisms of the theory. However he disagreed with this "mutation" since he considered it to be essentially based on the premise that variable efficiency of energy utilization can be interpreted as being either a process of stochastic regulation of energy balance leading to the range of homeostasis or a process of biological adjustment in response to changes in DEC. His disagreement was on the grounds that the process of regulation in energy balance, being justified by auto-correlation in the series of energy balance of a group of healthy and adequately nourished individuals (the army recruits), cannot be taken to be the same as the process of biological adaptation to extraneous DEC constraints. He argued that the consideration of the range of homeostasis as also reflecting the range of adaptation had to be based on independent evidence on the limits of adaptation and not on the autocorrelation exercise.

In examining the discussions on the subject of adaptation in the literature, Osmani found that the hypothesis of no risk adaptation was not yet substantiated by scientific evidence although its possibility could not be ruled out altogether. Hence he concluded that Sukhatme's theory and hence the use of the lower limit of the range of variation in requirement as cut-off point could not be justified in the light of current scientific evidence.

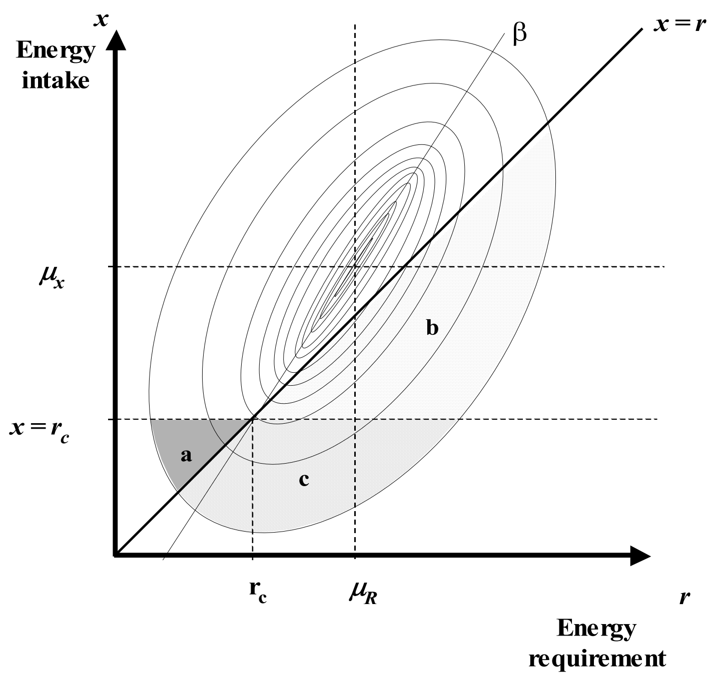
Nevertheless, Osmani indicated that the above conclusion did not imply a return to the use of the average DER (μ_R) as the cut-off point or norm for dietary energy adequacy since the well documented inter-individual variation in DER needs to be taken into account. The use of μ_R as cut-off point in this context would lead to errors of underestimation and overestimation which may not cancel each other out. Therefore he considered the issue of whether to use μ_R or some other norm becomes a matter of value judgement. However he suggested that a "value-neutral" way of tackling this problem would be shun the cut-off point approach and use instead a joint distribution, meaning the bivariate distribution approach.

2) The Anand and Harris article

Anand and Harris graphically illustrated, within the framework of the bivariate distribution model considered by Lörstad in the early 1970's, the implications of the use of a single energy requirement as cut-off point on the distribution of X in the estimation of the prevalence of undernourishment. The graphical illustration is as shown in Figure1, which is actually an adapted version reproduced from Svedberg (2003).

In Figure1, the joint density function of X and R, i.e. $f_{XR}(x,r)$, is depicted as an ellipse truncated at the edges and stretching out in a southwest to northeast direction in order to reflect the dependence of X on R and the positive correlation.

Figure 1: The bivariate distribution framework



The increasingly closer iso-contours as one moves towards the joint mean (μ_X,μ_R) indicates that the joint density assumes its highest value at this point. Or, conversely, as one moves away from the joint mean, the joint density becomes smaller. From the shape of the joint distribution it is clear that the joint distribution has been assumed to be bivariate normal. The β-line represents the regression line expressing the dependence of X on R while the line, $x=r$, represents the energy adequacy line.

The β-line, which represents the linear relationship between X and R, is actually expressed in terms of the parameters of the bivariate distribution as follows:

$$x = \mu_X + \rho\, \sigma_X\, (r - \mu_R)/\sigma_R.$$

Thus it was shown that the area of the joint distribution below the 45-degree line represents $P(X<R)$ while the area above the line refers to $P(X>R)$. Accordingly, Anand and Harris had pointed out that if a single requirement level, r_c, is used as a cut-off point on the marginal distribution of consumption, $f_X(x)$, the estimate would be subject to errors of misclassification: some of those whose DECs are actually below their DERs would be classified as not being undernourished (the area marked as **b**) while some of those whose DECs are actually above their DERs would be classified as being undernourished (the area marked **a**). On the other hand, if the minimum or lowest requirement, r_L (not shown in the chart) is used as the cut-off point, the resulting estimate has the advantage of reflecting "purity" in the sense it will include only undernourished persons but it also has the disadvantage of excluding a significant part of those whose DECs are below their DERs.

c) The Kakwani article

Given $f_X(x)$, Kakwani considered the problem of estimating the prevalence of undernourishment under the assumption that i) only μ_R is and ii) both μ_R and σ^2_R are known.

