Policy Impacts on Inequality

Decomposition of Income Inequality by Income Sources

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by

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1. SUMMARY

This tool illustrates how to decompose inequality measures by income sources. In particular, it discusses this decomposition in the context of the Gini and the Theil Indexes. A step-by-step procedure and numerical examples give operational content to the tool.

2. INTRODUCTION

Objectives

The objective of the tool is to provide the analytical and the practical framework to understand where inequality comes from. To this purpose, decomposition of the most suitable indexes will be provided.

For empirical applications, the knowledge of overall inequality may be insufficient to properly target public policies. Actual policies may have very differentiated impact on different income types (e.g. earned and unearned incomes). It is therefore essential to split overall inequality among different income sources.

Decomposing inequality indexes by income sources means exploring the **structure of inequality**, i.e. the disaggregation of total inequality in relevant factors.

Target audience

This module targets current or future policy analysts who want to increase their capacities in analysing impacts of development policies on inequality by means of income distribution analysis. On these grounds, economists and practitioners working in public administrations, in NGOs, professional organisations or consulting firms will find this helpful reference material.

Required background

Users should be familiar with basic notions of mathematics and statistics.

Links to relevant EASYPol modules, further readings and references are included both in the footnotes and in <u>section 7.2</u> of this module[1].

3. CONCEPTUAL BACKGROUND

Decomposing inequality by income sources is particularly useful to understand whether total inequality is concentrated in specific income items. Typical questions are: how

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[1] EASYPol hyperlinks are shown in blue, as follows:
  a) training paths are shown in **underlined bold font**;
  b) other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
  c) links to the glossary are in **bold**; and
  d) external links are in *italics*.

much of total inequality is explained by earnings inequality? And, for instance, how much by inequality in agricultural incomes?

This section will give tools to disentangle this issue. For the same reasons as above, attention will be confined to the most used indexes for decomposition. Again, formulas and examples will be provided for the Gini Index and for the Theil Index. Both have useful features for decomposing inequality by income sources.

3.1   The Gini Index

A useful decomposition of the Gini Index by income sources has been proposed by Lerman and Yitzhaki (1985). It is based on the covariance formula of the Gini Index[2]. Assuming an income distribution and *m* different income sources (e.g. earned income, agricultural income, etc.), we can write:

$$[1] \qquad G = \frac{2}{\bar{y}} Cov(y, F(y)) = \sum_{k=1}^{m} \frac{2}{\bar{y}} Cov(y_k, F(y))$$

Note that the Gini Index of the distribution of total income is equal to the sum (over *m* income sources) of the Gini coefficients calculated by using the covariance between the kth income source and the cumulative distribution function of **total income**. This is a very simple way of decomposing inequality by income sources. Furthermore, the Gini Index is in this case perfectly decomposable[3]

3.2   The Theil Index

In the case of the Theil index, the logic of the decomposition by income source is the same as in the case of the decomposition by population subgroups. In particular, for *m* income sources, the following formula applies:

$$[2] \qquad T = \sum_{k=1}^{m} \underbrace{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i^k}{\bar{y}} \right) \ln\left(\frac{y_i}{\bar{y}} \right)}_{\text{Contribution of the income source k}}$$

The basic difference with the Theil Index calculated on the whole income distribution is that under the contribution of the income source k, y_i^k now appears instead of y_i. Summing all contributions to inequality by *m* income sources gives the total Theil index.

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[2] See EASYPol Module 040: *Inequality Analysis: The Gini Index*

[3] Compare EASYPol Module 052: *Policy Impacts on Inequality: Decomposition of Inequality by Population Subgroups.*

4. A STEP-BY-STEP PROCEDURE TO DECOMPOSE THE GINI INDEX BY INCOME SOURCE

4.1 A step-by-step procedure to decompose the Gini Index by income source

Figure 1 illustrates the very simple procedure to decompose the Gini Index by income source. Step 1 asks us to identify income sources (e.g. earned and/or agricultural income, etc.) and to sort the income distribution by total income. Note that sorting must be done by total income and not by income sources.

Once sorted, Step 2 asks us to calculate the Gini Index of the distribution of total income[4].

In order to decompose the Gini Index, Step 3 asks us to consider each income source. For example, take the first income source (e.g. earned income) and calculate the covariance between this income source and the distribution function of total income. This step is quite important, as the relevant cumulative distribution function is that of total income (and not that of the specific income source), while the relevant income distribution is that of the specific income source.

Once this covariance has been calculated, we must calculate the Gini Index of the specific income source according to formula [1] (Step 4). Steps 3 and 4 must be replicated for each income source.

Step 5 we have to only check for the exactness of the decomposition by comparing the Gini Index of the total income distribution with the sum of the partial Gini Indexes for each income source.

