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ANALYTICAL TOOLS

Module 040

# Inequality Analysis

## The Gini Index

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## The Gini Index

by

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**About EASYPol**

EASYPol is a an on-line, interactive multilingual repository of downloadable resource materials for capacity development in policy making for food, agriculture and rural development. The EASYPol home page is available at: ***www.fao.org/tc/easypol*.**

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# 1. SUMMARY

This tool addresses the most popular inequality index, the Gini index. It discusses its characteristics and the link with another popular graphical tool of representing inequality, the Lorenz Curve. Extended version of the Gini Index with different weighting schemes are also discussed. The use of the Gini Index and of its generalised versions is explained through a step-by-step procedure and numerical examples.

# 2. INTRODUCTION

### Objectives

The objective of this module is to introduce readers to the use of both the Gini Index and the Generalised Gini Index, to compare income distributions and to discuss their relative merits as well as their relative disadvantages.

### Target audience

This module targets current or future policy analysts who want to increase their capacities in analysing impacts of development policies on inequality by means of income distribution analysis. On these grounds, economists and practitioners working in public administrations, in NGOs, professional organisations or consulting firms will find this helpful reference material. In addition, academics may find this material useful to support their courses in Cost-Benefit Analysis (CBA) and development economics. Furthermore, users can use this material to improve their skills in CBA and complement their curricula.

### Required background

Users should be familiar with basic notions of mathematics and statistics. In addition they should have mastered the concepts of:

- Income distribution and income inequality
- **Lorenz Curves**
- Inequality aversion.

Links to relevant EASYPol modules, further readings and references are included both in the footnotes and in section 9 of this module[1].

[1] EASYPol hyperlinks are shown in blue, as follows:
a) training paths are shown in **underlined bold font;**
b) other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
c) links to the glossary are in **bold**; and
d) external links are in *italics*.

# 3. CONCEPTUAL BACKGROUND

The Gini Index is an inequality measure that is mostly associated with the descriptive approach to inequality measurement. Lambert (1993) provides a summary of the analytical basis to link the Gini Index with **social welfare functions**, thus moving the Gini Index into the field of welfare analysis. In what follows, we will be mostly confined to the descriptive approach, leaving the welfare approach for more advanced tools.

The Gini Index is a complex inequality measure[2] and, as with many inequality measures, it is a synthetic index. Therefore, its characteristic is that of giving summary information on the income distribution and that of not giving any information about the characteristics of the income distribution, like location and shape.

With regard to the Gini Index, we apply the logic of the inequality axioms[3], as long as axioms are eligible criteria to evaluate the indicator performances.

## 3.1 The Gini Index

The Gini Index was developed by Gini, 1912, and it is strictly linked to the representation of income inequality through the **Lorenz Curve**. In particular, it measures the ratio of the area between the Lorenz Curve and the **equidistribution line** (henceforth, the **concentration area**) to the area of maximum concentration.

Figure 1 provides the visual representation of these areas, by drawing three Lorenz Curves from three hypothetical income distributions, labelled **A**, **B** and **C**. The shape of the Lorenz Curve based on income distribution A is the standard Lorenz Curve we find (you find) when analysing actual income distributions. The Lorenz Curve of income distribution **B** is an extreme case where all incomes are equal. In this case, the Lorenz Curve is also called the **equidistribution line**. Finally, the Lorenz Curve of income distribution **C** is another extreme case where all incomes are zero except for the last one.

In Figure 1, as **OP** is the equidistribution line, **ORP** is the area defined by the Lorenz Curve of the standard income distribution and the equidistribution line, what we called the **concentration area**. Finally, **OPQ** is the area of maximum concentration, i.e. the area between the Lorenz Curve of income distribution **C** and the equidistribution line.

It should be clear that the equidistribution line **OP** and the area **OPQ** represent the extreme values that the concentration area can assume in a Lorenz Curve representation. Either this area is zero (as in the case of the equidistribution line of distribution **B**) or this area is at its maximum (in the case of distribution **C**). For a standard income distribution, the concentration area would be some way between zero and the area of maximum concentration, as in Figure 1.

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[2] See EASYPol Module 080: ***Policy Impacts on Inequality: Simple Inequality Measures***.

[3] As discussed in EASYPol Module 054: ***Policy Impacts on Inequality: Inequality and Axioms for its Measurement***.

Now, the Gini Index measures the ratio of the concentration area to the maximum concentration area. Therefore, in Figure 1:

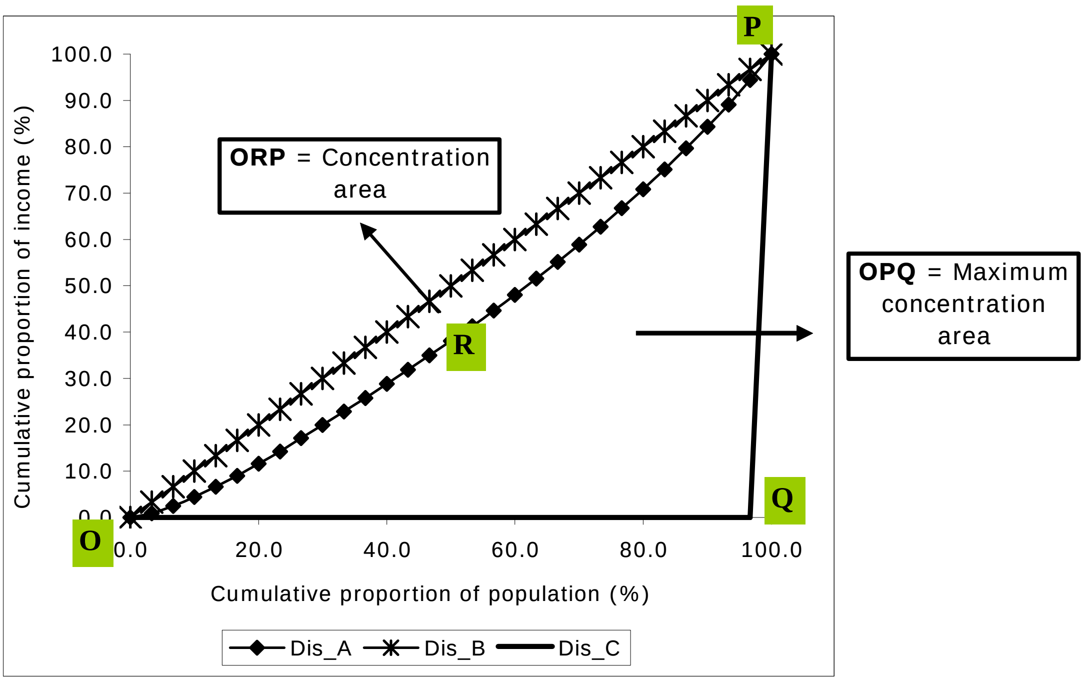
[1]

$$G = \frac{\text{concentration area}}{\text{maximum concentration area}} = \frac{ORP}{OPQ}$$

As the maximum concentration area is obtained by a distribution where total income is owned by only one individual, the Gini Index *G*, in general, measures the distance from the area defined by any standard income distribution to the area of maximum concentration.

