Abstract

National forest assessments are best conducted with sufficiently accurate and scientifically defensible estimates of forest attributes. This chapter discusses the statistical design of the sampling plan for a forest inventory, including the process used to define the population to be sampled and the selection of a sample intended to satisfy NFA precision requirements. The team designing a national forest inventory should include an experienced statistician. If such an expert is not available, this section provides guidance and recommendations for relatively simple sampling designs that reduce risk and improve chances for success.

1. Introduction

The sampling design chosen to support the technical programme used for an NFA requires a theoretical basis that can be implemented on the ground (see chapter on Organization and Implementation, p. 13). Understanding the basic concepts of statistical design and estimation methods is a key component of the overall process for information management and data registration for NFAs (see chapter on Information management and data registration, p. 93).

1.1 Objectives

The goal is to estimate the condition of forests for an entire nation using data collected from a sample of field plots. The basic objectives of an NFA are assumed to be fourfold: (i) to obtain national estimates of the total area of forest, subdivided by major categories of forest types and conditions, as well as the numbers and distributions of trees by species and size categories, wood volume by tree characteristics, non-wood forest products, estimates of change in these forest attributes and indicators of biodiversity; (ii) to obtain sufficiently precise estimates for selected geographic regions such as the nation, subnational areas, provinces or states and municipalities; (iii) to collect sufficient kinds and amounts of information to satisfy...
international reporting requirements; and (iv) to achieve an acceptable compromise between cost and precision, and geographic resolution of estimates.¹

**Assumptions and simplifying constraints**

Several assumptions underlie the discussion that follows: first, that expert statisticians experienced in designing natural resource inventories and analysing the data are not available; second, that ancillary data in the form of maps depicting features such as ecological regions, land cover, soils, elevation, political and administrative boundaries, and transportation systems are available; and third, that models for predicting attributes such as individual tree volumes from basic tree measurements are available. Even on the basis of these assumptions, a full discussion of all sampling design possibilities for an NFA is beyond the scope of this section. Thus, three constraints have been imposed: first, this chapter presents only relatively simple, multipurpose designs that can be used reliably with local expertise; second, the discussion is limited to designs that are flexible, yet reduce risks of bias and loss of credibility; and third, there is a focus on designs that feature equal probability samples, or in the case of stratified designs, equal probability samples within strata.

### 1.2 Why use sampling?

The most precise description of a population comes from accurate measurements of each member of the population, otherwise known as a census. However, a typical census is very difficult to undertake because of cost and logistical problems. Imagine trying to measure every tree in a 1 million hectare forest. Instead, a sample measures a portion of the population – in forestry this is usually a very small portion. Estimates based on data collected from the measured sample are then extrapolated to the entire population, the majority of which has not been measured.

This can be thought of as “guessing” or “estimating” the condition of a population based on sampling a few members of that population. If the sample is representative of the entire population, then the estimate will be accurate and less likely to deviate from the true population value. Otherwise, estimates will be inaccurate and misleading – a situation that may not be known because the true condition of the whole population will remain unknown. The best possible approach is to increase the chances of measuring a representative sample. This can be done by using scientifically rigorous rules to select the sample, maximizing the number of sample units observed or measured, and minimizing the errors in measuring each sample. It is not difficult to produce data. It is much more challenging to produce accurate data with known reliability that will be used to help make important decisions.

### 1.3 Defining the population

Scientifically defensible estimation of population attributes is based on a formal body of mathematical theory, which must be respected if it is to be used to defend the accuracy of sample-based estimates. The careful selection of a sampling frame, plot configuration and sampling design are crucial steps in the process and cannot be accomplished independently of each other. Each decision has an impact on the others. The mathematical theory begins with a precise definition of the population for which attributes will be estimated. For example, for a municipality of 5 million ha of which 1 million ha comprises forest, the statistical population could be described in several different but logical ways:

- Thousands of tree-stands and non-forest polygons
- Tens of millions of potential 0.1 ha sampling plots
- Ten million remotely sensed 30 m x 30 m pixels
- Billions of trees
- An infinite number of points.

¹ See chapter on *Observations and measurements*, Section 2, p. 42
There is no one best definition of a population for forest inventories. The key issue in basic applications of forest sampling is to define precisely the geographic boundaries of the targeted population, such as all lands, both forest and non-forest, within a nation that are outside the geopolitical boundaries of urban areas. It is not uncommon to discover that portions of a target population cannot be sampled. Examples include areas that are remote and inaccessible or unsafe to access. These areas should be identified precisely in a cartographic form, even though the true boundaries might not be obvious, and excluded from the sampled population. Scientifically defensible estimates must be limited to the sampled population only.

1.4 Choosing a sampling frame

Three terms can be distinguished: sampling frame, sampling design and plot configuration. Sampling frame refers to the set of all possible sample units; sampling design refers to the selection of a subset of sample units to represent the population; and plot configuration refers to the size, shape and components of the field plot.

Some advantages are gained with a sampling frame that considers a forest to be an infinite population of points. One approach to sampling with this type of sampling frame is to use the popular Bitterlich plot, which is efficient for estimating variables correlated with tree size. Alternative point-based plot configurations measure a support region and impute its attributes to a point. When near a boundary or stand edge, a point is more easily assigned to one side or the other, whereas plots with different designs can straddle edges or boundaries. A recommended approach is to consider the forest population to be an infinite set of points and to use physical measurements in a support region to describe conditions at a sample point.