When only μ_R is known, Kakwani indicated that the prevalence of undernourishment could be estimated by applying this requirement level as the cut-off point on the distribution of X as given by (17). On the other hand when both μ_R and σ^2_R are known he suggested an approach that uses the distribution of R to derive the probability of undernourishment over the whole range of X just as Lörstad attempted to do in the early 1970's.

Thus, using the data on distribution of X derived from the Indian National Sample Survey of 1971-72, i.e. those used by Dandekar (1981) and Sukhatme (1982), Kakwani had derived numerical estimates of the prevalence of undernourishment for urban and rural India according to (i) the approach based on μ_R as the cut-off point and (ii) the approach that assumes knowledge of both μ_R and σ^2_R and hence $f_R(r)$. In the latter case, X and R were assumed to be independent and $f_R(r)$ was first assumed to be uniform and then normal.

The results for urban and rural India in 1971-72 were as given below.

	Prevalence of undernourishment (%) based on		
	μ_R as cut-off point	$f_R(r)$ uniform	$f_R(r)$ normal
Urban India	67.5	64.2	64.5
Rural India	52.4	51.3	51.4

The above showed that, under the assumption that X and R are independent, the estimates based on μ_R as cut-off point would not lead to any significant difference as compared to those based on the distribution of R.

Kakwani also investigated into the effect of correlation in the context where μ_R and σ^2_R are known and $f_R(r)$ is assumed to be normal. The approach for this investigation turned out to be the same as conditional distribution framework used by Lörstad (1974) and discussed in **III(*b*)**. The results showed that by varying the coefficient of correlation, ρ, from *0.0 to 0.9*, while keeping μ_R and σ^2_R constant, the prevalence of undernourishment changed only marginally: in urban India it changed from 64.5 to 70.9 while in rural India the change was from 51.4 to 48.9. This rather weak sensitivity to correlation led Kakwani to conclude that the estimates would not be too biased if X and R were assumed to be independent. However, unlike Lörstad, he did not express any concern regarding the rather large estimates of the prevalence of undernourishment obtained.

4) The Srinivasan article

Srinivasan was one of the few economists who were predisposed to Sukhatme's theory of intra-individual variation in energy requirement and the range of homeostasis. Therefore in his paper he provided an explanation of the process approach to energy balance underlying Sukhatme's theory and the relationship with the range homeostasis and short-term adaptation.

However, he also had viewed Sukhatme's theory of intra-individual variation in requirement as referring to the variation in the efficiency of energy utilisation. Therefore he indicated that, after allowing for this intra-individual variation, the inter-individual variation due to sex, age, bodyweight and physical had still to be taken into account within a joint distribution framework in order to estimate the prevalence of undernourishment.

5) The Payne article

Payne, on whose advice FAO in the early 1970's initiated the approach of deriving the cut-off point based on the BMR and the PAL index for light activity, took a totally different view from the others. He felt that the controversy surrounding the theory of intra-individual variation and resulting uncertainty partly arise from the fact that undernutrition or undernourishment has so far been defined as any state that is below the optimum required for maintaining health, expressed in terms of ideal body-size or desirable level of energy expenditure. Thus in the context of the measure of undernourishment based on food consumption, the focus has been on a level of energy expenditure that is consistent with the maintenance of an ideal body-size and the performance of a desirable level of physical activity. In this connection each individual has been traditionally assumed to have a single optimum combination of body-size and physical activity. Therefore the concern in the nutrition community about the theory of intra-individual variation in energy requirement stems from the fact that it implied that this traditional view of a single optimum combination of body-size and desirable physical activity (and hence a single energy requirement) for an individual should be abandoned. After reviewing the limitations in the recommendations of the international expert groups on nutritional requirements, Payne in fact argued the case for abandonment of the traditional view and offered the alternative view that the measurement "should be concerned more with identifying those who have failed to avoid life-threatening risk even after making all possible adjustments to nutritional stress".

In reviewing the limitations in the international expert groups' recommendations, Payne highlighted the fact that, until the *1981 FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements*, all the recommendations of FAO/WHO expert groups on nutritional requirements were for prescriptive purposes in the sense that they primarily served the purposes of arriving at μ_R as an estimate of the average food energy supply at the national level that would be consistent with a nutritionally healthy population. The reports of the committees have also been emphatic that these recommended levels should not be applied to individuals, but only to groups. That is to say these recommended levels were not meant to be used as yardsticks for the detection of undernourishment, but only as a basis for planning food supplies. Thus he contended that most of the disputes about measuring the extent of undernourishment have been about how to apply standards that were never intended for that purpose

Payne however acknowledged that the report of the 1981 FAO/WHO/UNU Expert Consultation departed from the earlier FAO/WHO committees' reports in two ways. Firstly a clear distinction is made between the use of energy requirements for prescriptive purposes, as indicated above, and diagnostic purposes, e.g. to judge the probable adequacy or inadequacy of observed DECs. Secondly, at least for the adolescents and adults, the new recommendations defined DER as the energy expenditure for maintaining body-size that are consistent with good health and for performing desirable physical activity rather than the habitual intake of individuals presumed to be healthy. Moreover, as indicated in Section **IV(c)**, energy expenditure was expressed in terms of the BMR and the PAL index and a set of linear regression equations for estimating BMR by sex and age on the basis of body-weight were provided. Thus given body-weight and the PAL index the corresponding level of energy expenditure could be neatly derived. However he noted that, apart from commenting that recommendations for DER depended as much on value judgments about what body-sizes, growth rates (for children) and levels of physical activity are deemed to be desirable as on biological needs, the 1981 FAO/WHO/UNU Expert Consultation's report said almost nothing on the subject of how to use the new principles for the purpose of diagnosing undernourishment.