It is now important to understand how the formula in Figure 1 can be applied in practical terms. Let us start from the **denominator** of *G*. We have already explained[4] that the maximum coordinates of the Lorenz Curve are at the point **(1,1)**. The area **OPQ**, therefore, must be a triangle with base length of 1 and height length of 1. Its area is therefore equal to ½. The denominator of *G* is therefore ½.

**Figure 1: The Lorenz Curve and the Gini Index**




| | | | | |
|---|---|---|---|---|
| **GINI** | = | Concentration area / Maximum concentration area | = | **ORP** / **OPQ** |

[4] See EASYPol Module 000, ***Charting Income Inequality: The Lorenz Curve***.

What about the **numerator**? Instead of calculating the concentration area directly, we can exploit the fact that this area is given by the difference between the maximum concentration area and the area under the Lorenz Curve (this latter being given by **ORPQ**). The area under the Lorenz Curve is more easily calculated as follows.

First of all, let us recall the definition of the coordinates of the Lorenz Curve[5]. Given $y_1 \le y_2 \le ... \le y_n$, it must be that:

$$q_i = \frac{y_1 + y_2 + ... + y_i}{y_1 + y_2 + ... + y_n} = \frac{y_1 + y_2 + ... + y_i}{Y} \rightarrow \text{cumulative proportion of income}$$

$$p_i = \frac{i}{n} \rightarrow \text{cumulative proportion of population}$$

with $q_0 = p_0 = 0$ and $q_n = p_n = 1$.

Now, the area **ORPQ** under the Lorenz Curve is the sum of the areas of a series of polygons. Let us consider Figure 2, where a simplified Lorenz Curve is built for a population of four individuals. The first polygon is a *triangle* ($p_o q_1 p_1$), the other three polygons are *rotated isosceles trapeziums*. Each area can therefore be calculated separately, and separate results added to get the value of the overall area. Let us define the area of the *i*-th polygon as $Z_i$ and the total area so obtained by Z.

The area of the triangle is given by:

$$Z_1 = \frac{\overbrace{p_1}^{\text{base}} \ \overbrace{q_1}^{\text{height}}}{2}$$

while the area of each trapezium is given by:

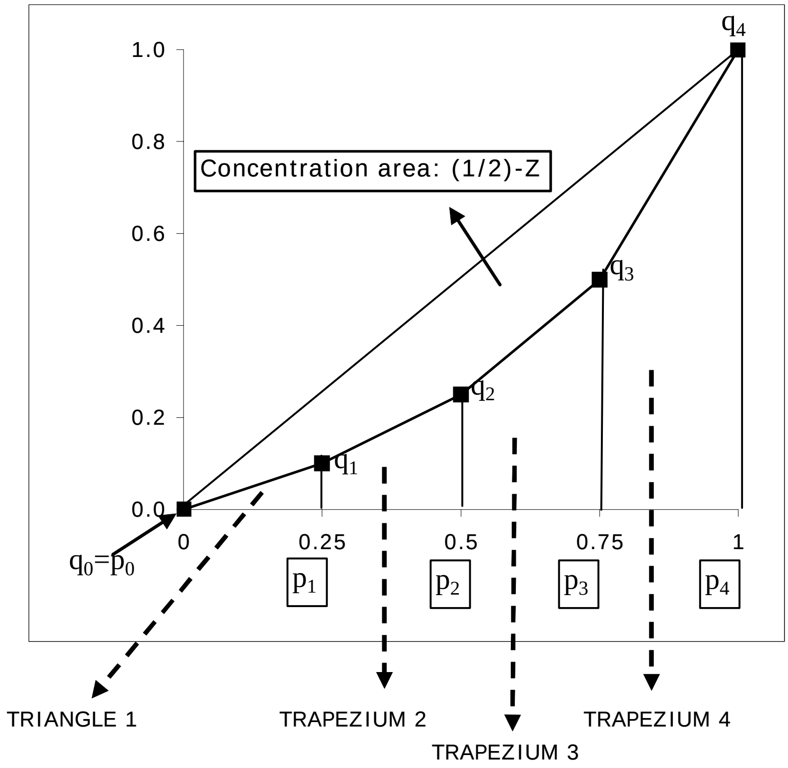
$$Z_i = \frac{\overbrace{(q_i + q_{i-1})}^{\text{long base + short base}} \ \overbrace{(p_i - p_{i-1})}^{\text{height}}}{2}$$

As $q_0 = p_0 = 0$, the sum of all these areas gives rise to:

$$Z = \sum_{i=1}^{n} Z_i = \frac{1}{2} \sum_i [(q_i + q_{i-1})(p_i - p_{i-1})] \qquad \text{for n=4}$$

[5] See EASYPol module 000, ***Charting Income Inequality: The Lorenz Curve.***

**Figure 2: How to calculate the concentration area**




However, Z is not the concentration area, but the area under the Lorenz Curve. To calculate the concentration area (the numerator of the Gini Index) it is now sufficient to subtract Z from the maximum concentration area (½ ) as follows:

$$\text{Concentration area} = \frac{1}{2} - Z = \frac{1}{2} - \frac{1}{2}\sum_i \left[(q_i + q_{i-1})(p_i - p_{i-1})\right]$$

According to [1], the Gini Index *G* is therefore equal to:

$$[2] \qquad G = \frac{\frac{1}{2} - \frac{1}{2}\sum_i \left[(q_i + q_{i-1})(p_i - p_{i-1})\right]}{\frac{1}{2}} = 1 - \sum_i \left[(q_i + q_{i-1})(p_i - p_{i-1})\right]$$

that can also be rewritten as:

$$[3] \qquad G = 1 - 2Z$$

The previous formula simply reveals that the Gini Index is equal to 1 minus twice the area under the Lorenz Curve.

This geometrical interpretation based on the Lorenz Curve, however, is only one of the possible methods to calculate the Gini Index. One way, that will prove particularly useful below, is to directly express the Gini Index in terms of the covariance between income levels and the cumulative distribution of income. In particular:

[4] $$G = Cov(y, F(y))\frac{2}{\bar{y}}$$

where *Cov* is the covariance between income levels *y* and the cumulative distribution of the same income *F*(*y*) and $\bar{y}$ is average income. In turn, it is worth recalling that the covariance is the expected value *E* of the products of the deviations from the mean of each variable. In the specific case:

[5] $$Cov[y, F(y)] = E[y - \bar{y}] \cdot \left[F(y) - \overline{F(y)}\right]$$

### 3.2 The generalised Gini Index ($G_v$)

In evaluating the policy impact on inequality, we have an inequality measure that is flexible enough to embody different policy-makers' preferences with regard to, say, the degree of inequality aversion. After all, the effects of a given policy might be evaluated differently by two policy makers having different attitudes towards inequality.

The Gini Index developed in the previous section (henceforth the *standard* Gini Index) does not allow for any variation in this attitude, i.e. the degree of inequality aversion.

A generalisation of the Gini Index by Yitzhaki (1983) makes the Gini index dependent on the specified degree of inequality aversion. The corresponding formula is the following:

[6] $$G(v) = -\frac{v}{\bar{y}} Cov\left[y, (1 - F(y))^{v-1}\right]$$

where all terms have the same meaning as in [4] and *v* is the degree of inequality aversion. Assigning different values to *v* may change the value of the Gini Index, by weighting differently incomes in different parts of the income distribution.