Figure 1: A step-by-step procedure to decompose Gini Index by income source

STEP	Operational content
1	Identify total income and income sources. Sort the income distribution by total income
2	Calculate the Gini Index of total income
3	Consider each income source. Calculate the covariance between the distribution of each income source and the distribution function of total income
4	Calculate the Gini Index of each income source according to the formula in the text
5	Check for the decomposition

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[4] See EASYPol Module 040 : *Inequality Analysis: The Gini Index*

4.2 A step-by-step procedure to decompose the Theil Index by income source

Figure 2 illustrates the step-by-step procedure to decompose the Theil Index by income source. Step 1, as usual, asks us to identify total income and the various income sources. The income distribution must then be sorted by total income.

Figure 2: A step-by-step procedure to decompose Theil Index by income source

STEP	Operational content
1	Identify total income and income sources. Sort the income distribution by total income
2	Calculate the Theil Index of total income
3	Consider each income source. Calculate the Theil Index of each income source according to the formula in the text
4	Check for the decomposition

Step 2 asks us to calculate the Theil Index of total income, according to the step-by-step procedure[5].

Step 3 must be repeated as many times as income source are identified. For each income source, one must calculate the Theil Index according to formula [2] above.

Step 4, finally, requires to check for decomposition by comparing the Theil Index calculated on total income and the sum of the partial Theil Indexes calculated on each income source.

5. A NUMERICAL EXAMPLE OF HOW TO DECOMPOSE INEQUALITY INDEXES BY INCOME SOURCE

5.1 A numerical example of how to decompose the Gini Index

Table 1 shows an example of how this decomposition works and which variables are needed in order to perform it. We can now start from an income distribution where total income derives from two sources: earned income and agricultural income.

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[5] Discussed in EASYPol Module 051: *Policy Impacts on Inequality The Theil Index and the Other Entropy Class Inequality Indexes.*

Table 1: Decomposing the Gini Index by income source

	STEP 1 and 2				STEP 3 and 4 Isolate the distribution of the first income source. Calculate the covariance between this distribution and the fractional rank of total income. Calculate the Gini of the income source according to formula		STEP 3 and 4 Isolate the distribution of the second income source. Calculate the covariance between this distribution and the fractional rank of total income. Calculate the Gini of the income source according to formula		STEP 5 Check the decomposition	
	Identify income sources and sort the income distribution by total income. Calculate the Gini index of the distribution of total income									
Individual	Cumulative distribution function	Total income	Earned income	Agricultural income	Earned income	Cumulative distribution function	Agricultural income	Cumulative distribution function		
1	0.200	1,000	400	600	400	0.200	600	0.200	Total Gini	0.267
2	0.400	2,000	1,700	300	1,700	0.400	300	0.400		
3	0.600	3,000	2,500	500	2,500	0.600	500	0.600	Gini (earned)	0.213
4	0.800	4,000	2,500	1,500	2,500	0.800	1,500	0.800	Gini(agricultura	0.053
5	1.000	5,000	4,000	1,000	4,000	1.000	1,000	1.000	Total Gini	0.267
Totals		15,000			Covariance		Covariance			
Means		3,000			(earned, total)	320.0	(agricultural, total)	80.0		
Covariance (total,total)	400.0									
Total Gini	0.267				Gini (earned)	0.213	Gini (agricultural)	0.053		

Step 1 and 2 identify total income and the two income sources. The cumulative distribution function of **total income** is also calculated. The Gini Index of total income is **0.267**.

Steps 3 and 4 are repeated twice in the example, one for each income source. Take the first case, earned income. In this case, the distribution of earned income (not sorted) is put in relation with the cumulative distribution function of total income. This is in line with expression [1]. The calculated covariance (earned, total) is 330.0 and the application of formula [1] gives a Gini of earned income equal to 0.213.

The same steps must be repeated for agricultural incomes. It yields a covariance of 80.0 and a Gini Index of 0.053.

Now, check for this decomposition in Step 5. The total Gini index is 0.267 and the sum of the two partial Gini Indexes is again 0.267. Given these results, we could say that 80 per cent of total inequality (0.213/0.267) is explained by earned incomes, while the rest (20 per cent) is explained by the distribution of agricultural income.

5.2. A numerical example of how to decompose the Theil Index

Table 2 shows how the decomposition works for the Theil Index. Step 1 illustrates the distribution of total income and the distribution of two income sources, earned and agricultural, as in the case of the Gini Index.