In view of the above limitations in the recommendations of the international expert groups on nutritional requirements, Payne felt that the way forward should be to set aside for the time being the problem of defining the state of a perfectly adequate nutrition and "to concentrate instead on identifying a critical limit of DEC below which there is clear evidence of risk of loss of functional capacity, measured in some objective terms, or, indeed, life threatening risk". He indicated that adoption of this minimalist position did not imply that policies and interventions would be restricted to "worst case situations" but rather a strategy of "putting the last first".

In considering a critical limit of DEC below which an individual is subject to life threatening risk, Payne referred to the various of ways that an individual, faced with nutritional stress or DEC constraints, could adjust his or her energy expenditure to maintain balance with DEC. He considered such adjustments, implying changes in body-size and/or physical activity, as not being without cost. However over a wide range of adjustment, i.e. the range of variation of DER, those costs would be balanced by increased chances of survival and hence could be considered as acceptable. Therefore he suggested that the lower limit of the range of variation of R, i.e. r_L, be taken as the critical limit of DEC below which an individual is subject to life threatening risk.

VIII. METHODOLOGICAL REVIEW IN CONNECTION WITH THE SIXTH WORLD FOOD SURVEY

In the course of the preparatory work for the *Sixth World Food Survey* (FAO, 1996) the debate and development outside FAO, as discussed in the previous section, were reviewed. In this connection the following points were noted:

- The controversy within the nutrition community surrounding Sukhatme's theory intra-individual variation due variation in the efficiency of energy utilization was not so much on the existence of such a variation but rather on whether it could be as large as that claimed by Sukhatme (a coefficient of variation of about 20%).
- Osmani had concluded that Sukhatme's theory could not be justified in the light of current scientific evidence.
- Even if the intra-individual variation due to efficiency of energy utilization is accounted for by a reduction in the mean DER, the issue of how to account for the inter-individual variation still remained. The views of Osmani as well Srinivasan in this connection were that this variation should ideally be considered within a joint (bivariate) distribution framework.
- Anand and Harris had ignored the intra-individual variation in DER and had graphically illustrated the problem of estimating the prevalence of undernourishment within the bivariate distribution framework.
- Kakwani also had ignored the intra-individual variation theory and applied alternative approaches for estimating the prevalence of undernourishment on the basis of DEC data for India. However these approaches, which yielded similar results, had already been considered and rejected by FAO.
- Beaton, who had long been a supporter of the bivariate distribution model, had recognized that the major problem in applying this model concerned the insensitivity to the effect of correlation that is assumed to exist between DEC and DER. But he had interpreted this insensitivity to be an indication that the probability approach cannot be applied in estimating the prevalence of undernourishment.
- In the light of the dispute surrounding the theory of intra-individual variation in DER Payne had indicated that the way forward should be to define a critical limit of DEC below which an individual could be considered to be at risk of loss of functional capacity or indeed life threatening risk. The critical limit of DEC that he suggested in this connection boiled down to being practically the same as the cut-off point used by FAO.

In view of the above diverse and somewhat conflicting views, it was decided to review the probability framework for estimating the prevalence of undernourishment. The detailed considerations in this connection were not included in *The Sixth World Food Survey* but the essential aspects were referred to in a separate staff paper (Naiken, 1998). However for the sake of clarity, the points considered are discussed below in a more logical manner. Following this the conclusions that emerged and the approach taken in the *Sixth World Food Survey* are briefly indicated.

(a) The Issue Boils Down to the Status of the Consumptions Overlapping the Range of Variation of Requirement

The measure of undernourishment implies a comparison of an individual's value of X, i.e. x, with the individual's value of R, i.e. r. In this context while x is assumed to be known, r is not known but presumed to be given within a range, i.e. the range of variation of R in the population. The fact that r is unknown implies that one needs to consider the probability corresponding to three possible events concerning the status of the given x in relation to the unknown r: the probability that x is below r i.e. $P(x<r)$, the probability that x is in balance with r, i.e. $P(x=r)$, and the probability that x is above r, i.e. $P(x>r)$. At the population level these probabilities, which are expressed as $P(X<R)$, $P(X=R)$ and $P(X>R)$ respectively, need to be conceived as an average of the individual probabilities over the whole range of X as follows:

$$P(X<R) = \int_{-\infty}^{\infty} f_X(x)\, P(x<r)\, dx \quad \text{.......... (27)}$$

$$P(X=R) = \int_{-\infty}^{\infty} f_X(x)\, P(x=r)\, dx \quad \text{.......... (28)}$$

$$P(X>R) = \int_{-\infty}^{\infty} f_X(x)\, P(x>r)\, dx \quad \text{.......... (29).}$$

Thus, given $f_X(x)$, the evaluation of the three population level probabilities depends on the behaviour of the respective individual probabilities over the whole range of X.

If r_L and r_U, represent the lower and limits respectively of the range of R, it is clear that,

$P(x<r) = 1$ for all x below r_L

and

$P(x>r) = 1$ for all x above r_U.