Note that with *v*=2, expression [6] collapses to the standard Gini Index (expression [4]).

In order to capture the meaning of the generalised Gini Index, let us just recall the following expanded definition of the covariance term in [6]:

[7] $$Cov[y, (1 - F(y))] = E[y - \bar{y}] \cdot \left[(1 - F(y)) - \overline{(1 - F(y))}\right]$$

The second square bracket on the right hand side of expression [7] is best interpreted as the weight that should be assigned to each income level, i.e. to each deviation from the mean income level (the first square bracket).

For lower incomes (below mean income) the first square bracket is negative. For lower incomes, instead, the second square bracket is positive. For higher incomes (above the mean) the situation is reversed. Deviations from mean incomes are positive, while the term in the second square bracket will be negative.

Note that one property of the **cumulative distribution function** (CDF) is that its mean is equal to its median value (½). Therefore, the value in the second square bracket will be positive until the median value of the CDF. This means that the median value of the income distribution will have a zero weight, as the median income is that income level where $F(y) = ½$.

We now have to understand what happens to incomes before and after the median when the value of $v$ increases.

Figure 3 investigates this issue. The red line depicts the difference of [1-F(y)] from its mean for the standard case of $v$=2. The red line intersects the $x$-axis in correspondence to the median level of this hypothetical income distribution.

If we now consider the case $v$=3 (the bold black line), it is relatively easy to see that there is a fraction of richer people (to the right of the intersection with $v$=2 in the south-east area of the graph) that has (in absolute terms) a lower weight than in $v$=2. This regards all incomes for which the bold black line lies above the red line in the graph.

At the same time, there is a fraction of poorer people (to the left of the intersection with $v$=2 in the north-west area of the graph) that has a greater weight than in $v$=2. This regards all incomes for which the bold black line lies again above the red line in the graph.

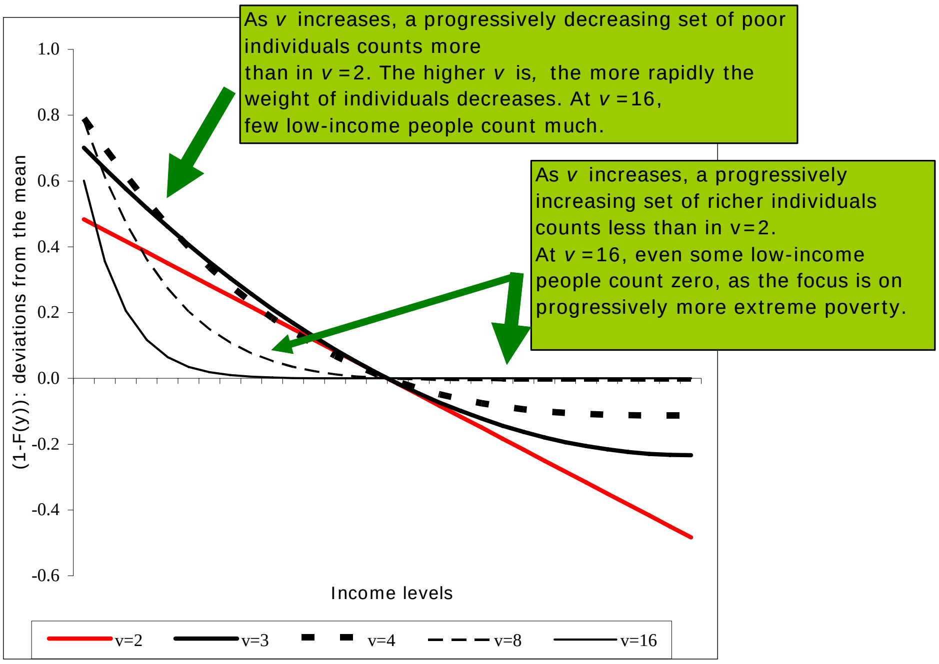
By increasing $v$, the fraction of richer people increases whose income deviation from the mean receives less weight than in $v$=2. At the same time, the fraction of poorer people decreases whose income deviation from the mean receives more weight than in $v$=2. Figure 3 reports the case for $v$=2, $v$=4, $v$=8 and $v$=16.

Therefore, as $v$ increases, there is a smaller number of low incomes having a large weight and an increasing number of people having a zero weight.

In calculating the Gini Index, therefore, increasing $v$ means focusing more on inequality in a progressively lower fraction of the income distribution[6].

[6] This is the reason why $v$ is often thought of as an «**inequality aversion parameter**».

**Figure 3: Weighting schemes in the Gini Index**




Hence, the generalised Gini Index gives more flexibility to the evaluation of development programs and policies than the standard Gini Index, because of the possibility to embody different degrees of inequality aversion.

To best interpret this issue, it is worth using the example described in Table 1.

**Table 1: An example of the implicit weighting scheme in the generalised Gini Index**

| Individuals | Incomes | F(y) | 1-F(y) | [1-F(y)]v-1 *v* =2 | [1-F(y)]v-1 *v* =4 | Incomes: deviations from the mean | [1-F(y)]v-1 v=2 : deviations from the mean | [1-F(y)]v-1 v=4 : deviations from the mean | Implicit weighting scheme with v=2 | Implicit weighting scheme with v=4 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1,000 | 0.20 | 0.80 | 0.80 | 0.51 | -2,000 | 0.40 | 0.45 | 2.0 | 8.0 |
| 2 | 2,000 | 0.40 | 0.60 | 0.60 | 0.22 | -1,000 | 0.20 | 0.15 | 1.5 | 3.4 |
| 3 | 3,000 | 0.60 | 0.40 | 0.40 | 0.06 | 0 | 0.00 | 0.00 | 1.0 | 1.0 |
| 4 | 4,000 | 0.80 | 0.20 | 0.20 | 0.01 | 1,000 | -0.20 | -0.06 | 0.5 | 0.1 |
| 5 | 5,000 | 1.00 | 0.00 | 0.00 | 0.00 | 2,000 | -0.40 | -0.06 | 0.0 | 0.0 |
| Mean | 3,000 | 0.60 | 0.40 | 0.40 | 0.06 | | | | | |

Table 1 reports a hypothetical distribution of income with five individuals. For each level of income, the third and the fourth column report the calculation of $F(y)$ and $(1-F(y))$, respectively. The fifth and the sixth columns give the result of applying $v$=2 and $v$=4 to the outcome of the fourth column.

Note, that $v$=2 corresponds to the standard Gini Index, while $v$=4 should correspond to weighting more people with low incomes.

The seventh column reports the deviation of each income from average income. For low incomes, this deviation is negative, while it is positive for higher incomes. We must just recall that this is a part of the covariance term in [7].

The eighth and the ninth columns calculate the deviations from the mean of the other part of the covariance term in formula [7]. What should we look for when comparing these columns? We can easily see that the «weight» assigned to the lowest incomes is greater with *v*=4 than with *v*=2. At the same time, the weight of the richest individual goes rapidly to zero with *v*=4.