1.5 Choosing a plot configuration

The plot configuration consists of the plot size and shape and determines the variables to be measured at each sample plot location. Choices for plot configurations include variable area plots, fixed-area plots, subdivisions of plots into subplots and cluster plots, all of which require size and shape considerations. Variable area plots using Bitterlich sampling are particularly effective for obtaining precise estimates of forest attributes relating to tree size. Fixed-area plots, while not necessarily optimal for any particular forest attribute, are an excellent compromise when sampling is intended to produce estimates of a wide variety of forest attributes, and tend to be more compatible with ancillary data. Cluster sampling reduces travel between plots while providing a sufficient number of plots. The optimal shape and size may be addressed using sampling simulation and prior information, although circular plots are often used in forest inventories.

Issues concerning the selection of a plot configuration are also discussed in the chapter on Observations and measurements, p. 41.

1.6 Measuring sample plots

The chapter on Observations and measurements summarizes the major considerations relevant to measuring sample plots. For more detailed information, see Schreuder et al. (2004). This section discusses two aspects of this issue: the use of remotely sensed data for measuring plots and temporary versus permanent plots.

First, remotely sensed data from medium-resolution satellites and high-altitude aerial photography (1:24 000 to 1:60 000 scales) provide cost-effective measurements for coarse indicators of forest conditions, mostly forest area changes. However, most measurements of detailed forest conditions are impossible with these sensors (see chapter on Remote sensing, p. 77). More detailed measurements of forest conditions may be obtained with low-altitude aerial photography.
and sensors such as Lidar. All these sensors are currently expensive and have narrow fields of view that are not currently capable of producing border-to-border coverage of an entire nation. However, in principle, these sensors could be used to measure a sample of locations in a national survey. For example, it may be cost efficient to measure a plot initially with data from a remote sensor to determine if the plot has accessible forest land cover or forest land use. If not, field crew visits to such locations may not be warranted.

Second, estimating changes and trends in a country’s forests is often an important part of an NFA. If the locations of sample plots are sufficiently documented, then the same plots can be remeasured over time to obtain more precise estimates of forest change, such as tree growth, mortality, harvesting, regeneration, and changes in the areas of forest conditions and land-use categories. Remeasurement of plots increases estimation efficiency and contributes to better understanding of the components of change. However, if permanent plots are used, their locations must be very accurately documented. To do this, researchers can drive a pin into the ground at the centre or corner of a plot, and carefully document how to find the pin from a convenient starting location, perhaps several kilometres distant. The pin should be hidden from normal view to keep the plot truly representative of thousands of hectares that will never be measured. A sample plot will not be representative if it receives special treatment, such as protection from harvesting or other disturbances. An obvious pin in the ground could also influence how others treat the location.

Although remeasurement of the same trees produces the most precise estimates of change, this approach is more costly because the same plot centres and trees must be relocated at the time of each measurement. Alternatives for estimating change from temporary plots include estimation of tree growth from increment borings, and gross estimation of forest area and volume change by comparing independent estimates obtained from measurements of different sets of temporary plots at different points in time. However, harvest, mortality and regeneration are difficult to estimate using data from temporary plots. Thus, where possible, it is recommended to use permanent plots or a combination of permanent and temporary plots (e.g. Ranneby et al., 1987).

2. Sampling design

There are two general sampling approaches: subjective or purposive sampling and probability sampling. Subjective sampling attempts to use professional judgement to select sample units believed to be representative of the entire population. These units are often convenient to measure, which reduces cost. Although data gathered in this way accurately describe the conditions on the sampled sites, they may not accurately characterize the entire population. Supporters of subjective sampling trust the ability of experts to select a representative sample and argue that this approach is good enough for practical purposes. In some simple situations, this may be true. However, some users of the data may lack the same confidence in the experts. Expensive data can become worthless because the sampling design is not sufficiently robust under scientific criticism. In addition, convenient sampling sites are often near roads, which are frequently associated with unique landforms, land uses, management histories and landscape patterns. Are such sites truly representative of the entire population? The answer is debatable. It is far easier to discredit the accuracy of population estimates from a subjective sample than prove otherwise.

Probability sampling replaces subjective judgements with objective rules based on known probabilities of selection for each member of a population. For example, if a 1 million ha forest comprises a population of 10 m x 10 m plots, there would be 100 million of those plots in the population. The selection of one of these plots at random amounts to
a probability of \(1/100\,000\,000\). Selection of a simple random sample of 1,000 plots to estimate conditions in the entire 1-million ha population would give each member of that population a probability of selection of approximately \(1,000/100\,000\,000 = 1/100\,000\), and each plot measured in the sample could be seen as representing 99,999 other unmeasured plots. The important lesson is that probability sampling is an objective method with precise rules and a mathematical foundation for estimating population attributes based on a sample. The probability that an expert will select any one potential sample plot is unknown, and the mathematics of subjective sampling cannot be applied in a scientifically defensible way. Thus, this chapter recommends probability rather than subjective sampling, and further recommends equal probability sampling in which possible sampling unit locations have equal probabilities of selection for the sample.