Therefore, in using the univariate distribution formula with r_L as the cut-off point, only the range of X where $P(x<r) = 1$ has been considered in the evaluation $P(X<R)$. Since $P(x<r)=0$ for the range of X above r_U, this upper range of X is obviously irrelevant for the

evaluation of $P(X<R)$. Hence, the issue boils down to the probability assessment for the values of X within the range of variation of R, i.e.

$$r_L < X < r_U.$$

1) The Probability Statement in the Report of the 1981 FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements

The report of 1981 *FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements* had in fact addressed the probability assessment for the values of X overlapping the range of R (ref. pages 15-19). In this discussion it is stated that, if X and R were independent (uncorrelated), $p(x<r)$, will steadily decrease from practically 1 to practically 0 and, conversely, $p(x>r)$ will steadily rise from practically 0 to practically 1 as x increases from r_L to r_U. This statement in fact results from the formulation of the probability (risk) functions in terms of the distribution function of R as follows:

$$P(x<r) = 1 - F_R(x)$$
$$P(x>r) = F_R(x).$$

Thus independence or zero correlation implies that the probability of energy adequacy, i.e. $P(x=r)$, is uniformly 0 over all x. This situation is theoretically the same as under the assumption that the variance of R is random and the distribution R is normal which, as was explained in Section **IV*(a)***, would justify the use of μ_R as the cut-off point in the univariate distribution formula.

However, as R varies within the range of variation of X, the two variables cannot be assumed to be independent and hence a correlation should be expected between X and R. In this context the 1981 *FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements* stated the following:

"Most people have the ability to select their food intake in accordance with their energy requirement over the long term, since it is believed that regulatory mechanisms operate to maintain a balance between energy intake and energy requirement over long periods of time. This implies that one would expect there to be a correlation between energy intake and energy requirement if sufficient food is available in the absence of interfering factors ....If self-selection is allowed to operate, it is to be expected that individuals will make selections according to need and the probability of inadequacy or excess will be low across the whole range ....if the average of a class were equal to the average requirement of the class, almost all individuals would be at low risk because of processes regulating energy balance and the resultant correlation between intake and requirement."

Thus, it is indicated that, because of the existence of processes regulating energy balance, individuals tend to consume food according to their respective needs with the consequence that $p(x<r)$ and $p(x>r)$ are expected to be uniformly low, and consequently, $p(x=r)$ uniformly high for the values of X within the range of R, i.e.

$r_L < X < r_U$.

It is also indicated that the correlation expected between X and R is a reflection of the tendency for individuals to consume food according to their respective needs. It follows from this that the correlation actually refers to $P(X=R)$ so that a perfect correlation implies that $P(X=R)=1$. Thus the condition for a perfect correlation, i.e. $\rho=1$, is that $\sigma_X=\sigma_R$ and by implication $\mu_X = \mu_R$.

The reference to the fact that the correlation between X and R implies a high probability of energy balance (adequacy) for the values of X within the range of R had, however, not caught the attention of most observers. This is probably because the FAO/WHO/UNU Expert Consultation's statement was made in the context of justifying the representation of the DER for a group (or class) of individuals by the average (the justification being that the average represents the average DEC of a perfectly nourished population) rather than in the context of assessing food inadequacy or excess.

2) The relationship between the 1981 FAO/WHO/UNU Expert Consultation's statement and Sukhatme's theory of intra-individual variation

It may be noted that Sukhatme also had referred to the regulatory mechanisms or processes operating to maintain balance between energy intake and energy requirement in the long-term rather than every-day. However, he had interpreted the resulting high probability of energy balance (adequacy) for the values of X within the range of R in the context of the day-to-day variation in energy balance within individuals. This is clear from his statement that "although energy intake may not be equal to energy expenditure even when averaged over a week, man is in balance every day in a probabilistic sense with varying intervals of peaks and troughs and varying amplitudes in daily balance".

The fact that the correlation resulting from the effect of regulatory mechanisms or processes operating to maintain energy balance refers to $P(X=R)$ implies that the presence of the regulatory mechanisms or processes would be reflected by an auto-correlation in a time series pertaining to the difference (X-R). This suggests that, by testing for auto-correlation in the daily series pertaining to (X-R), Sukhatme was in fact attempting to demonstrate the presence of the regulatory mechanisms or processes that explains the existence of a probability for energy balance or adequacy, i.e. $P(X=R)$.

It was thus realized that Sukhatme's theory of (intra-individual) variation in the efficiency of energy utilization was a biological explanation for the mechanisms or processes regulating energy balance that lead to a high probability of energy balance for the values of X overlapping the range of R rather than an additional source of variation in the level of the DER as interpreted by most observers. Therefore the high probability of energy adequacy for the values of X overlapping the range of R is valid irrespective of whether the variance of R is defined in context of the day-to-day variation within individuals or the variation between individuals.

However, it must be noted that, although both the report of the 1981 FAO/WHO/UNU Expert Consultation and Sukhatme had referred to the effect of the regulatory mechanisms or processes that operate to maintain balance between energy intake and energy requirement, the purpose was different. In the former case the purpose was to justify the representation of the DER for a group of individuals by μ_R (since it represents μ_X in a group or population where everyone is in the state of energy adequacy) while in the latter case it was to justify the use of the lower limit of the range of *R* as the norm for diagnosing undernourishment.

3) The statistical explanation for the high probability of energy adequacy for the values of X overlapping the range of R and the correlation between X and R

In the case of both the 1981 FAO/WHO/UNU Expert Consultation's statement and Sukhatme's analysis, the distribution referred to individuals who are similar in so far as not only the demographic (sex-age) but also the body-weight and physical activity characteristics (i.e. individuals of a given "class" or of the "reference" type) and hence the variance of *R* was considered in the context of the variation between such similar individuals. In view of this the variation due to body-weight and physical activity did not arise.