One way to derive the implicit weighting scheme of the Gini Index is to set the ratio of the value of the function $(1-F(y))^{v-1}$ at any income level compared with the value of the same function at the median level of income.

For *v*=2 and *v*=4, this calculation is reported in the last two columns of Table 1. At *v*=2, twice as much contribution to the calculation of the Gini Index, would be attached to the lowest income (compared with the median income). With *v*=4, eight times as much contribution would instead be attached to the lowest income. Also note that the contribution of the highest incomes is lower at *v*=4.

# 4. A STEP-BY-STEP PROCEDURE TO CALCULATE THE GINI INDEX

## 4.1 The Gini Index

To illustrate the step-by-step procedure to build the Gini Index, we can refer to Figure 4. Note that this Figure is built by referring to formula [2], above.

The starting point is to sort the income distribution by income level (Step 1).

In Step 2 we must calculate the cumulative income distribution.

In Step 3 we can obtain the cumulative proportion of income ($q_i$) by dividing each cumulative income by total income.

Step 4 gives the cumulative proportion of population ($p_i$). We must rank individuals in an increasing order assigning rank 1 to the individual with the lowest income and rank "n" to the one with the highest income, and then dividing by n.

In Step 5 we must compute the areas of the polygons $Z_1$,$Z_2$, $Z_3$....$Z_n$. The first one is a triangle, the rest are trapeziums (apply the formula in the text).

In step 6, all the areas are summed up to obtain the area under the Lorenz Curve (Z), then the Gini Index G=1-2Z is computed.

Figure 4 presents the usual flow-chart to implement this step-by-step procedure.

**Figure 4: A step-by-step procedure to calculate the Gini Index**

| STEP | Operational content |
|---|---|
| 1 | If not already sorted, sort the income distribution by income level |
| 2 | Calculate the cumulative income distribution |
| 3 | Calculate the cumulative proportion of income by dividing each cumulative income by total income |
| 4 | By assigning rank 1 to the lowest income and rank *n* to the highest income, calculate the cumulative proportion of the population by dividing each rank by *n* . |
| 5 | Compute the area of polygons by applying formulas for areas of triangle and trapezium in the text |
| 6 | Sum up all the areas to obtain Z. Then calculate G=1-2Z |

However, an alternative formula to calculate the Gini Index directly has been provided in the text (see formula [4]), based on the covariance term. It is therefore worth explaining the step-by-step procedure to calculate the Gini Index in this alternative way. Figure 5 illustrates.

**Figure 5: A step-by-step procedure to calculate the Gini Index with the covariance formula**

| STEP | Operational content |
|---|---|
| 1 | If not already sorted, sort the income distribution by income level |
| 2 | Calculate the cumulative distribution function F(y) |
| 3 | Calculate the covariance Cov(y,F(y)) and the mean income level |
| 4 | Calculate Gini, by applying formula [4] in the text |

In particular, Step 1, as usual, asks us to sort the income distribution by income levels.

Step 2 asks us to calculate the CDF F(y)[7].

Step 3 only asks us to take the covariance between the distribution of income levels and the cumulative distribution function and the mean income level, which is used in the denominator of formula [4].

Finally, Step 4 is the direct application of formula [4] in the text.

### 4.2 The generalised Gini Index

Figure 6 reports the steps required to calculate the generalised version of the Gini Index.

**Figure 6: A step-by-step procedure to calculate the generalised Gini Index**

| STEP | Operational content |
|---|---|
| 1 | If not already sorted, sort the income distribution by income level |
| 2 | Calculate the cumulative distribution function F(y) |
| 3 | For each income, calculate 1-F(y) |
| 4 | Choose the value of the inequality aversion parameter *v* . |
| 5 | Calculate $[1-F(y)]^{v-1}$ |
| 6 | Calculate the covariance: $Cov[y, (1-F(y))^{v-1}]$ and the mean of the income distribution |
| 7 | Calculate GINI, by applying the covariance formula [6] in the text |

The steps are very similar to those required to calculate the standard Gini Index by the covariance formula. In particular, Step 1 and Step 2 are identical.

For convenience, because of the way in which the formula is expressed, Step 3 asks us to calculate, for each income level, the value of one minus the value of the cumulative distribution function at that point.

Step 4 is the most characteristic element of this procedure, as it asks us to choose the value of the parameter of inequality aversion *v*. We must just recall that higher values of *v* indicate more inequality aversion.

[7] See EASYPol Module 007: ***Impacts of Policies on Poverty: Basic Poverty Measures***

Step 5 asks us to calculate one element of the covariance term, namely the value of $(1-F(y))^{v-1}$.

In Step 6 we have to calculate the full covariance and the average income. This opens the way to apply formula [6] in the text to get the generalised Gini Index (Step 7).

# 5. A NUMERICAL EXAMPLE OF HOW TO CALCULATE THE GINI INDEX

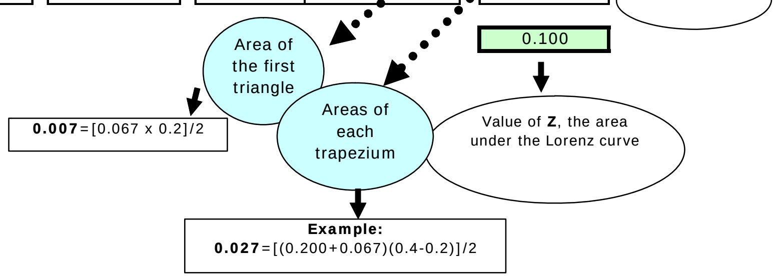
## 5.1 The standard Gini Index with the Lorenz derivation

The calculation of the Gini Index in its Lorenz Curve derivation may be appreciated by looking at Table 2. In this Table, we assume the existence of an income distribution with five individuals (and five incomes).

Step 1 just asks us to sort this income distribution by income level. Step 2, instead, asks us to calculate the cumulative distribution of income. The final point of this calculation is therefore 15,000 units of income, i.e. total income in the economy.

**Table 2: A numerical example to calculate the Gini Index (Lorenz derivation)**

| STEP 1 | | STEP 2 | STEP 3 | STEP 4 | | STEP 5 | STEP 6 |
|---|---|---|---|---|---|---|---|
| Sort the income distribution | | Calculate cumulative income | Calculate the cumulative proportion of income ($q_i$) | Assigning rank to incomes | Calculate the cumulative proportion of population ($p_i$) | Compute the areas of polygons (Z) | Calculate the Gini index |
| Individual | Income distribution | Cumulative income | Cumulative proportion of income | Ranks | Cumulative proportion of population | Area of polygons | Gini index |
| 1 | 0 | 0 | 0.000 | 1 | 0.2 | 0.000 | |
| 2 | 0 | 0 | 0.000 | 2 | 0.4 | 0.000 | 0.800 |
| 3 | 0 | 0 | 0.000 | 3 | 0.6 | 0.000 | |
| 4 | 0 | 0 | 0.000 | 4 | 0.8 | 0.000 | |
| 5 | 15,000 | 15,000 | 1.000 | 5 | 1.0 | 0.100 | G = 1 - 2Z |
| | | | | | | 0.100 | |




Step 3 transforms the cumulative income distribution into a cumulative proportion of income. The final point is now 1. We must just recall that these values are the $q_i$'s introduced in section 3.1 above.

