Resources for policy making

ANALYTICAL TOOLS

EASYPol MODULE 130

Social Accounting Matrix (SAM) for analysing agricultural and rural development policies

Conceptual aspects and examples

by

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1. SUMMARY

This document offers a methodological introduction to Social Accounting Matrices (SAMs) for analysing socio-economic impact of agricultural and rural development policies. Firstly, the concept and the structure of an SAM are presented. Next, the concept of policy "instrument" and "objective" variables are introduced and applied for SAM-based policy impact analysis. The module concludes by discussing the multipliers matrix as an instrument for analysing policy impact and presenting some examples.

2. INTRODUCTION

This paper is part of a set of materials focussed on analysing the impact of policy options. It will prove useful for an analyst who has to provide information on the likely socio-economic impact of measures such as, private investment support, export subsidies or import protection, tax reforms, infrastructural investment policies, sector or specific sub-sector policy measures. This module is intended to be used by different categories of users in different contexts:

- **policy analysts** will find it a useful reference document when performing their professional work;
- **academics** will be able to use it as support material for classes delivered to undergraduate students of national accounting, macroeconomics, economic policy, development economics and related fields;
- **Trainers** will be able to make use of it for capacity building activities, such as training policy analysts in using SAMs in their work;
- other users like ONGs, political parties, professional organisations or consultancy firms wishing to reinforce their expertise in impact analysis of policies on economic development and poverty.

The reader should be familiar with some basic concepts in economics such as: intermediate consumption and factors of production; value added and payments to factors; national accounting and balance of payments; concepts of policy impact simulation. The reader should also be comfortable working with matrix algebra.

Links to EASYPol modules on related topics are listed at the end of the document.

3. DEFINITION OF SOCIAL ACCOUNTING MATRIX

A Social Accounting Matrix (SAM) is a summary table, which refers to a given period, representing the production process, income distribution and redistribution which occurs between sectors, factors of production, actors in an economic system and the "Rest of the World" (ROW), meaning, all actors outside the economic system being studied)[1]. Since the

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[1] The SAM is described and standardised in the United Nations System of National Accounts (SNA), 2008. United Nations, Statistical Division, 2008. *System of National Accounts*, at: http://unstats.un.org/unsd/nationalaccount/docs/SNA2008.pdf

SAM represents the whole economic system, it highlights the interlinkages and the circular flow of payments and receipts among the different components of the system such as goods, activities, factors, and institutions.

The SAM has three main aims: 1) organise the information on the social and economic structure of a country for a given period; 2) provide a synoptic view of the flows of receipts and payments in an economic system; and 3) form a statistical basis for building models of the economic system, with a view to use this to simulate the socio-economic impact of policies.

3.1. Structure and content of a SAM

From an accounting perspective, the SAM is a two-entry square table which presents a series of double-entry accounts whose receipts and outlays are recorded in rows and columns respectively. Accounts usually refer to:

a) Goods and services: these accounts depict the origin of final goods available in the economic system (production activities and imports) and their destination (activities as intermediate inputs and institutions[2]).

b) Production activities: these are basically the production activities of the economy being analysed and generally refer to the defined sectors.

c) Factors of production: these accounts depict receipts from productive activities, which pay for factor services, and payments to institutions, which provide those services. They are usually distinguished in labour and capital, but may refer also to natural resources, such as land and water.

d) Institutions (economic agents), normally comprising households, companies (corporations) and the government. These accounts record incomes of institutions along the rows and expenditure on the columns.

e) The capital account or saving-investment or accumulation account, which records allocation of resources for capital formation and use of these resources for the purchase of investment products and building up stocks of goods.

f) The rest of the world account or external account, in which the row records payments received by the rest of the world from the economic system and the column records the outlays of the rest of the world towards the economic system.

Each category is then normally split into several more detailed accounts which are shown in specific rows and columns.

Here, it must be stressed that the sequence of accounts in rows and columns are identical.

Regarding recording different flows in one SAM, all receipts from an account are recorded in one row (*i*) and expenditure in one column (*j*). In this way, all monetary flows (s_{ij}) in a cell

[2] In some SAMs, no distinction is made between goods and services accounts and production activities accounts

of the matrix represent expenditure in the (*j*) column account and a receipt in the (*i*) row account.

The figure below shows an SAM in table format and illustrates the content of an SAM.