However, the above is a reflection of the past practice of defining DER on the basis of the intakes of the "reference" man and woman. As indicated in Section **IV(c)**, the *1981 FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements* has discontinued this approach and instead defined DER as the energy expenditure of an individual having a body-size and a physical activity level that are consistent with good health and that will allow for the maintenance of economically necessary and socially desirable physical activity. As a consequence of this new definition the variation of *R* has been defined in terms of the variation in the normatively specified body-weight and physical activity of the individuals. Nevertheless the fact remains that DER represents the DEC of an individual who is in the state of energy balance or adequacy. Hence the distribution of *R* represents the distribution of *X* in a population where every one's DEC is in balance with DER, i.e. $x=r$, so that the variance of *R* represents the variance of *X* in a population consisting individuals in the state of energy adequacy. It therefore follows that the distribution of *R* reflects the realization of the joint distribution of *X* and *R*., i.e.

$$f_{XR}(x,r) = f_R(r) \qquad (30)$$

so that when

$$f_X(x) = f_R(r)$$

and hence

$$P(X=R) = 1.$$

This above means that for the region of X overlapping the range of R,

$$P(x=r) = 1$$

and hence

$$P(x<r) = P(x>r) = 0.$$

Thus the statistical explanation for the high probability of energy adequacy associated with the values of X overlapping the range of R is that the distribution of R represents the realization of the joint distribution of X and R.

The fact that the distribution of R represents the realization of the joint distribution of X and R, also means that the variance of R represents the covariance, i.e.

$$\sigma^2_R = \sigma_{XR},$$

so that

$$\rho = \sigma_R / \sigma_X.$$

As σ_R is fixed, it follows that, under the assumption that the distribution of R is normal, $\sigma_X=\sigma_R$ would imply that $\rho=1$ while $\sigma_X>\sigma_R$ would imply that $\rho<1$. Since $\sigma_X>\sigma_R$ would be reflected by the extension of the lower and upper tails of $f_X(x)$ beyond the limits of $f_R(r)$, it follows that

$$\rho = P(X=R) \ldots\ldots (34).$$

(b) The Univariate Probability Distribution Framework for Estimating the Prevalence of Undernourishment and Overnourishment

It follows from the above that, as $P(x<r)$ is uniformly 1 for the region of X below r_L, $P(x=r)$ is uniformly 1 for the region overlapping the range of R and $P(x>r)$ is uniformly 1 for the region above r_U, the three population level probabilities given by formulae (27), (28) and (29) reduces to the following:

$$P(X<R) = \int_{-\infty}^{r_L} f_X(x)\, dx \ldots\ldots (31)$$

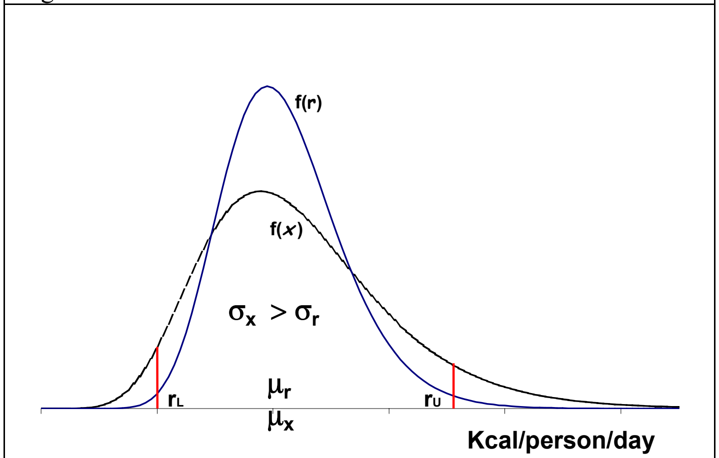
$$P(X=R) = \int_{r_L}^{r_U} f_X(x)\, dx \ldots\ldots (32)$$

$$P(X>R) = \int_{r_U}^{\infty} f_X(x)\, dx \ldots\ldots (33).$$

Note that (31) is equivalent to the univariate distribution formula given earlier by (4).

The above means that the problem of estimating the prevalence of undernourishment or overnourishment has to be viewed within the univariate distribution framework illustrated in Figure 2.

Figure 2: The univariate distribution framework



The distribution of X is shown to be wider that that of R since the latter is expected to be located within the range of variation of X and hence variance of X is expected to be larger than that of R (i.e. unless $f_X(x)= f_R(r)$). The larger variance of X is due the fact that it includes, in addition to the variance of R, the variance due to income and residual factors.

In the above figure the means of X and R are assumed to be equal solely for simplicity and the purpose of explaining the extension of the distribution of X beyond the limits of the distribution of R in terms of higher variance or standard deviation. It is obvious that in most cases, where the two means are not likely to be equal, the extension of the distribution of X beyond the limits of the distribution of R would have to be explained in terms of higher CV.

As the extension of two tails of the distribution X beyond the limits of the distribution of R mainly reflects the effect of income, this distribution is shown to be skewed to the right just as the income distribution. The distribution of R also is likely to be skewed but much less than that of X. Moreover, the true lower and upper limits of the distribution of R, i.e. r_L and r_U, are actually not known. The positions that they are shown in the figure reflect the fact that they have been taken to correspond to the 5th and 95th percentiles respectively of the distribution of R.

Thus, the area corresponding to $P(X=R)$ is represented by part of the distribution of X ranging from r_L to r_U while that corresponding to $P(X<R)$ is represented by the part below r_L and $P(X>R)$ by the part above r_U. It follows from this that the long-term food policy objective should be to reduce the two tails of the distribution of X so that the latter tends towards the fixed distribution of R. In other words food and nutrition policies should address not only the problem of undernourishment but also that of overnourishment which is growing in the developing countries also.