According to the sort of the income distribution made in Step 1, an increasing rank (from 1 to $n$) is assigned to each income (Step 4). These ranks are then transformed in the cumulative proportion of the population. This calculation gives the $p_i$'s discussed in section 3.1 above.

Having both $q$ and $p$ for all income levels, we can calculate the area of the polygons below the Lorenz Curve. We must just remember that the first is a triangle and the others are trapeziums. Step 5 accomplishes this task by applying the formulas developed in section 3.1. The sum of all these areas gives ***Z***, the total area under the Lorenz Curve.

Finally, Step 6 is the mechanical application of formula [6] in the text. The resulting Gini Index is **0.267**.

### 5.2 The standard Gini Index with the covariance formula

Table 3 reports a numerical example to calculate the Gini Index according to the covariance formula [4], included in the text. In this case, the steps are reduced.

Step 1 is the same as before: just sort the income distribution by income level. Step 2 asks us to calculate the cumulative distribution function $F(y)$. Step 3 asks us to calculate the two essential parameters to apply the covariance formula: the covariance between income levels and the cumulative distribution function (whose value is 400 in the example); and the mean income level, which is 3,000 income units.

**Table 3: A numerical example to calculate the Gini Index with the covariance formula**

| STEP 1 | | STEP 2 | STEP 3 | | STEP 4 |
|---|---|---|---|---|---|
| Sort the income distribution | | Calculate the cumulative distribution function F(y) | Calculate the covariance Cov (y, F(y)) | Calculate the mean income level | Calculate Gini by using formula [4] |
| Individual | **A** - A typical income distribution | Cumulative income distribution | Covariance | Mean income | Gini |
| 1 | 1,000 | 0.2 | | | |
| 2 | 2,000 | 0.4 | | | |
| 3 | 3,000 | 0.6 | | | |
| 4 | 4,000 | 0.8 | | | |
| 5 | 5,000 | 1.0 | 400 | 3,000 | **0.267** |

By applying formula [4] in the text, we can obtain a value of the Gini Index equal to **0.267** (of course, the same as in Table 2!).

### 5.3 The generalised Gini Index

Using the covariance formula, we have explained that the standard Gini Index may be made sensitive to a given degree of inequality aversion. Table 4 reports an example of how to cope with this generalisation, using the same income distribution as in Tables 2 and 3.

**Table 4: A numerical example to calculate the generalised Gini Index**

| STEP 1 | | STEP 2 | STEP 3 | STEP 4 | STEP 5 | STEP 6 | | STEP 7 |
|---|---|---|---|---|---|---|---|---|
| **Sort the income distribution** | | **Calculate the cumulative distribution function F(y)** | **Calculate 1 - F(y)** | **Choose v** | **Calculate $[1 - F(y)]^{(v-1)}$** | **Calculate the covariance Cov (y, F(y))** | **Calculate the mean income level** | **Calculate Gini, by using formula [4]** |
| Individual | **A** - A typical income distribution | Cumulative income distribution | 1 - F(y) | ***v*** | $[1 - F(y)]^{(v-1)}$ | Covariance | Mean income | Gini |
| 1 | 1,000 | 0.2 | 0.8 | | 0.51 | | | |
| 2 | 2,000 | 0.4 | 0.6 | | 0.22 | | | |
| 3 | 3,000 | 0.6 | 0.4 | | 0.06 | | | |
| 4 | 4,000 | 0.8 | 0.2 | | 0.01 | | | |
| 5 | 5,000 | 1.0 | 0.0 | 4 | 0.00 | -246 | 3,000 | **0.246** |

Steps 1 and 2 are identical to those already discussed in Table 3. Step 3, instead, asks us to calculate the values of one minus the cumulative distribution function. Step 4 introduces the parameter of inequality aversion *v*, which we have chosen to be **4**. Any amount calculated in Step 3 is therefore raised at the power of (**v**-1), i.e. **3** (Step 5). Step 6 calculates the two essential parameters: the covariance term (-246 in the example) and the mean income (3,000). Step 7 applies formula [6] in the text, giving rise to a Gini Index of **0.246**. Note that this value is different from that obtained with the standard Gini Index, as different weights have now been attached to the same incomes.

## 6. THE MAIN PROPERTIES OF THE GINI INDEX

This section will describe the main properties of the Gini Index in terms of the axioms it respects[8].

As most of the properties are common to both the standard and the generalised Gini Index, the discussion will be carried out under a common heading. The main differences among the two indices will however be underlined.

The main properties of the Gini Index are:

- ***G* has zero as lower limit for any *v*.** When all incomes are equal, the covariance between income levels and the cumulative distribution function is zero. The Gini Index is therefore zero. With regard to the geometrical interpretation of the standard Gini Index, note that when all incomes are equal, the Lorenz Curve is equal to the equidistribution line. Therefore, the sum of areas of the polygons (***Z***) is equal to ½, i.e. the sum of the triangle under the Lorenz Curve. Therefore, the Gini Index (**1–2*Z***) is equal to zero.

[8] See EASYPol Module 054 ***Policy Impacts on Inequality: Inequality and Axioms for its Measurement***, for a discussion of axioms in inequality measurement.

- The standard Gini Index ***G*** **has** $\frac{n-1}{n}$ **as upper limit**. The limit of this value, for very large populations, is **1**. When all incomes are zero except for the last, the last income is also equal to total income, *y*=*Y*. It means that there is only one area to calculate, i.e., the last trapezium. However, for very large populations, this area tends to be smaller. In the limit (i.e. in a continuous framework) the value of the area ***Z*** tends towards zero. Therefore, the Gini Index tends towards 1. As a generalisation, ***G(v)*** has $\frac{2}{v}\frac{n-1}{n}$ as upper limit. Remember, that the standard Gini Index is one in which *v*=2.

- The Gini Index is **scale invariant**. By multiplying all incomes by a factor α, the value of the Gini Index ***G*** does not change. Intuitively, when all incomes are scaled by a common factor, the cumulative distribution of income does not change, as a given fraction of population still holds the same fraction of total income. The areas under the Lorenz Curve, therefore, do not change. With regard to the covariance formula, the application of a common factor to all incomes makes the covariance and the average income increase by the same factor. The Gini Index does not change. The same is true for ***G(v)***.

- On the other hand, the Gini Index ***G*** is not **translation invariant**. By adding/subtracting the same amount of money to all incomes, the Gini Index would increase (decrease) accordingly. The same is true for ***G(v)***.

- **The Gini Index satisfies the principle of transfers for any *v***. If income is redistributed from relatively richer individuals to relatively poorer individuals, both ***G*** and ***G(v)*** decrease. The opposite holds true if income is redistributed from relatively poorer to relatively richer individuals. With regard to the standard Gini Index, we note that the size of its change, following a change in any income, depends on the **rank** of the individuals involved in redistribution and on the sample size. It does not depend on the level of individual incomes involved in redistribution, but it depends on total income. In particular, the Gini Index reacts more to redistribution occurring among individuals who have a greater difference in ranks. The same amount of redistribution, indeed, generates a much lower effect if the two individuals have a close rank.