Table 2 – Decomposing Theil Index by income source

	STEP 1			STEP 2			STEP 3		STEP 3		STEP 4	
	Identify income sources and sort the income distribution by total income.			Calculate the Theil Index of total income (see analytical tool 3)			Calculate the Theil Index of each income source		Calculate the Theil Index of each income source		Check for decomposition	
Individuals	Total income	Earned income	Agricultural income	**TOTAL income**			**EARNED income**		**AGRICULTURAL income**			
				A: y_i/y	**B**: $\ln(y_i/y)$	**C=AxB**	**D**: $y_i(k)/y$	**E=DxB**	**F**: $y_i(k)/y$	**G=FxB**		
1	1,000	400	600	0.333	-1.099	-0.366	0.133	-0.146	0.200	-0.220	Theil (total)	0.120
2	2,000	1,700	300	0.667	-0.405	-0.270	0.567	-0.230	0.100	-0.041		
3	3,000	2,500	500	1.000	0.000	0.000	0.833	0.000	0.167	0.000	Theil (earned)	0.109
4	4,000	2,500	1,500	1.333	0.288	0.384	0.833	0.240	0.500	0.144	Theil (agricultural)	0.011
5	5,000	4,000	1,000	1.667	0.511	0.851	1.333	0.681	0.333	0.170	Theil (total)	0.120
Total income	15,000			Theil (total)		0.120	Theil (earned)	0.109	Theil (agricultural)	0.011		
Mean income	3,000											

Step 2 illustrates the calculation of the Theil Index for total income. The Theil Index is equal to 0.120.

Step 3 is repeated twice, one for each income source. Columns illustrates the parameter needed to calculate the Theil Index according to formula [2] in the text. The Theil Index of earned income is 0.109, while the Theil Index of agricultural income is 0.011.

The sum of the two partial Theil Indexes gives the total Theil Index (0.120), as reported under Step 4.

Again, this decomposition tells us that about 91 per cent (0.109/0.120) of total inequality as measured by the Theil Index is caused by inequality in earned incomes.

6.   A COMPREHENSIVE STEP-BY-STEP PROCEDURE TO USE THE DECOMPOSITION OF INEQUALITY

At this stage, it is useful to describe the procedure to use inequality decomposition by using a flow chart diagram as in Figure 3. This procedure embodies the case of inequality decomposition by population subgroups[6]. It is worth considering the two types of decomposition together in order to appreciate the differences in the route followed to decompose inequality.

Indeed, the route is slightly differentiated if either subgroup or income source decomposition is the aim of the analysis. Let us describe the following steps:

Step 1: Choose the type of decomposition. It obviously depends on the aim of the analysis. Will infrastructural projects increase incomes among rural individuals? Does a tax on capital income reduce income inequality? These two are simple example stressing the need of defining the right question to answer.

Step 2: According to the type of decomposition, select the most suitable indexes. Just note that decomposition issues leaves less space in choosing indexes. This choice is basically restricted to the Gini Index or to the generalised entropy class.

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[6] See EASYPol Module 052: *Policy Impacts on Inequality: The Decomposition of Inequality by Population Subgroups*

Step 3: Whatever the aim of the analysis and the type of decomposition, **calculate overall inequality** with the selected index. This will provide a benchmark to check the decomposition.

If the type of decomposition you are working with is by **subgroups**, then:

Step 4a: First, calculate the **WITHIN** element in the way dictated by the selected index.

Step 5a: If you are not using the Gini Index, the **BETWEEN** element can be calculated as a difference between total inequality T (step 3) and the WITHIN element W (step 4), i.e. $B=T-W$. If the selected index is the Gini Index, then calculate the BETWEEN element directly.

Step 6a: If the selected index is the Gini Index, the **residual** must be calculated as the difference between overall inequality (step 3) and the sum of within (step 4) and between (step 5) elements. If groups are non overlapping, the residual is zero. In this case, check the decomposition with overall inequality. If the selected index is not Gini, then calculate the sum of within and between and check again with overall inequality.

If the type of decomposition is by **income sources**, steps from 4a to 6a must be replaced by:

Step 4b: First, calculate sub-indedxes for each income source. Whatever the selected index, these sub-indexes are required by the corresponding formulas.

Step 5b: Calculate any other parameter required by the formula and figure out the specific contribution of each income source. Sum these contributions and check with overall inequality.