2.1 Selecting a probability sampling design

Many of the difficulties associated with selecting a sampling design arise from two factors: first, sampling units are distributed in space and observations of them may be spatially correlated; and second, different sampling designs have different costs. Spatial correlation among observations of variables of interest strongly influences selection of sampling designs. Ecological, climatic and soil factors, and forestry management practices, cause observations from plots that are near to each other to be, on average, more similar than observations from plots that are farther apart. The result is that, in a strict sense, construction of a completely optimal sampling design is an impossible task because the numerous NFA-measured and derived variables vary quite differently in space. Thus, because optimal sampling designs are different for different variables, optimization may necessitate a focus on minimizing the standard error of a single important variable, such as wood volume, or on a weighted function of the standard errors for a small number of variables. One partial solution is to minimize the effects of spatial correlation by establishing sampling locations as far apart as possible. This also accommodates the fact that sample plot observations that deviate more from each other bring more information to the sample. In forest sampling, this often suggests hexagonal sampling designs. The primary sampling costs are attributed to travelling to and from the sampling unit location and measuring the unit. These costs, in turn depend on the structure of landscape and forests, measurements to be taken, and topographic, economic and transportation conditions.

A common starting point in selecting a sampling design is knowledge of the acceptable upper bounds for standard errors of estimates and an upper bound for cost. Optimizing the sampling design, given the sampling frame and plot configuration, involves selecting a procedure for spatially distributing the sampling unit locations in such a way that standard errors are minimized, while not exceeding the total allowable costs. Sometimes this will not be possible, and compromises may be necessary.

2.2 Simple random sampling

Figure 1a shows a simple random sample with sample plots placed randomly within the sampled population. Although there are spatial clusters and voids in the plot distribution, this remains a valid probability sample. The geographic coordinates for each sample plot in a random sample may be selected with a random number generator with the allowable coordinates restricted to the sampled population. Otherwise, no consideration is given to safety, difficulty of measuring plots or travel to and from plot locations. This is the least risky equal-probability sampling design, but it is also usually the least efficient with respect to both cost and the precision of estimates,
partially because of spatial correlation among observations.

2.3 Systematic sampling

A systematic sample uses a fixed grid or array to assign plots in a regular pattern (Figure 1b). The advantage of systematic sampling is that it maximizes the average distance between plots and therefore minimizes spatial correlation among observations and increases statistical efficiency. In addition, a systematic sample, which is clearly seen to be representative in some sense, can be very convincing to decision-makers who lack experience with sampling. Systematic samples may be based on rectangular grids or hexagonal arrays. For example, a sample plot could be established at the intersections of a 2 km x 2 km grid. A random number is used to select the starting point and orientation for this grid, but no other random numbers are required. This sampling design is common in forestry. The greatest risk is that the orientation of the grid may, by chance, coincide with or be parallel to natural or man-made features, such as roads or gravel ridges resulting from melting glaciers. For very large geographic areas, orientation of gridlines along lines of longitude should be avoided. In higher latitudes the converging nature of these north-south gridlines may cause sample plot locations to be closer together in higher latitudes than in lower latitudes. Sampling designs based on hexagonal arrays alleviate this problem (White et al., 1992).

Systematic unaligned sampling designs combine features of both simple random and systematic sampling designs. With these designs, a sample plot is assigned to a randomly selected location within each grid or array cell (Figure 1c).

2.4 Cluster sampling

For practical reasons, such as increasing cost efficiency and reducing field crew travel,
sample plots may be organized into clusters, thus leading to systematic cluster sampling and stratified systematic cluster sampling. In systematic cluster sampling, the clusters are distributed throughout the population using grids or polygons such as hexagons.

Several questions are relevant when planning a cluster-based sampling design: (i) what is the spacing between clusters? (ii) what is the shape of the cluster? (iii) what is the number of plots per cluster? and (iv) what is the sample plot configuration? To answer these questions, preliminary information about the spatial distribution and correlation of the variables of interest is needed. Correlation as a function of distance between field plots, estimated using variograms, can be used to compare the efficiencies of different sampling designs.

2.5 Stratified sampling

Stratified sampling entails first dividing the population into non-overlapping subpopulations called strata that together comprise the entire population, and then drawing an independent sample from each stratum. If the sample in each stratum is a simple random sample, the whole procedure is described as stratified random sampling. Numerous reasons may be given as justification for stratified sampling (Cochran, 1977; Schreuder et al., 1993). First, stratification is used to increase the precision of population estimates. To understand the potential gain in precision that may be achieved with stratification, some notation and formulae are necessary. With simple random sampling (SRS), the estimate of the population mean is

$$\bar{y}_{SRS} = \frac{1}{n} \sum_{i=1}^{n} y_i,$$  

[1]

and the estimate of the variance of the mean is

$$var(\bar{y}_{SRS}) = \frac{s^2}{n} ,$$  

[2]

where \( n \) is the sample size, \( y_i \) is an observation, and

$$s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2 ,$$  

[3]

is the sample estimate of the population variance. Cochran (1977) provides basic formulae for stratified estimation. Ignoring finite population correction factors and estimation errors in stratum weights, an unbiased estimator of the population mean and variance are,

$$\bar{y}_{Str} = \sum_{h=1}^{L} W_h \bar{y}_h,$$  

[4]

and

$$var(\bar{y}_{Str}) = \sum_{h=1}^{L} W_h^2 s_h^2 / n_h ,$$  

[5]