Table 5 illustrates how the main properties of the Gini Index work. In the left panel of the table, there are three simulated income distributions. The first is a «typical one» ranked by income level. The second is where all individuals have the same income. The third is where all individuals have zero income except for the last.

For the typical income distribution **A**, the Gini Index is 0.267. For the equi-distributed incomes **B**, the Gini Index is zero, while for the most concentrated income distribution **C** the Gini Index is 0.8 ($\frac{n-1}{n}=\frac{4}{5}=0.8$).

If all incomes were increased by 20 per cent, the fifth column of the table shows that the Gini Index would be unchanged (0.267). This is the property of **scale invariance**. This is not the case when all incomes are increased by the same absolute amounts (e.g. 2,000 income units in the text). In this case, Table 5 shows that the Gini Index would be lower (0.160). The Gini Index is not **translation invariant**.

The Gini Index satisfies the **principle of transfers**. If we redistribute, say, 500 income units from the richest to the poorest individuals, the Gini Index would be lower (0.213) in the text. Note that the same transfers, when occurring between individuals having closer ranking (the last column of Table 5), still gives rise to a lower Gini Index (0.253) but higher than in the previous case (0.213). This shows that the Gini Index is more sensitive to transfers occurring among individuals having distant ranks.

**Table 5: The main properties of the Gini Index**

| Individual | **A** - A typical income distribution | **B** - Income distribution with equal incomes | **C** - Income distribution with only one individual having income | Original income distribution with all incomes increased by 20 per cent | Original income distribution with all incomes increased by US$ 2,000 | Original income distribution with a redistribution of US$ 500 from the richest to the poorest | Original income distribution with a redistribution of US$ 200 from two individuals with a close rank |
|---|---|---|---|---|---|---|---|
| 1 | 1,000 | 3,000 | 0 | 1,200 | 3,000 | 1,500 | 1,000 |
| 2 | 2,000 | 3,000 | 0 | 2,400 | 4,000 | 2,000 | 2,500 |
| 3 | 3,000 | 3,000 | 0 | 3,600 | 5,000 | 3,000 | 2,500 |
| 4 | 4,000 | 3,000 | 0 | 4,800 | 6,000 | 4,000 | 4,000 |
| 5 | 5,000 | 3,000 | 15,000 | 6,000 | 7,000 | 4,500 | 5,000 |
| Total income | 15,000 | 15,000 | 15,000 | 18,000 | 25,000 | 15,000 | 15,000 |
| **GINI** | **0.267** | **0.000** | **0.800** | **0.267** | **0.160** | **0.213** | **0.253** |

**UNCHANGED** **DECREASED** **DECREASED** **DECREASED**

**The Gini index reacts less to transfers among individuals with a close rank**

## 7. LORENZ INTERSECTION AND THE GINI INDEX

We have explained that the Gini Index may be Lorenz-derived. In other terms, the Gini Index is Lorenz consistent. However, it is worth recalling that the ordering provided by Lorenz Curves, in particular by **Lorenz dominance**, is a *partial ordering*, as when Lorenz Curves do not cross anything we can say which income distribution has more inequality.

The Gini Index, instead, provides for a *complete ordering*, as it reduces the whole income distribution to a single number.

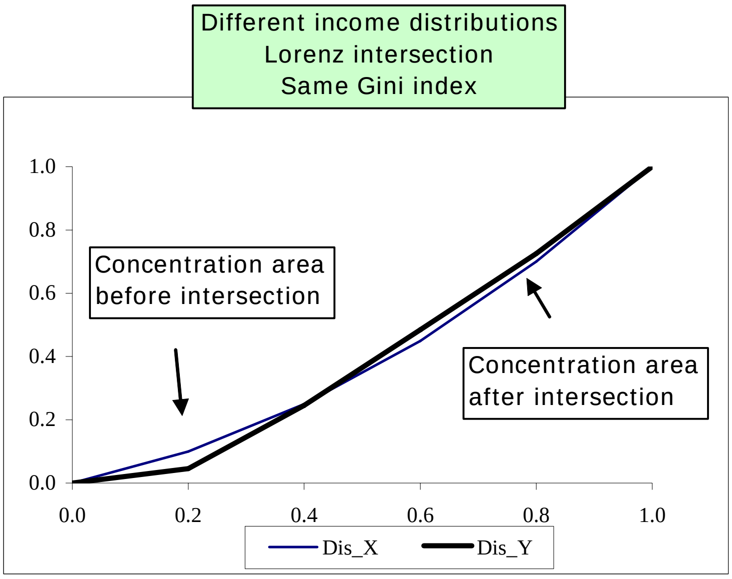
The basic difference between the two approaches can be appreciated by considering a case in which Lorenz Curves cross. For example, let us consider the two income distributions reported in Table 6. Incomes are distributed differently, but the Gini Index is the same (0.200).

**Table 6: Two income distributions with the same Gini Index**

| Individuals | Income distribution **X** | Income distribution **Y** |
|---|---|---|
| 1 | 2,000 | 900 |
| 2 | 3,000 | 4,000 |
| 3 | 4,000 | 4,800 |
| 4 | 5,000 | 4,800 |
| 5 | 6,000 | 5,500 |
| **Total income** | 20,000 | 20,000 |
| **Gini** | **0.200** | **0.200** |

These two income distributions give rise to Lorenz Curves as depicted in Figure 7. Lorenz Curves intersect, therefore they cannot be used to rank income inequality among these two income distributions. But the way they intersect is such that the area before the intersection and the area after the intersection are of the same value. This give rise to the same Gini Index, even in the presence of quite different income distributions.

**Figure 7: An intersection of Lorenz Curves**



## 8. A SYNTHESIS OF THE MAIN PROPERTIES OF THE GINI INDEX AND OF ITS GENERALISED VERSION

Table 7 gives a synthetic picture of the main properties of the Gini Index and its generalised version. These properties reflect the structure of axioms[9].

It is worth first noting that the lower limit of the both the standard Gini Index and of its generalised version, is zero, while the higher limit is differentiated. The latter limit is very close to 1 (for very large populations) in the case of the standard Gini Index; it tends towards $2v$ (again for very large populations) in the case of the generalised Gini Index.

Both versions of the Gini Index respect the principle of transfers. However, it is worth recalling that the Gini Index is more sensible for income transfers among individuals having a wide distance in their relative position (ranks) in the income distribution.

Both versions are scale invariant and not translation invariant and respect the principle of population. This structure makes the Gini Index and the generalised version of the same index two indices belonging to the class of relative inequality indexes. In applied works, the characteristics of these indices are very useful, hence the reason why they merit a high appeal.

**Table 7: The Gini Index and its desirable properties**

| | LOWER LIMIT | UPPER LIMIT | Principle of transfers | Scale invariance | Translation invariance | Principle of population | Index of relative inequality (RII) | Appeal |
|---|---|---|---|---|---|---|---|---|
| **Gini** | 0 | $(n-1)/n$ | YES, more sensible if individuals have distant ranks | YES | NO | YES | YES | High |
| **$G_v$** | 0 | $(2/v)(n-1)/n$ | YES, more sensible if individuals have distant ranks | YES | NO | YES | YES | High |

## 9. READERS' NOTES

### 9.1 EASYPol links

Selected EASYPol modules may be used to strengthen readers' background and to further expand their knowledge on inequality and inequality measurement.