Figure 3:   A comprehensive step-by-step procedure to decompose inequality
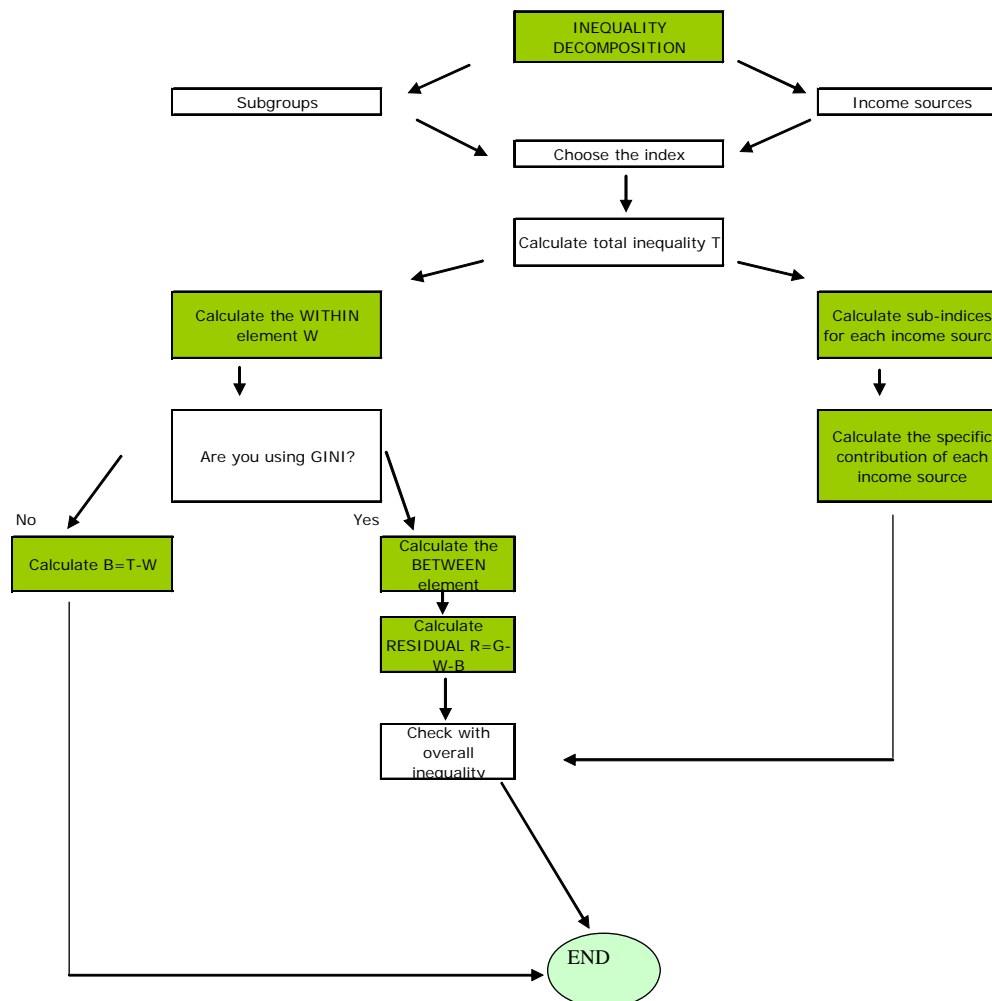
7.   READERS' NOTES

7.1.   Time requirements

Time required to deliver this module is estimated at about four hours.

7.2.   EASYPol links

Selected EASYPol modules may be used to strengthen the readers' background knowledge and to further expand their understanding on inequality and inequality measurement.

This module belongs to a set of modules which discuss how to compare on inequality grounds alternative income distributions generated by different policy options. It is part of the modules composing a training path addressing **Analysis and monitoring of socio-economic impacts of policies.**

The following EASYPol modules form a set of materials logically preceding the current module, which can be used to strengthen users' background knowledge:

- ✓   EASYPol Module 000: ***Charting Income Inequality: The Lorenz Curve***

- ✓   EASYPol Module 001: ***Social Welfare Analysis of Income Distribution: Ranking Income Distribution with Lorenz Curves***

- ✓   EASYPol Module 040: ***Inequality Analysis: The Gini Index***

- ✓   EASYPol Module 051: ***Policy Impacts on Inequality: The Theil Index and the Other Entropy Class Inequality Indexes***

- ✓   EASYPol Module 052: ***Policy Impacts on Inequality: The Decomposition of Inequality by Subgroups***

7.3.   Frequently asked questions

- How do we decompose inequality indexes?

- How to work out whether some income sources contribute more to total inequality than others?

- Is decomposability by income sources perfect?

8.   REFERENCES AND FURTHER READING

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4. Summary

This tool illustrates how to decompose inequality measures by income sources. In particular, it discusses this decomposition in the context of the Gini and the Theil Indexes. A step-by-step procedure and numerical examples give operational content to the tool.

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6. Author(s)

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7. Module type	☐ Thematic overview ☐ Conceptual and technical materials ☒ Analytical tools ☐ Applied materials ☐ Complementary resources
8. Topic covered by the module	☒ Agriculture in the macroeconomic context ☐ Agricultural and sub-sectoral policies ☐ Agro-industry and food chain policies ☐ Environment and sustainability ☐ Institutional and organizational development ☐ Investment planning and policies ☒ Poverty and food security ☐ Regional integration and international trade ☐ Rural Development
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10. Training path	**Analysis and monitoring of socio-economic impacts of policies**
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