Table 1: SAM structure and content

			Goods and services	Production Activities	Factors		Resident Institutions			Savings-Investments	Rest of the world	Total
					Labour	*Capital Services*	*Households*	*Firms*	*Public sector*			
			(1)	(2)	(3)		(4)			(5)	(6)	
Goods and services	(1)		Trade/transp. marg.	Intermediate consumption			Final cons.hous.		Final cons.of PS	Investment & var.stocks	Exports	Demand of goods
Production Activities	(2)		Domestic production						Subsidies to production			Inflows of activities
Factors	(3)	*Labour*		Wages and Salaries							labour inc. from ROW	Labour incomes
		Capital Serv.		Earn.b.taxes (EBT)								Capital incomes
Resident Institutions	(4)	*House holds*			Wages and Salaries		Intra-hous. transfers	Distributed profits	Transfers to households		Transferts from ROW	Households incomes
		Firms				Earn.b.Taxes (EBT)					Transferts from ROW	Firms incomes
		Public sector	Taxes on goods/serv	Taxes on activities			Taxes/social security	Taxes	Transfers within PS	Budget deficit	Transferts from ROW	PS income
Savings-Investment	(5)		Decreases of stocks	Depreciation of capital			Savings of households	Savings of firms	Budget surplus		Deficit bal.of payments.	Financial resources
Rest of the world	(6)		Imports		Remun.of ext.labour		Transfers to ROW	Transfers to ROW	Transfers to ROW	Surplus bal. of payments		Outlays to ROW
Total			Supply of goods and services	Domestic production	Payments for labour	Payments for capital services	Households expenditure	Use of EBT	Public expenditure	Total investment	Payments of ROW	

One of the accounting principles of the social accounting matrix is that total receipts must equal total expenditure in each account.

3.2. Reading rows and columns

In this section, we will analyse the flows of receipts (reading per account row) and payments (reading per account column) of each account compared to the next. The numbering used for rows and columns refers back to Table 1 above.

READING THE ROWS

Goods and services accounts (row 1)

The rows for goods and services accounts, record payments made at market prices, which include indirect taxes (VAT etc.) due to intermediate consumption of production activities, end consumption by households, the government and investment, represented by changes in stock, and gross fixed capital formation and exports.

Activity accounts (row 2)

If we read production activity accounts, by row, we can see that activities receive payments for: 1) goods and services produced (output from domestic production activities), net of tax and product subsidies; 2) export subsidies and 3) exported goods and services. These elements make up the total production value.

Factor accounts (row 3)

The receipts in factor of production accounts, such as capital and labour, are made up of payments from production activities which use factors of production. These receipts make up the total added value.

Institution accounts (row 4)
Institutions' receipts are split out according to the specific institution.

Households receive payments from factor accounts for provision of labour services and transfers from the state, transfers between households, profits paid by companies for services from entrepreneurs, capital services, and payments from workers abroad. Companies receive gross operating income paid by the capital factor account.

Public administration receives payments from goods and services accounts (taxes on products), activities accounts (such as taxes on activities and social contributions on labour employment) and other institutions, in particular, taxes on household revenues and on company profits.

Companies, receive gross operating income and government transfers.

The **public administration account**, records government revenue from indirect taxes on products, value-added tax (VAT) on activities, import tariffs and taxes on goods and services exports, social security contributions, taxes on profits, taxes on household revenue and capital account payments, when there is a public budget deficit;

Capital account (row 5)
The capital account receives payments of household, corporate and Government savings. It also records receipts due to net stock reductions of goods held in stock (negative changes in stock over the period being analysed). Furthermore, it records the capital transfers (namely, balance of the balance of payments) received from the rest of the world. This arises when the balance of payments is negative, in other words, there is a deficit.

The Rest of the World account (ROW, row 6)
The rest of the world account receives payments from the country's imports of goods and services and transfers from the rest of the world. It must be noted that when the balance of payments is positive (balance of payments surplus), the rest of the world account is credited with the corresponding amount.

READING THE COLUMNS

By reading the matrix column by column, the payments made by each account to other accounts can be identified.

Goods and services accounts payments (column 1)
Goods and services pay the value of goods and services produced by activities (domestic production) into the activities accounts and pay the value of imported products to the rest of the world account. This account also records payments due to net stock reductions of goods held in stock (negative changes in stock over the period being analysed). The prices used for evaluating goods and services are market prices, which include indirect taxes but exclude consumption subsidies.

Activities accounts payments (column 2) This column represents the account for domestic production activities. Activities pay: 1) intermediate consumption to the goods and services accounts; 2) labour and capital services to the factors accounts; 3) indirect taxes

(VAT) to public administration; 4) physical capital consumption (depreciation) to the capital account;

Factors accounts payments (column 3)
Factors pay salaries to the household accounts and operating income to companies. Additionally, they pay foreign labour and capital services to the ROW account.