As a perfect correlation, i.e. $\rho =1$, implies that the requirements of all the individuals in the population are met and the condition for a perfect correlation is

$$f_X(x) = f_R(r)$$

it follows that the dietary energy adequacy norm for a population or a group is represented by $f_R(r)$ so that the assessment of the prevalence of undernourishment and overnourishment implies the comparison of $f_X(x)$ with $f_R(r)$ and the evaluation of the two parts falling outside the limits of $f_R(r)$.

However, $\underline{\mu_R}$ refers to the average DEC of an adequately nourished population. As such it represents the level of consumption to which those below the lower limit and those above the limit of the range of R should be raised and diminished respectively so that

$$f_X(x) = f_R(r)$$

and thus everyone be in the state of energy adequacy. Therefore $\underline{\mu_R}$ represents the adequacy norm for those classified in the inadequate and excess categories and is thus useful in estimating the food deficit of the group of undernourished and the food excess of the group of overnourished.

(c) Conclusion of the Review and the Approach Taken in the Sixth World Food Survey

The review led to the following conclusions:

- Sukhatme's theory of regulatory processes operating to maintain balance energy balance in individuals in the long-term rather than every day actually referred to the high probability of energy balance associated with the values of X overlapping the range of variation of R. However, his interpretation of this high probability for energy balance over a range of individual consumption levels as reflecting the capacity of an individual to vary his or her efficiency of energy utilization in response to food constraints, has led to the unnecessary controversy and debate in the literature over the issue of intra- individual and inter-individual variation in R. The issue concerning the variation of R in estimating the prevalence of food inadequacy or excess is in fact not whether it is of inter-individual or intra-individual nature but whether it is random or systematic.
- The 1981 *FAO/WHO/UNU Expert Consultation on Energy and Protein Requirements* had also referred to the regulatory processes operating to maintain energy balance in individuals and the resulting correlation between X and R. However, it was not realized that the correlation actually referred to $P(X=R)$ and that the condition for a perfect correlation is $f_X(x) = f_R(r)$ with the consequence that an imperfect correlation is reflected by an extension of the two tails of $f_X(x)$ beyond the limits of $f_R(r)$.
- The fact that $f_R(r)$ represents $f_X(x)$ in a perfectly nourished population implies that $f_R(r)$ reflects the realization of $f_{XR}(x,r)$ with the consequence that the probability of energy adequacy is high for the values of X within the range of R. Hence only the region of X below the lower limit and the region above the upper limit of the

distribution of *R* can be considered to be in the inadequate and excess categories respectively.

- While the lower and upper limits of $f_R(r)$ represent the norms for the diagnosing food inadequacy and excess respectively, the mean, i.e. μ_R. represents the energy adequacy norm for those diagnosed as being in the inadequate and excess categories. In other words μ_R is the norm for prescribing the consumption level to be attained by those diagnosed as being in the inadequate and excess categories.
- The use of the lower and upper limits of the range of variation of *R* as the criteria for diagnosing undernourishment and overnourishment respectively does not necessarily imply, as suggested by Payne, the abandonment of the traditional view of a single optimum combination of body-size and desirable physical activity and hence a single DER for an individual but rather the recognition of the fact that, since individuals tend to consume according to their needs, the probability of achieving energy adequacy is high for the individuals with consumptions overlapping the range of *R*. In other words, the range of variation of *R* needs to be regarded as a range of acceptable or tolerable consumption levels for an individual.

The above conclusions had led to FAO's use of the univariate distribution formula given by (4) with greater confidence in the *Sixth World Food Survey* and the subsequent reports on the *State of Food Insecurity in the World*. As regards the cut-off point, r_L, the alternative cut-off point in the *Fifth World Food Survey*, involving a reduction in the *BMR* to account of intra-individual variation due to efficiency in energy utilization, was discontinued. This is due to the fact that, as indicated under **c)** above, the variation in the efficiency of energy utilization referred to by Sukhatme was a biological explanation of the effect of the mechanisms or processes leading to the probability of energy balance rather than an additional source of systematic variation in *R*.

The *Sixth World Food Survey* also included an exercise where the food gap or the amount of extra food required to eliminate undernourishment was calculated by considering μ_R as adequacy norm for the undernourished. In this context μ_R was derived by calculating the average dietary energy requirement (ADER) by sex-age groups and averaging over the sex-age groups using proportion of the population as weights just as in the case of the derivation of r_L described in section **IV(*c*)**. However, for the purpose of calculating the sex-age specific ADERs, body-weight was taken to correspond to the median (50th percentile) of the relevant WHO reference weight for given height distributions and physical activity was reflected by the PAL index for moderate activity or lifestyle.