where

$$\bar{y}_h = \frac{1}{n_h} \sum_{j=1}^{n_h} y_{hj},$$  

[6]

$$s_h^2 = \frac{1}{n_h - 1} \sum_{j=1}^{n_h} (y_{hj} - \bar{y}_h)^2 ,$$  

[7]

are the within stratum means and variances, respectively; \( h=1, 2, \ldots, L \) denote strata; \( j \) denotes observations within strata; \( n_h \) denotes the number of sample observations within the \( h^{th} \) stratum with \( n_1 + n_2 + \ldots + n_L = n \); and \( W_h \) is the stratum weight representing the proportion of the population in the \( h^{th} \) stratum. The effects of stratification and stratified estimation on precision are often assessed using relative efficiency, \( RE \), defined as,

$$RE = \frac{var(\bar{y}_{SRS})}{var(\bar{y}_{Str})} ,$$  

[8]

where \( RE > 1 \) indicates a beneficial effect. Relative efficiency may be interpreted as the increase in the overall sample size necessary to achieve the same precision using estimation based on simple random sampling, as is achieved through using stratification and stratified estimation. From a quantitative perspective, precision gains are realized when
variances of estimated stratum means are substantially less than the variance of the overall estimated mean (i.e. \( \frac{s^2}{n_h} < \frac{s^2}{n} \)) and/or when strata with large \( \frac{s^2}{n_h} \) represent small proportions of the population (i.e. when \( W_h \) is small). From a qualitative perspective, precision gains are realized when heterogeneous populations are divided into more homogenous subpopulations. This typically means that the strata have substantially different means, variances or both.

A second reason for stratification is that it may contribute to avoiding bias, depending on the estimator selected. For example, NFA field crews generally are granted access to plot locations on publicly owned lands. However, if the permission of private landowners is required to measure sample plots on their lands, inevitably some will deny access. In extreme cases, the ratio of privately owned to publicly owned plots in the accessible portion of the sample may be considerably less than the ratio of privately owned to publicly owned forest lands in the population. If the species compositions and/or management practices are substantially different on privately owned and publicly owned forest lands, bias may occur. One solution is to stratify lands by ownership, thus leading to independent sample estimates for the two ownership strata (McRoberts, 2003).

A third reason for stratification is to accommodate different sampling protocols or different estimation procedures for different subpopulations. For example, a substantial portion of sampling costs may be attributed to travel to and from plot locations. If data from remote sensors may be used to determine that some plots are located on non-forest land, then travel costs may be substantially reduced by not sending field crews to these plot locations. As a result of the different measurement technique, however, a different estimator may be required for these strata.

The greatest benefits of stratified estimation are realized when the population is stratified and stratum sample sizes are determined before sampling is conducted. The process of determining stratum sample sizes or, equivalently, allocating samples to strata, may be accomplished in several different ways and for several different purposes. Frequently, samples are allocated to strata in proportion to some attribute of the strata. An easily implemented approach is to allocate sample plots to strata in proportion to strata sizes. If simple random or systematic sampling is used within strata, then this approach leads to equal probability samples within strata, which may simplify estimation. However, with this approach, the variances of stratum means may differ greatly. If comparably precise estimates of stratum means are desired, then samples may be allocated to strata in proportion to stratum variances. A potential disadvantage of this approach is that good estimates of stratum variances are necessary before samples are allocated to strata. Finally, it may be that estimates of means for some strata are more important than others. In this case, samples may be allocated to strata in proportion to a subjective assessment of strata importance.

Often the sampling objectives prohibit stratified random sampling. For example, a systematic sampling design may be used as a means of optimizing the precision of estimates for multiple variables simultaneously. Even though the greatest benefits of stratification may not be realized for any particular variable, the beneficial effects of increasing precision and precluding estimation bias may still warrant post-sampling stratification and stratified estimation. Thus, even if stratified sampling is not used, consideration of post-sampling stratified estimation is recommended because large increases in precision may often be realized with little additional cost or effort.

Almost any source of data can be used to create strata as long as two tasks can be accomplished in a consistent manner. First, stratum weights, calculated as the proportion of the population represented by each stratum,
Since forest inventory estimates are frequently either means or totals for either area or volume, the relevant derived variables in forest inventory are often of the form

\[ M = \frac{X}{Y} \tag{9} \]

where \( X \) and \( Y \) are expectations of random variables, \( x \) and \( y \). As an example, consider estimation of mean forest area per land-use stratum for sample plots that may intersect multiple strata, all within the category of forest land. One method for accommodating this phenomenon that is particularly useful with point sampling is to use the information from the centre point only. Let \( x_i = 1 \) when the centre point of the plot belongs to the stratum in question and \( x_i = 0 \) otherwise, and let \( y_i = 1 \) when the centre point is on forest land and \( y_i = 0 \) otherwise. Then the ratio estimator for mean area is

\[ m = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} y_i} = \bar{x} / \bar{y} \tag{10} \]

where \( n \) is the number of sampling units. Let \( E(\cdot) \) denote statistical expectation; then,

\[ E(m) = \frac{\bar{X}}{\bar{Y}} = \frac{\bar{X}}{\bar{Y}} = M \tag{11} \]

means that \( m \) is approximately unbiased when \( n \) is large.