This module belongs to a set of modules that discuss how to compare, on inequality grounds, alternative income distributions generated by different policy options. It is part of the modules composing a training path addressing **Analysis and monitoring of socio-economic impacts of policies.**

9 As discussed in EASYPol Module 054: ***Policy Impacts on Inequality: Inequality and Axioms for its Measurement.***

The following EASYPol modules form a set of materials logically preceding the current module, which can be used to strengthen users' backgrounds:

- ✓ EASYPol Module 000: ***Charting Income Inequality: The Lorenz Curve***,
- ✓ EASYPol Module 001***:*** ***Ranking Income Distribution with Lorenz Curves***,
- ✓ EASYPol Module 007: ***Impacts of Policies on Poverty: Basic Poverty Measures***
- ✓ EASYPol Module 054: ***Policy Impacts on Inequality: Inequality and Axioms for its Measurement.***

Issues addressed in this module are further elaborated in the following modules

- ✓ EASYPol Module 002: ***Social Welfare Analysis of Income Distributions: Ranking Income Distribution with Generalised Lorenz Curves.***
- ✓ EASYPol Module 041: ***Social Welfare Analysis of Income Distributions: Social Welfare, Social Welfare Functions and Inequality-Aversion***

A case study presenting the use of the Gini Index to measure inequality impacts in the context an agricultural policy impact simulation exercise with real data is reported in the EASYPol Module 042: ***Inequality and Poverty Impacts of Selected Agricultural Policies: The Case of Paraguay.***

# 10. APPENDIX I – ALTERNATIVE WAYS TO CALCULATE THE GINI INDEX

### *The geometrical derivation of the Gini Index and an alternative formula*

The Lorenz derivation of the Gini Index has a direct correspondence with another, rather cumbersome, way to calculate the Gini Index:

[A.1] $$G = 1 + \frac{1}{n} - \left(\frac{2}{n}\right)\left(\frac{y_n}{Y} + 2\frac{y_{n-1}}{Y} + 3\frac{y_{n-2}}{Y} + ... + n\frac{y_1}{Y}\right)$$

Note the peculiarity of the last round bracket, where each income share, from the highest to the lowest, is multiplied by the rank of individuals in the income distribution from the lowest to the highest, so that the largest share has rank 1 and the smallest share has rank *n*.

This correspondence is best shown by an example with *n*=3. By recalling the definition of *q*'s and *p*'s, we get:

$$q_0 = \frac{y_0}{Y} = 0; \qquad p_0 = \frac{0}{n} = \frac{0}{3} = 0$$

$$q_1 = \frac{0 + y_1}{Y}; \qquad p_1 = \frac{1}{3}$$

$$q_2 = \frac{0 + y_1 + y_2}{Y}; \qquad p_2 = \frac{2}{3}$$

$$q_3 = \frac{0 + y_1 + y_2 + y_3}{Y} = 1; \qquad p_3 = \frac{3}{3} = 1$$

By substituting these values into equation [2] in the text would yield:

[A.2] $$G_1 = 1 - \left[\frac{1}{3}\frac{y_1}{Y} + \frac{1}{3}\left(\frac{y_1 + y_2}{Y} + \frac{y_1}{Y}\right) + \frac{1}{3}\left(\frac{y_1 + y_2 + y_3}{Y} + \frac{y_1 + y_2}{Y}\right)\right] = 1 - \left[\frac{1}{3}\frac{y_3}{Y} + \frac{3}{3}\frac{y_2}{Y} + \frac{5}{3}\frac{y_1}{Y}\right]$$

Let us call this definition of the Gini Index $G_1$.

Now, let us also rewrite expression [A.1] as:

[A.3] $$G_2 = 1 + \frac{1}{3} - \frac{2}{3}\left[\frac{y_3}{Y} + 2\frac{y_2}{Y} + 3\frac{y_1}{Y}\right]$$

Let us call this definition of the Gini Inde x $G_2$.

Now, let us rewrite [A.2] and [A.3] in a more convenient way, by manipulating the square brackets:

[A.4] $$G_1 = 1 - \left[ \frac{1}{3}\left( \frac{y_3 + y_2 + y_1}{Y} \right) + \frac{2}{3}\frac{y_2}{Y} + \frac{4}{3}\frac{y_1}{Y} \right] = 1 - \frac{1}{3} - \frac{2}{3}\left[ \frac{y_2}{Y} + 2\frac{y_1}{Y} \right]$$

[A.5] $$G_2 = 1 + \frac{1}{3} - \frac{2}{3}\left[ \left( \frac{y_3 + y_2 + y_1}{Y} \right) + \frac{y_2}{Y} + 2\frac{y_1}{Y} \right] = 1 + \frac{1}{3} - \frac{2}{3} - \frac{2}{3}\left[ \frac{y_2}{Y} + 2\frac{y_1}{Y} \right] = 1 - \frac{1}{3} - \frac{2}{3}\left[ \frac{y_2}{Y} + 2\frac{y_1}{Y} \right]$$

as the expression in round brackets in both equations is equal to 1 for $n$=3. It is quite easy to verify that the two expressions give the same result ($G_1$=$G_2$). The formula [A.1], which is often used in operational applications, is therefore entirely based on the geometrical derivation of the Gini Index.

### 10.1 The Gini Index with the covariance formula

In the text, we have shown that the Gini Index might be directly calculated if we know the mean income and the covariance between income levels $y$ and the cumulative distribution function $F(y)$.

Analytically:

[A.6] $$G = \frac{2}{\bar{y}} Cov(y, F(y)).$$

This formula is also equivalent to formula [A.1]. Let us see why.

Using the fact that $Cov[y, F(y)] = -Cov[y, (1 - F(y))]$, since the expected value of F(y) is ½, and the expected value of [1-F(y)] is also ½, we can rewrite the expression [A.6] as: $G = -\frac{2}{\bar{y}} Cov[y, (1 - F(y))]$.