IX. THE DEVELOPMENTS FOLLOWING THE SIXTH WORLD FOOD SURVEY

The *Sixth World Food Survey's* estimate of about 840 million undernourished people in the developing world (20 % of the population) in 1990-92 was presented at the World Food Summit held in 1996. Following its deliberations this Summit set the goal of halving the number of undernourished by the year 2015. Thus, as part its effort in monitoring progress towards this goal, FAO had in 1999 initiated the process of issuing annual updates of estimates of the prevalence of undernourishment through its new publication entitled the

State of Food Insecurity in the World (SOFI). In this connection the estimates presented referred not only to the broad regional/global aggregates as in the World Food Surveys, but also to the individual countries. This new development was accompanied by criticisms of the data as well as methodology underlying the FAO estimates of the prevalence of undernourishment. The major criticism in this connection came from Svedberg who in a paper (Svedberg, 2003), circulated in 2000, used the bivariate distribution framework originally conceived by Sukhatme to demonstrate that FAO had overestimated the prevalence of undernourishment due to flaws in the data used. In the light of such criticisms FAO had organized the *International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition* that was held in Rome in 2002. At this symposium the FAO methodology as well as the data used for estimating the prevalence of undernourishment was presented in detail in a keynote paper (Naiken, 2004) and in an appendix of the paper the flaw in the approach based on the bivariate distribution framework used by Svedberg as well as the others before him was addressed. However, since the reason for the flaw was apparently not sufficiently clear the issue is addressed again here following a discussion of the Svedberg study and the keynote paper presented at the *International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition*.

(a) The Svedberg Study

Svedberg, using the arguments of Anand and Harris (1992), considered the univariate distribution formula with r_L as cut-off point as leading to an underestimate of the prevalence of undernourishment and is therefore “biased”. He considered the estimate originally formulated within the bivariate distribution framework by Sukhatme to be “unbiased” and therefore wondered why FAO had refrained from applying it. Thus taking the view that this formula was probably not applied by FAO because of lack of data on the joint distribution, $f_{XR}(x,r)$, he, just as Lörstad, applied it by modeling the joint distribution of X and R. In this connection he noted that, assuming a normal distribution and knowledge of μ_X, μ_R, $\sigma^2{}_X$ and $\sigma^2{}_R$, the only missing parameter for evaluating the bivariate distribution formula was ρ. Therefore, using the regional averages of the values for μ_X, μ_R, $\sigma^2{}_X$ and $\sigma^2{}_R$ either explicitly or implicitly used by FAO for the estimates published in the *Sixth World Food Survey* (FAO, 1996), he derived estimates for the four developing regions under the assumption of alternative values for ρ. He had actually assumed the joint distribution of X and R to be lognormal. Thus the only difference between this approach and that used by Lörstad in his 1970 study is that the correlation between X and R was assumed to be log-linear rather than simply linear.

Svedberg noted, as the others who had previously attempted to apply the bivariate distribution formula, that the estimates were insensitive to the effect of correlation. Just as Kakwani, he had taken this as an indication that a correct specification of ρ was not of significant importance. He also noted, just as Lörstad, that the estimates were unrealistically high. However, he had attributed the unrealistically high estimates to systematic errors in the “FAO input data” i.e. the estimates of the parameters used. In particular, he considered the estimates of σ_X and μ_R to be too high and σ_R to be too small. Therefore, on the basis of rather subjective reasoning, he made adjustments in these parameters in order to arrive at "realistic" estimates. These estimates were found to be generally lower than the FAO estimates for the four developing regions in the *Sixth World Food Survey*.

Thus he concluded that FAO's estimates of the prevalence of undernourishment presented in the *Sixth World Food Survey* were subject to two types of biases: a downward "methodological bias" due to the fact they are based on the "biased" univariate distribution formula rather than the "unbiased" formula based on the bivariate distribution; and an upward "data bias" resulting from the systematic errors in the input data. However, as the estimates that he obtained on the basis of the "corrected" input data were lower than the FAO estimates, he considered the "data bias" to be greater than the "methodological bias" and hence considered that the estimates in the *Sixth World Food Survey* to be "overestimates".

(b) The FAO Keynote Paper Presented at the International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition

This paper focused on the FAO methodology for estimating the prevalence of undernourishment by presenting all the details regarding the procedures for estimation of the distribution of DEC, $f_X(x)$, and the cut-off point, r_L, as well as the data used. However, since Svedberg had wondered why FAO had not attempted to apply the bivariate distribution framework by modeling joint distribution, $f_{XR}(x,r)$, the flaw in the approach was addressed in an appendix of the paper while justifying the FAO approach.

In Appendix A of the paper it was indicated that all those who had attempted to apply the bivariate distribution framework by modeling the joint distribution of X and R had failed to take into account the fact that, as the distribution of R is located within the range of the distribution of X, the covariance reduces to the variance of R with the consequence that $\rho = \sigma_R / \sigma_X$. Thus, as the values for ρ were imputed without considering the given values for σ_R and σ_X , the flaw in the approach was attributed to a failure to correctly interpret the correlation between X and R. The univariate distribution formula with r_L as the cut-off point was explained as being the result of a correct interpretation of the correlation. That is to say, with the correct interpretation of correlation, the bivariate distribution formula reduces to the univariate distribution formula.

However, in a unpublished commentary following the *International Scientific Symposium on the Measurement of Food Deprivation and Undernutrition*, Svedberg (2002), had contested the above argument by indicating that the bivariate distribution formula would reduce to the univariate distribution formula only under the condition that the joint distribution X and R is zero in the two regions of X outside the limits of the distribution of R and the issue of whether this was realistic or not was left to the reader to ponder. However, from the discussion in section **VIII*(d)***, it is now clear that the joint distribution is indeed zero outside the limits of the range of R simply because it is represented by the distribution of R.

(c) The Irrelevance of the Bivariate Distribution Framework

However, the fact of the matter is that the bivariate distribution framework is irrelevant for considering the problem at hand and therefore the estimate of the prevalence of undernourishment should not have been formulated within this framework. In other words the flaw actually lies in Sukhatme's original formulation of the estimate within the bivariate distribution framework. This point is explained below.