The estimation of standard errors is complicated by spatial correlation that may arise from trend-like changes in variables and either systematic or cluster sampling. Matérn (1947; 1960) suggested the error variance, \( E(m-M)^2 \), as a measure of the reliability of the estimator and also proposed a variance estimator. Let \( i \) denote field plots; let \( r \) denote clusters of field plots; and consider the cluster residuals \( z_r = y_r - m \), where \( x_r = \sum_{i \in r} x_i \) and \( y_r = \sum_{i \in r} y_i \).
Assume that the residuals form a realization of a second order stationary (weakly stationary) stochastic process. The variance of the process can be estimated by means of quadratic forms

\[ T = \sum_r \sum_s c_{rs} z_r z_s \]

where \( c_{rs} = c_{sr} \), \( \sum_r \sum_s c_{rs} = 0 \) and

\[ \sum_r c_{rr} = 1 \]

where \( r \) and \( s \) both refer to clusters of field plots. Estimators of this form are unbiased if the process \( z \) is spatially uncorrelated and conservative if the process is positively correlated (Matérn, 1960). This approach has been used in the Swedish and Finnish inventories (cf. Ranneby, 1981; see also Tomppo et al., 1997, and Heikkinen, 2006) and is applied by sampling strata as follows. Within each stratum, the group \( g \) of four field plot clusters \( (r_1, r_2, r_3, r_4) \) is composed in such a way that each cluster belongs to four different groups (Figure 2).

![Figure 2](image.png)

Groups of clusters and clusters of sample plots.

The deviance of the cluster mean, \( \bar{y}_r \), from the stratum mean \( \bar{y} \) is computed for each cluster \( r \). Denote \( zr = (\bar{y}_r - \bar{y})n_r \), where \( n \) is the number of relevant sample points in cluster \( r \) (for this example, \( n_r = 4 \)). The weights \( c_{r1} = c_{r4} = c_{r2} = c_{r3} = \frac{1}{2} \) are often used. The quadratic forms can then be expressed as \( T_g = (z_{r1} - z_{r2} - z_{r3} - z_{r4})^2/4 \) and the resulting standard error estimators for each stratum are

\[ s = \sqrt{\frac{k \sum_g T_g}{\sum_i y_i^2}} \]

where \( g \) denotes a group of clusters in the stratum, \( i \) denotes plots in the stratum, and \( k \) is the number of clusters in each cluster group (for this example, \( k = 1 \)). The standard error estimators for the entire study area can be obtained by combining the stratum-specific estimators with the usual formula for stratified sampling (eqs. [4] and [5]). This procedure is relevant for strata having large numbers of field plots, preferably at least several hundred.

### 3. Sample size

Determination of sample size is one of the most important steps in constructing a sampling design. If the sample is too small, then uncertainty will be great; if the sample is too large, then the cost will be unnecessarily high. It is possible to quantify the expected confidence in future estimates made from a valid probability sample. As the number of sample plots increases, the variance of the estimation error decreases, the precision of the estimate increases, and more confidence can be placed in the estimate. Usually, the exact value of the estimate is known but not the true condition of the forest. With probability samples, the probability that an estimate is within a specified distance from the true value may be determined. These are the roles of the “confidence interval”, an estimated range of proportions likely to include the true, but unknown, proportion of forest, and the “confidence coefficient”, the probability that similar confidence intervals constructed using different samples will contain the true proportion of forest.

The simplest case is that of estimating proportions with a simple random sample,
such as estimating the proportion of a nation that is forested. For example, an NFA covers a sampled population of 5 million ha, and in a simple random sample with \( n = 1,000 \) plots, 400 are forested. The estimated proportion of forest is 40 percent. What level of confidence can be placed in this estimate? If a confidence coefficient of 80 percent is acceptable, for 80 sample plots the true but unknown percentage of forest is within the confidence interval. From available tables and figures (Czaplewski, 2003), with \( n = 1,000 \), and an estimate of 40 percent forest, the confidence interval is 38.0 to 42.0 percent. As another example, suppose that a rare forest type exists in the population, but the exact amount is not known. However, none of this rare forest was observed in the simple random sample of \( n = 1,000 \) plots, and the estimated percentage of the nation in this rare forest condition is 0. For the same 80 percent confidence coefficient, the confidence interval for this estimate is 0.0 to 0.2 percent. Thus, the estimate of the area of this rare forest type in the entire 5-million ha nation is 0 ha to 10,000 ha. The final example is a 100,000 ha municipality for which measurement of a sample of \( n = 20 \) of the 1,000 plots reveals that 18 are forested. The estimate for this municipality is 90 percent forest cover with a confidence interval of 75.5 percent to 97.3 percent, or 75,500 to 97,300 ha. Other calculations of sample sizes are possible with interactive “sample size calculators”, available online. These examples demonstrate that accurate national estimates for common types of forest cover are possible with relatively few sample plots. However, larger sample sizes are often needed if the NFA requires estimates of rare forest types or small portions of the nation. It is the sample size that determines the precision of estimates in an NFA, not the size of the entire sampled population.