The equivalence between formula [A.6] and formula [A.1] can be shown again for a simplified case, $n$=3. First of all, it is worth recalling that $Cov(y, F(y)) = E(yF(y)) - E(y)E(F(y))$, where $E$ denotes the expected value (the mean) of a given variable. Second, it is worth defining the individual components of the covariance:

$$E(yF(y)) = \frac{\frac{3}{3}y_3 + \frac{2}{3}y_2 + \frac{1}{3}y_1}{3}; \quad E(y) = \frac{Y}{3}; \quad E(F(y)) = \frac{\frac{1}{3} + \frac{2}{3} + \frac{3}{3}}{3} = \frac{2}{3}.$$

Therefore, taking into account that $Y = y_1 + y_2 + y_3$, we can yield:

$$Cov(y, F(y)) = \frac{3}{9}y_3 + \frac{2}{9}y_2 + \frac{1}{9}y_1 - \frac{2}{9}y_3 - \frac{2}{9}y_2 - \frac{2}{9}y_1 = \frac{1}{9}[y_3 - y_1].$$

By considering that $\bar{y} = \frac{Y}{n}$, the covariance formula for Gini Index becomes:

$$G = \frac{2 \cdot 3}{Y} \cdot \frac{1}{9}[y_3 - y_1] = \frac{2}{3}\left[\frac{y_3}{Y} - \frac{y_1}{Y}\right].$$

Now, considering that, for *n*=3,

$\frac{y_1 + y_2 + y_3}{Y} = 1$, expression [A.1] can be rewritten:

$$\frac{y_1}{Y} + \frac{y_2}{Y} + \frac{y_3}{Y} + \frac{1}{3}\left(\frac{y_1 + y_2 + y_3}{Y}\right) - \frac{2}{3}\frac{y_3}{Y} - \frac{2}{3}\frac{y_2}{Y} - \frac{2}{3}\frac{y_2}{Y} - \frac{2}{3}\frac{y_1}{Y} - \frac{2}{3}\frac{y_1}{Y} - \frac{2}{3}\frac{y_1}{Y} = \frac{2}{3}\left[\frac{y_3}{Y} - \frac{y_1}{Y}\right]$$

which is the same as that obtained with the covariance formula.

## 10.2 The main properties of the Gini Index

GINI HAS ZERO AS A LOWER LIMIT

We can see that for the simplified case where *n*=3. In the specific case, *Y*=3*y*. Then, the formula [A.1] ] would yield:

$$G = 1 + \frac{1}{3} - \frac{2}{3}\left(\frac{y}{Y} + \frac{2y}{Y} + \frac{3y}{Y}\right) = 1 + \frac{1}{3} - \frac{2}{3}\left(\frac{6y}{3y}\right) = 1 + \frac{1}{3} - \frac{4}{3} = 0.$$

GINI HAS (N-1)/N AS UPPER LIMIT

This can again be shown for *n*=3 to be: Assuming again *n*=3, when all incomes are zero except for the last, expression [3b.4] would yield:

$$G = 1 + \frac{1}{3} - \frac{2}{3}\left(\frac{0}{Y} + \frac{0}{Y} + \frac{y}{Y}\right) = 1 + \frac{1}{3} - \frac{2}{3} = \frac{2}{3} = \frac{n-1}{n}$$

as in this case *y*=*Y*.

GINI IS SCALE INVARIANT

We can see this by scaling formula [A.1] by α. For example, with n=3, G would become: $G = 1 + \frac{1}{3} - \frac{2}{3}\left(\frac{\alpha y_3}{\alpha Y} + \frac{\alpha 2 y_2}{\alpha Y} + \frac{\alpha 3 y_3}{\alpha Y}\right) = 1 + \frac{1}{3} - \frac{2}{3}\left(\frac{y_3}{Y} + \frac{2y_2}{Y} + \frac{3y_3}{Y}\right)$.

GINI IS NOT TRANSLATION INVARIANT

This can again be seen through equation [A.1] in the case of n=3. Suppose all incomes are increased by Δy = $ 2,000, which means that total income will be increased by nΔy, i.e. $3 \cdot 2{,}000$. Equation [A.1] would become:

$$G = 1 + \frac{1}{3} - \frac{2}{3}\left[\left(\frac{y_3 + 2{,}000}{Y + (3 \cdot 2{,}000)}\right) + 2\left(\frac{y_2 + 2{,}000}{Y + (3 \cdot 2{,}000)}\right) + 3\left(\frac{y_1 + 2{,}000}{Y + (3 \cdot 2{,}000)}\right)\right].$$

As the numerator and the denominator of all round brackets grow in different ways, their ratios are different from those of the original formula. There is therefore no reason to expect the same Gini Index.

GINI SATISFIES THE PRINCIPLE OF TRANSFERS.

This can be easily seen by considering the derivative of the Gini Index with respect to the i-th income:

$$\frac{\partial G}{\partial y_i} = -\left(\frac{2}{n}\right)\underbrace{(n+1-i)}_{\text{individual rank}}\left(\frac{1}{Y}\right).$$

GINI REACTS LESS TO TRANSFERS OCCURRING AMONG INDIVIDUALS WITH CLOSER RANKS.

Let us explain this again by assuming n=3 and assuming first that income is redistributed from the richest person (rank 3) to the poorest person (rank 1). By totally differentiating [A.1] with respect to y, would yield:

$$dG = \overbrace{-\left(\frac{2}{3}\cdot\frac{3}{Y}\right)dy}^{\text{variation of G due to increasing } y_1} - \overbrace{\left(\frac{2}{3}\cdot\frac{1}{Y}\right)(-dy)}^{\text{variation of G due to decreasing } y_3} = -\frac{1}{Y}\left[\frac{4}{3}\right]dy\,.$$

Now, let us assume that income is redistributed from the richest individual (rank 3) to the immediately less richer individual (rank 2). In this case, differentiation would yield:

$$dG = \overbrace{-\left(\frac{2}{3}\cdot\frac{2}{Y}\right)dy}^{\text{variation of G due to increasing } y_2} - \overbrace{\left(\frac{2}{3}\cdot\frac{1}{Y}\right)(-dy)}^{\text{variation of G due to decreasing } y_3} = -\frac{1}{Y}\left[\frac{2}{3}\right]dy$$

which is clearly lower than dG in the first case. This property can be generalised by saying that, given the rank of the donor (in our example, from income y3), the reduction of the Gini Index is higher the more distant the rank of the receiver is from that of the donor. As a consequence, the Gini Index is more sensible to transfers occurring around the mode of the income distribution, where there is higher density of individuals.

## 11. REFERENCES AND FURTHER READING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## Module metadata

| | |
|---|---|
| **1. EASYPol module** | 040 |
| **2. Title in original language** | |
| **English** | Inequality Analysis |
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| **Other language** | |

**4. Summary**

This tool addresses the most popular inequality index, the Gini Index. It discusses its characteristics and the link with another popular graphical tool of representing inequality, the Lorenz Curve. Extended version of the Gini Index with different weighting schemes are also discussed. The use of the Gini Index and of its generalised versions is explained through a step-by-step procedure and numerical examples.

**5. Date**

December 2006

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|---|---|
| **7. Module type** | ☐ Thematic overview<br>☐ Conceptual and technical materials<br>☒ Analytical tools<br>☐ Applied materials<br>☐ Complementary resources |
| **8. Topic covered by the module** | ☒ Agriculture in the macroeconomic context<br>☐ Agricultural and sub-sectoral policies<br>☐ Agro-industry and food chain policies<br>☐ Environment and sustainability<br>☐ Institutional and organizational development<br>☐ Investment planning and policies<br>☒ Poverty and food security<br>☐ Regional integration and international trade<br>☐ Rural Development |
| **9. Subtopics covered by the module** | |
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**11. Keywords**

capacity building, agriculture, agricultural policies, agricultural development, development policies, policy analysis, policy impact anlaysis, poverty, food security, analytical tool, inequality, inequality analysis, inequality index, inequality measures, cost-benefit analysis, gini index, generalised gini index, lorenz curve, social welfare analysis