Institutions accounts payments (column 4) Institutions' payments are split out according to the specific institution, as already illustrated for receipts.

The **households** column records allocation of household revenue. They pay: 1) purchase of end consumption goods and services to the goods and services accounts; 2) intra-household transfers, including payments for services provided to households by other households, to other households; 3) social contributions and direct taxes on income to public administration; 4) savings from that period to the capital account; and 5) transfers abroad to the ROW.

The **companies** column records corporate payments of: 1) shared profits to household accounts; 2) taxes on profit to public administration accounts; 3) corporate savings to the capital account; 4) transfers abroad to the ROW.

The **public administration** column records payments to: 1) goods and services accounts, for end consumption; 2) activities accounts for subsidies to domestic production activity, 3) household accounts, for transfers, such as pensions and consumption support; 4) the capital account for savings from the central administration, when there is a public budget surplus; and 5) payments to the ROW account, for servicing external debt.

Capital account payments (column 5)
The capital account pays: 1) the goods and services account for positive changes in stock (stock formation) and for investment goods (formation of physical capital stock); 2) the ROW account when there is a balance of payments surplus, which is considered a foreign investment.

ROW account payments (column 6*)* The ROW account pays: 1) the goods and services account for exports; 2) the factors accounts, for labour services and capital returned by national actors abroad; 3) the capital account, when there is a balance of payments deficit (an outward net deficit position is offset by a transfer from outside the workings of the economy to the national economy).

THE DIFFERENT BLOCKS IN SAM

The different blocks of the SAM are made up of intermediate consumption, added value, production, end consumption, salaries and profits paid out to institutions, imports and exports, transfers, gross fixed capital formation and taxes.

a) The intermediate consumption block
All purchases made by the activities of intermediate consumption goods and services for use in their production process. In the SAM, they are translated into monetary flows of the production activities accounts to the various goods and services accounts.

b) Value Added

The value added block refers to payment of factors of production. This payment comprises salaries and capital payment (machines, buildings and other equipment). In general, the value added for each production activity is calculated by taking the difference between the value of total production shown in the total row and the value of intermediate consumption used. The value added is shown in the SAM by monetary flows from production activity accounts in columns to the labour and capital accounts in rows.

c) Domestic sales

This block deals with payments made from the goods and services accounts to production activities accounts. These domestic sales refer to the share of goods and services intended for the domestic market; exports of goods and services are therefore not included.

d) End consumption

This block covers household and State expenditure on food, non-food products and services for end consumption. In the SAM, end consumption is shown by monetary flows from household and State accounts to accounts of consumed goods and services.

e) Imports and exports

This refers to all agricultural and non-agricultural products traded abroad. In the SAM, imports are represented by payments made by imported goods and services accounts to the rest of the world account. Exports are represented by monetary flows from the rest of the world account to the exported goods and services accounts.

f) Salaries and profits

This refers to all monetary flows from factors of production accounts to household accounts. They are made up salaries received in exchange for work and as revenue from capital.

g) Transfers

They represent monetary flows which exist between the various institution accounts. Theses are payments between household accounts, payments from corporate accounts to household accounts, payments from the State account to household accounts and corporate accounts, payments from the Rest of the World account (emigrates) to household account and payments from household accounts to the rest of the world account.

h) Gross capital formation

This refers to all payments made by the savings and investments account to the goods and services account. It is made up of changes in stock and of GFCF.

i) Taxes

These are payments without anything in form of exchange which household, corporate and goods and services accounts make to the State account. They are made up of general income tax, of production taxes (VAT), income and earnings taxes (corporate tax and income tax), local taxes (patents, urban taxes and council tax), registration fees and stamp duty and other payments without anything directly being exchanged.

Table 2: Example of a simplified social accounting matrix

	Agric. goods	Ind. goods	Agric. Sect.	Ind. Sect	Labour	Capital	Rur. hous.	Urb. Hous.	Corpor.	Govt.	S-I	ROW	TOTAL
Agricultural goods			10	50			40	45		5	5	10	165
Industrial goods			30	20			10	60		10	10	10	150
Agricultural sector	150												150
Industrial sector		135											135
Laboue			60	30									90
Capital			40	20									60
Rural households					50			10					60
Urban households					40	30			10	55		20	155
Corporations						30							30
Government	5	5	10	15				20					55
Savings-Investment							10	20	20	-15			35
Rest Of the World	10	10									20		40
TOTAL	165	150	150	135	90	60	60	155	30	55	35	40	1125

4. THE SAM AND THE STRUCTURE OF THE ECONOMY

The main objective of the SAM is organising information about the economic and social structure. Through this organising of information, the SAM offers a summary of the social and economic structure of the country since it provides a synoptic description of production activities, composition and use of household income, consumption, saving, investment and international trade. The SAM enables some useful economic structure indicators to be calculated. This is shown in the example below.