Determining the required sample size requires an estimate of the standard deviation of the differences between individual plot-level values and their average value. This standard deviation may be estimated with a pilot study or inventory that measures a small sample of forest plots to determine the variability among them. For example, assume the pilot inventory includes 60 plots, and wood volume is measured on each plot. Further, suppose that the mean volume is \( = 100 \text{ m}^3/\text{ha} \), the variance among plots is \( = 2 \text{ $500 \text{ m}^3/\text{ha}^2} \), and the standard deviation is \( = 50 \text{ m}^3/\text{ha} \). If observations from the pilot plots are normally distributed, about 1/6th of the plots will have \( 100 - 50 = 50 \text{ m}^3/\text{ha} \) or less, and another 1/6th of the 60 plots will have \( 100 + 50 = 150 \text{ m}^3/\text{ha} \) or more. Assume the precision requirement for the NFA is to estimate mean wood volume per hectare to within a ±5 percent “tolerance” or “maximum allowable difference” (\( D_{\text{max}} = 0.05 \)) with a 66 percent confidence coefficient. The required sample size \( n \) is approximately 100 sample plots.

\[
n = \left( \frac{\hat{\sigma}}{D_{\text{max}}} \right)^2 \cdot \left( \frac{50}{100 \times 0.05} \right)^2 = 100,
\]

[15]

If this NFA precision requirement is for the entire nation, then 100 sample plots are sufficient. If this NFA accuracy precision is for each of 10 subnational units, then a total of 1,000 sample plots is necessary. Sample sizes increase greatly as the acceptable tolerance becomes smaller. A tolerance of ±1 percent would require the sample size to increase from \( n = 100 \) to \( n = 2,500 \) sample plots (eq. 15) in this example. The required sample size increases for larger confidence coefficients. For example, it requires four times more sample plots to improve precision from a 66 percent confidence coefficient to the 95 percent level. More exact and detailed calculations of required sample sizes are possible with the aforementioned interactive sample size calculators.

4. Comparing sampling designs

An effective way to compare sampling designs is via simulation if a forest area model is available. The model may be obtained from a previous inventory or from satellite image-based estimation of variables of interest.
An example of the standard errors obtained from sampling designs for estimating mean growing stock volume is shown in Figure 3. The test site is in north Finland and has a land area of 6.47 million ha, a forest land area of 4.19 million ha, and a mean volume on forest land of 52.7 m³/ha.

A pixel level, border-to-border forest map has been produced using field data from the preceding inventory, satellite images and digital map data (Tomppo, 2004; Tomppo and Halme, 2004). Satellite images of different resolution provide one information source, in addition to existing maps. A pilot inventory may also be used to collect information for planning the final sampling design. Representative subareas can be selected from the population where pilot inventories may be conducted. However, these pilot inventories must be acknowledged and accepted as less than optimal. In addition, new sampling designs can be created using information from previous inventories, as has been the case in countries where forest inventories have been conducted since the 1920s and 1930s (e.g. Ilvessalo, 1927).

Figure 3. Mean volume RMSE, %

5. Sampling considerations for tropical forest inventories

In recent years, concern for the effects of climate change and actions to mitigate those effects have motivated intense interest in forest inventories in tropical countries for purposes of estimating carbon and carbon change. Such inventories, often characterized as Measurement, Reporting and Verification (MRV) systems when targeted to REDD purposes, are similar to national forest inventories (NFIs), although the MRV emphasis may be restricted to biomass-related variables, and the MRV population of interest may be restricted to lands that are subject to human-induced greenhouse gas emissions. However, because of the similarities between MRVs and NFIs, tropical developing countries often design their NFIs so that they can also serve as MRVs, or design their MRVs in such a manner that they can easily be extended to a complete NFI. Thus, the guidance articulated below pertains equally to MRVs as to NFIs.

By definition, a monitoring programme includes emphasis on change and trends. In addition, in recent years NFIs have come to place increased emphasis on change and trends. Therefore, selection of plot configurations, sampling designs and perhaps stratification schemes are driven at least partially by the desire to estimate change.

5.1 Plot configuration

Selection of a plot configuration is based on multiple general principles, many of which are the same for boreal, temporal and tropical inventories, although some are different. Precise estimation of change is known to be more difficult than precise estimation of current conditions, particularly when the change is only for a small area. However, remeasuring the same plots on successive occasions can increase the precision of change estimates. In addition, the land area
of interest could be stratified for variance reduction purposes using a variable relating to the likelihood of change. Thus, the emphasis on estimation of change in tropical inventories argues in favour of a relatively large proportion of permanent plots which, in turn, argues in favour of marking or determining the locations of trees, so that they can be relocated for successive inventories. Although establishment and measurement of a temporary plot is less expensive than establishment and measurement of a permanent plot, establishment and measurement of different temporary plots on two occasions is not necessarily less expensive than establishment, measurement and re-measurement of a single permanent plot.

Although no strong consensus exists regarding plot shape, circular plots are often preferred because they require only single control points, the plot centres. Rectangular plots require four control points, one at each corner. In addition, for a given plot area, a circular plot has a smaller perimeter meaning that fewer decisions will be necessary as to whether particular trees are or are not on the plot. Also, determining coordinates for individual trees, which may be necessary for assessing their change, may be easier for circular plots which have only a single control point, than for rectangular plots which have four control points. However, if tree densities are exceptionally large, then long, narrow, rectangular plots may be a more feasible alternative.