In Figure 1, which assumes that the bivariate distribution is normal, it is indicated that the *45-degree* line represents the event $x=r$ but there is no probability space assigned to this event. This is a consequence of the fact that in the bivariate distribution framework, the joint distribution of X and R and hence the covariance and correlation is defined under the condition that

$$f_X(x) \neq f_R(r).$$

But the probability of perfect dietary energy adequacy in the population, which is reflected by

$$\rho = P(X=R) = 1$$

in fact results from the condition that

$$f_X(x) = f_R(r).$$

Thus, as the above condition is excluded, the bivariate distribution by definition implies that

$$P(X=R) = 0$$

and therefore

$$P(X<R) + PX>R) = 1.$$

The above means that the existence of the probability for dietary energy balance or adequacy is denied with the consequence that the bivariate distribution refers to a population consisting of individuals whose consumptions are either below or above their respective requirements, i.e. the undernourished and overnourished.

The irrelevance of the bivariate distribution framework in the present context is demonstrated by the unrealistic relationship between X and R that results from this model. The relationships expressing dependence of X on R and vice versa are given by the following regression equations:

$$x = \mu_X + \rho\, \sigma_X (r - \mu_R)/\sigma_R. \qquad (35)$$

$$r = \mu_R + \rho\, \sigma_X\, (x - \mu_X)/\sigma_R \quad \ldots\ldots (36).$$

One would normally expect x to be 0 when r is equal to 0 and vice versa but in the above relationships it is clearly not so for any value of ρ.

In fact the bivariate normal distribution is relevant in the context of estimating the correlation and relationship between two variables whose values are not expected to be equal, e.g. height and weight, and therefore the condition of equality in distribution and hence perfect correlation does not arise. In the present context where food policy goal should be towards a perfect correlation the bivariate distribution is obviously not relevant.

Thus, by denying the existence of the probability of achieving energy balance or adequacy, the bivariate distribution framework is bound to overestimate the prevalence of undernourishment and overnourishment, i.e. $P(X<R)$ and $P(X>R)$. This also explains the anomaly built into the model as well as the insensitivity of $P(X<R)$ to the effect of correlation referred to in section **IV*(a)*** and acknowledged by Beaton. Therefore the estimates of the prevalence of undernourishment derived by Lörstad, Reutlinger and Alderman, Kakwani and more recently by Svedberg (on the basis of FAO input data) are gross exaggerations resulting from a flawed probability distribution model. By the same token the conclusions reached by Svedberg regarding the FAO methodology and input data are not valid.

X. CONCLUDING REMARKS

The history of the uncertainty and debate concerning the probability framework for considering the variation in DER in the estimation of the prevalence of undernourishment has been plagued by misunderstandings and confusion primarily stemming from Sukhatme's formulation of the estimate within the bivariate distribution framework and his initial consideration of the estimate formulated within the univariate distribution framework as being an approximation due to lack of data pertaining to the joint distribution of DEC and DER. In other words the impression given was that the bivariate distribution formula is the appropriate mathematical expression for the estimate. His subsequent justification of the univariate distribution framework by invoking the theory of intra-individual in DER in effect meant that the bivariate distribution framework was irrelevant. However, he had never acknowledged this. Therefore, since the theory of intra-individual variation itself had led to controversy and uncertainty rather than an understanding of the relevance of the univariate probability distribution framework, the myth of the bivariate distribution framework has been kept alive and thus encouraging others to apply it by using flawed joint distribution or conditional distribution models.

The univariate distribution framework is actually justified by the fact that, as the distribution of DER reflects the distribution of DEC in a population consisting of perfectly nourished individuals, the probability of energy balance or adequacy is high for the individuals with DEC falling within the limits of the distribution of DER. Consequently only the individuals with DEC falling below the lower limit of the distribution of DER can be classified as undernourished and only those with DEC above the upper limit classified as overnourished. This means that the adequacy norm for assessing the prevalence of food inadequacy and excess in a population is actually the distribution of DER rather than the average DER. However, since the average DER reflects the average DEC of a perfectly nourished population, it represents the DEC level to which the DECs of the undernourished should be raised and the DECs of the overnourished be reduced so that everyone in the population would be in the state of energy balance or adequacy *in the probability sense*. In other words, while the distribution of DER enables the assessment of the proportion of the population undernourished and overnourished, the average DER enables the assessment of the food deficit of the undernourished and the food excess of the overnourished.

Thus, for the purpose of assessing the prevalence of undernourishment and overnourishment and the implied food deficit and food excess, estimates of not only the average DER but also the distribution around the average are required. However, the problem in this context is that the focus of the international expert groups on nutritional requirements has so far been on recommendations regarding the average DER only. This has therefore led to the unwarranted use of the average DER as the dietary energy adequacy norm for making inference on food inadequacy or excess in a population or group. However, realizing the importance of the lower limit of the distribution around the average, in connection with its effort to estimate the prevalence of undernourishment, FAO has been relying on the advice of ad hoc technical groups convened by the Statistics Division for the specific purpose of defining and calculating this lower limit. The technical group convened by the Statistics Division in 2005 for considering the approach to be taken in the light of the principles set by the *2001 FAO/WHO/UNU Expert Consultation on Human Energy Requirements* had in fact made recommendations not only regarding the lower limit but also the upper limit of the distribution of DER that could eventually be used for the purpose of estimating the prevalence of overnourishment. Thus the way forward should be the setting of an international group of experts to consider and formalize these limits and the underlying distribution and subsequently issue appropriate guidelines for countries regarding the use of the principles of energy requirements in connection with the estimation of undernourishment and overnourishment.

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