Figure 1 shows a SAM of a simple 2-sector economy (agriculture and industry) and two institutions (households and government)[3]. The values are expressed in monetary units (mu).

Figure 1: A simple SAM (matrix S)

	Agricult	Industry	Households	Government	total
Agricult	50	20	25	15	110
Industry	30	30	15	5	80
Households	20	10	0	15	45
Government	10	20	5	2	37
total	110	80	45	37	272

If we look at matrix S, column by column, it can be seen that to produce 110 mu of output, the agricultural sector must pay 50 mu to the agricultural sector and 30 mu to the industrial sector for intermediate consumption. It must also pay 20 mu in salaries to households and 10 mu in taxes to the government (see "Agriculture" column).

Similarly, the industrial sector, in order to produce 80 mu of output, must pay 20 mu to the agricultural sector and 30 mu to the industrial sector for intermediate consumption. Furthermore, it must pay 10 mu in salaries to households and 20 mu in taxes to the government. (see "Industry" column).

[3] The goods accounts are aggregated with those of the productive sectors. The factors, capital and rest of the world accounts are omitted for simplicity.

The third column shows household expenditure. Households spend 25 mu in final consumption of agricultural products, 15 mu of industrial products and pay 5 mu in taxes.

The fourth column shows government expenditure: 15 mu are allocated to the agricultural sector and 5 mu to the industrial sector through subsidies for production. Additionally, 15 mu are transferred to households (for example, in the form of transfers as income support for poor households). Finally, 2 mu are made as an internal transfer to the public administration.

In mathematical terms, this is defined as follows:

a) SAM elements. Each element in matrix S is indicated s_{ij}, where $i = 1,2,.....n$; is the row index, $j = 1,2,.....n$ is the column index. For example, for SAM in figure 2, where $i = 1,2,3,4$ and $j = 1,2,3,4$, $s_{22} = 30$, $s_{13} = 25$.

b) Column sums $s_{\bullet j} = \sum_{i=1}^{n} s_{ij}$ are the column totals. In our example, $s_{\bullet 1} = 110$, $s_{\bullet 3} = 45$

c) Row sums. $s_{i\bullet} = \sum_{j=1}^{n} s_{ij}$ are the row totals. In our example, $s_{2\bullet} = 80$, $s_{3\bullet} = 45$

As already mentioned, for a given k account, expenditure is equal to receipts and is shown by the fact the sum of the row is equal to the sum of the column. Shown as a formula:

$$\underbrace{\sum_{i=1}^{n} s_{ih}}_{\substack{\text{Total expenditure in account } h \\ \text{(column sum)}}} = \underbrace{\sum_{j=1}^{n} s_{hj}}_{\substack{\text{Total receipts in account } h \\ \text{(row sum)}}} \qquad (1)$$

i.e.: $s_{\bullet h} = s_{h\bullet}$

If you divide each element in matrix S, s_{ij} by the total of the corresponding column $s_{\bullet j}$, you will get the column ratios or coefficients $c_{ij} = \frac{s_{ij}}{s_{\bullet j}}$. All these coefficients make up matrix C showing the column coefficients.

$$C = \begin{bmatrix} c_{11} = \frac{s_{11}}{s_{\bullet 1}} & c_{12} = \frac{s_{12}}{s_{\bullet 2}} & \dots & c_{1n} = \frac{s_{1n}}{s_{\bullet n}} \\ c_{21} = \frac{s_{21}}{s_{\bullet 1}} & c_{22} = \frac{s_{22}}{s_{\bullet 2}} & \dots & c_{2n} = \frac{s_{2n}}{s_{\bullet n}} \\ \dots & \dots & \dots & \dots \\ c_{n1} = \frac{s_{n1}}{s_{\bullet 1}} & c_{n2} = \frac{s_{n2}}{s_{\bullet 2}} & \dots & c_{nn} = \frac{s_{nn}}{s_{\bullet n}} \\ \hline 1 = \frac{s_{\bullet 1}}{s_{\bullet 1}} & 1 = \frac{s_{\bullet 2}}{s_{\bullet 2}} & \dots & 1 = \frac{s_{\bullet n}}{s_{\bullet n}} \end{bmatrix}$$

The column coefficients describe the "structure" of the economy[4] (Figure 2).