For purposes of logistical efficiency, monitoring and inventory programmes typically configure plots in clusters. Because of expected access problems, configuring plots in clusters may be even more crucial for tropical programmes. Thus, individual plot size and the number of plots within clusters are subject to multiple important considerations, all of which are generally related to logistical, cost and precision considerations (Scott, 1993; Tomppo et al., 2010a; 2011). First, plots should be small enough and few enough within clusters to allow a field crew to measure the entire cluster in a single day. The greatest proportion of the cost of measuring a plot in boreal and temporal forests is travel to and from the plot location; this proportion is likely to be even greater for tropical forests for which many regions are remote and nearly inaccessible. Thus, greater efficiency is achieved if field crews are not required to return to the same plot location on multiple days. Second, plot features such as radiiuses for circular plots or lengths for rectangular plots must be measured on a horizontal plane, not along irregular terrain. Because measurement on a horizontal plane is more difficult for larger plots, particularly in hilly and mountainous terrain, smaller plots are again preferable. Third, establishment of permanent rather than temporary plots to facilitate estimation of change usually requires either marking or determining coordinates for individual trees. The latter approach is more difficult for large plots in dense tropical forests because more trees will be located between the tree of interest and control points. An argument in favour of larger plots for tropical inventories is that tropical forests are typically more diverse than boreal and temperate forests, implying that the total area inventoried at each sampling location should be greater to capture the greater diversity. However, this greater size could be achieved by increasing the number of small plots in the same plot cluster. This approach is cost efficient when the spatial correlations among observations of the variables of interest are large but decrease with increasing distance.

Greater sampling efficiency is also achieved by using small subplots for measurement of smaller diameter trees. For circular plots, the subplots are usually nested (i.e. they take the form of concentric circles all with the same centre). The particular sizes of the subplots and the diameter thresholds corresponding to the subplots should be based on the expected number of trees to be found on the subplots, the expected similarities of trees, and the travel time between subplots of the same plot or plots in the same cluster.
Finally, the remote and mostly inaccessible nature of many tropical forests means that inventories may have to rely on a combination of plot and remotely sensed data. Thus, remote sensing considerations may be necessary when selecting a plot configuration. As an example, a plot should be large enough to constitute an adequate sample of the trees on the ground element corresponding to the remotely sensed element (e.g. satellite image pixel, Lidar footprint) that contains the ground element centre. In addition, a desire to align different plots in the same cluster with different remotely sensed elements may require distances between plots to be at least as great as the dimensions of the remotely sensed elements.

5.2 Sampling design

Selection of a sampling design for a tropical forest inventory entails consideration of multiple principles. First, spatial balance is generally a preferred feature of sampling designs, which means that large geographic regions of the population do not remain unsampled. Spatial balance is often achieved by incorporating a systematic component into the sampling design. This may take the form of a network of perpendicular grid lines or a tessellation of the population into regular polygons. Spatially aligned designs establish plots at grid intersections or centres of polygons, whereas spatially unaligned designs establish plots at random locations within the rectangles formed by the grid lines or the regular polygons.

Remote sensing considerations may also be appropriate when selecting a sampling design. For example, tropical forests are often characterized as having relatively few days without cloud cover. Thus, cloud-free imagery for satellite-based sensors, such as Landsat or SPOT, may be difficult to obtain. Lidar data, which are currently acquired from airborne platforms and use laser techniques, are often proposed as an alternative. In addition, laser pulses penetrate forest canopies and produce useful information for estimating volume, biomass and the carbon content of trees. If plots are located at the intersections of perpendicular grids, acquisition of Lidar data from airborne platforms in strips is facilitated because straight flight lines can be used.

Finally, when constructing grid networks and tessellations, consideration should be given to use of equal area projections. If not, then plots located at greater distances from the equator will represent less population area than plots located closer to the equator. Although weighting schemes can be used with unequal area projections, they are often complex and bothersome.

As previously noted, the greatest proportion of the cost of measuring a plot is travel to and from the plot location. This proportion may be very large in tropical forests with remote and inaccessible regions (Tomppo et al., 2011). Thus, cost efficiency dictates that plots be established in clusters rather than singly. Multiple approaches to cluster sampling are popular. One approach is to configure a plot as multiple subplots in a regular pattern and in close proximity to each other (McRoberts et al., 2005). With this approach, the data for all subplots may be aggregated and attributed to the plot centre. A second approach is to establish plots in clusters configured as rectangles, half-rectangles or other geographic shapes (Tomppo, 2006). A third approach is two-stage cluster sampling. With this approach, primary sampling units such as polygons in the form of large rectangles are first randomly selected, and then multiple secondary sampling units in the form of plots are established within the selected polygons at randomly selected locations. When using cluster sampling, consideration should be given to the spatial correlation among observations for plots within the same cluster. If distances between pairs of plots are less than the range of spatial correlation, observations will tend to be similar and the sampling will tend to be less efficient.
5.3 Stratification

Stratified approaches to sampling are used for multiple reasons, but primarily to vary sampling intensities to accommodate selected criteria. For example, for an MRV that emphasizes geographic regions subject to human-induced carbon emissions, lesser sampling intensities may be acceptable for remote, inaccessible regions less likely to be developed or harvested. In addition, the cost associated with greater sampling intensities in remote regions may be prohibitive. Nevertheless, sampling, albeit with lesser intensities, must be conducted in these regions to achieve spatial balance.

Multiple principles also guide stratified approaches to sampling. First, strata with stable boundaries are generally preferable. Otherwise, changes to boundaries of strata with different sampling intensities lead to different sampling inclusion probabilities and complicate estimation. In addition, stratified estimation requires that a plot be assigned to one and only one stratum. If the stratum to which a plot is assigned changes between measurements, then difficulties arise as to the stratum to which a plot change observation should be assigned. Thus, strata defined by topography, climatic zones, biomes or political boundaries may be preferable to strata defined by forest attributes such as density or perhaps forest type.

Stratified sampling is most often implemented using one of three plot allocation schemes. With equal allocation, the same number of plots is allocated to all strata, regardless of strata sizes. This scheme is preferred if the objective is estimates for individual strata. With optimal allocation, sampling intensities selected for strata are based on optimization criteria, such as measurement costs, and/or within-stratum variation of observations of variables of interest, such as volume or biomass or their likely changes. Greater sampling intensities are selected for strata with greater variation and/or lesser measurement costs. With proportional allocation, sampling intensities selected for strata are proportional to strata sizes. Cochran (1977) provides a comprehensive discussion regarding alternative strategies. For tropical countries with large, remote and nearly inaccessible regions, some form of optimal allocation will usually be necessary to mitigate the excessive costs associated with sampling these regions. Proportional and optimal allocation can be easily implemented using sampling designs based on networks of perpendicular grid lines. With proportional allocation, plots or plot clusters are established at grid intersections without regard to the stratum associated with the grid intersection. With optimal allocation, sampling intensities can be increased or decreased for different strata by selection of grid intersections at which plots are established. For example, if the sampling intensity is to be reduced by a factor of four, plots can be established at the intersections of every second grid line only in each direction.

5.4 Case study – Tanzania

For a sampling design for Tanzania, Tomppo et al. (2010a) used double sampling for stratification and optimal allocation of plots to strata. The first-phase sample consisted of an office assessment of a dense grid of field plots for assignment to volume and cost classes. Based on these assessments, strata were constructed using predicted cluster-level average volume of growing stock and estimated cost to measure a plot cluster. Volume classes were based on volume predictions using satellite imagery, observations for ground plots outside Tanzania, and robust models whose predictions were calibrated using areal volume estimates for Tanzania. Neyman allocation (Cochran, 1977) was used to select boundaries for the volume classes, so as to maximize the precision of the overall volume estimate assuming a fixed sample size. Cost classes were based on GIS analyses and local expert opinion of the number of days (one, two, more than two) necessary to measure a plot cluster.
Selection of the class intervals, which affects the gain that can be achieved with stratification, requires greater investigation. The second-phase sample consists of field measurement of plots where within-strata sampling intensities were selected using optimal allocation (Cochran, 1977). With optimal allocation, sampling intensities are proportional to the quantity \( \frac{\sigma_h}{\sqrt{c_h}} \) where \( \sigma_h \) is the within-stratum standard deviation for observations of the variable of interest (mean growing stock volume) and \( c_h \) is the average cost in terms of measurement time for a plot cluster in stratum \( h \). More details concerning the sampling design can be found in Tomppo et al. (2010a).

In the tropics, use of available vegetation maps to delineate land into forest and non-forest is sometimes appealing. However, if plot clusters are not established on delineated non-forest land in the same manner as on delineated forest land, map errors could contribute to bias, as forest land erroneously delineated as non-forest land will not be sampled. However, allocating lesser sampling intensities to these lands can decrease the costs associated with sampling-delineated non-forest land. In addition, field measurement of plot clusters entirely outside forest and without growing stock can often be avoided by assessing such clusters with land-use information obtained from other reliable sources, such as those proposed for Brazil (Tomppo, 2009).

The lack of transportation routes, other than rivers, presents a special challenge for tropical forest inventories, such as in the Amazonian Biome. For example, roads may be available only a part of the year (approximately six months in the case of the Amazonian Biome). In addition, some forests may be designated for nature conservation purposes or for the sole use of indigenous peoples. Stratification based on relevant variables such as the likelihood of changes and measurement costs promote both cost efficiency and adherence to sound statistical inventory principles.

6. Summary

Construction of an appropriate sampling design for an NFA, NFI or MRV is a crucial step if estimates are to be sufficiently precise and scientifically defensible. One of the first steps in this process is to define the target population and select a sampling frame. The recommended option is an infinite population sampling frame in which observations and measurements of a field plot support area are attributed to the point at the field plot centre. Because inventories are often expected to produce estimates of change, it is recommended that the sampling design include at least some permanent plots. The next step is to distribute the field plots throughout the population to be sampled. This chapter has presented information on and discussed several popular sampling designs: simple random sampling, stratified sampling, systematic sampling and cluster sampling. If the sampling design includes a systematic component, caution is recommended when using rectangular grids for target populations with large north-south components. Although the selection of the particular sampling design depends on a variety of considerations, if stratified sampling is not used, consideration should be given to post-sampling stratification and stratified estimation. Finally, additional information on these and more complex sampling design issues is available in the reference material.

Self-study exercises

1. Describe the differences, advantages and disadvantages of simple random, systematic, stratified and cluster sampling designs.
2. Explain why a stratified sampling design may be superior to a simple random or systematic sampling design. Describe ancillary data that may be useful for constructing strata.
3. What role does spatial correlation among observations or measurements of forest attributes play in the selection
of a sampling design and estimation of population variances?
4. Describe the criteria and information necessary to determine an appropriate sample size.
5. Identify sampling issues and constraints unique to inventories in tropical forests.

References and technical resources