Figure 2: The column coefficients matrix (Matrix C)

	Agricult	Industry	Households	Government
Agricult	0.455	0.250	0.556	0.405
Industry	0.273	0.375	0.333	0.135
Households	0.182	0.125	0.000	0.405
Government	0.091	0.250	0.111	0.054
total	1.000	1.000	1.000	1.000

Each coefficient c_{ij} represents average payment from an account j to another account i per unit of expenditure in account i.

The economic interaction of the coefficients varies according to account group. For example, for production activities, the coefficients represent the cost of different inputs and factors per unit of production; whereas, for institutions, they represent the proportion of income allocation. For example, in the matrix in figure 3, in order to produce one mu of output, the agricultural sector must pay on average 0.455 mu to the agricultural sector itself, 0.273 mu to the industrial sector, 0.182 in salaries to households and 0.091 in taxes to the government. On the other hand, households devote 55.6% of their income to consumption of agricultural products, 33.3% to consumption of industrial products, do not transfer any money to the other households and pay on average 11.1% of their income to the government.

5. From the SAM to a model of the economy

5.1. The structure of an economic model

Information on the structure of the economy contained in a SAM, namely the matrix S and the derived matrix C, can be used for the quantitative analysis of the socio-economic impact of policies. Indeed, we can build a model of the economic system which:

a) uses the information about the structure of the economy contained in the SAM;
b) enables the expected socio-economic impact of certain policy options to be calculated.

In order to build this model, two types of variables are identified[5]:

a) **endogenous variables**: the values of endogenous variables are determined by the economic model;
b) **exogenous variables**: the values of exogenous variables are determined outside the model.

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[4] The approach is very similar to that of calculating "technical coefficients" in the input-output analysis.

[5]Those aspects, generally valid for any quantitative policy analysis model, are described for example in: Fox, K.A., J.K. Sengupta & E. Thorbecke (1996): *The Theory of Quantitative Economic Policy*, North Holland, Amsterdam

In the context of policy analysis, on the one hand, within the endogenous variables there are variables which are **"policy objectives",** i.e, target variables, which policy measures aim to change, for example, the level of added value of a given sector, income of poor households, budgetary revenues etc.

On the other hand, within exogenous variables there are **"policy instrument** “variables. These are variables which can be directly or indirectly manipulated by decision makers and which enable policy measures to be driven inside the model.

In an economic system, changes introduced through policy instruments have an impact on target variables through linkages which exist among the different parts of the system. For instance, an exogenous increase in the demand of a good or service, in presence of excess production capacity leads to an expansion of the production of that good, increasing on the one side the demand of intermediate inputs and, on the other side, the demand of factors. The increased demand of intermediate inputs activates other production sectors, which in turn require additional intermediate inputs and factors. The increased demand of factors activates factors’ supply, which in turn increases the factor incomes (wages, profits, rents). Factor incomes, allocated to households and, through taxes, to government, activate the demand of consumption goods, which in turn activates the production sectors producing final consumption goods. These processes keep feeding each other through a ***multiplicative process***, until the effects of the original increase in the demand of a good or service are fully absorbed by the economic system.

In a model, these links are represented by a system of equations. We will now look at how, based on information contained in the SAM, it is possible to:

a) identify endogenous and exogenous variables;
b) calculate the coefficients matrix of endogenous accounts;
c) identify equations which express links between the different variables in an economic system;
d) solve the model, i.e. determine the changes in the value of endogenous variables (policy objectives) corresponding to changes in the values of exogenous variables (policy instruments)

5.2. Identifying endogenous and exogenous variables in a SAM

In SAM-based models, normally the following accounts: a) value of goods produced; b) output from activities; c) payment from factors; and d) households' income; are assumed to be **endogenous**.

If we assume that the SAM presents k endogenous accounts, we will call e_{ij} (i = 1,….k; j=1….k) the elements of the SAM which belong to k endogenous accounts and E the square sub-matrix of (k x k) dimensions of these elements.

Exogenous accounts, on the other hand, are normally the a) public administration accounts, b) capital variations; and c) rest of the world accounts. These accounts are normally aggregated because of the fact that expenditure from those accounts is all exogenous.

Figure 4 shows the matrices S and C from the example above in which the endogenous and exogenous accounts are identified. In Figure 3, k=3, notably, the agriculture and industry sectors as well as the households are assumed to be endogenous. The government instead is assumed to be exogenous, i.e. to be the channel through which changes are introduced into the economic system. In figure 3 the exogenous account, i.e. the government account, is highlighted in grey.

We will call expenditure from exogenous accounts d_i. For example, in Matrix S, d_1=15, d_2 =5, d_3=15, d_4=2.

Figure 3: Endogenous and exogenous accounts

Matrix S

	Agricult	Industry	Households	Government	total
Agriculture	50	20	25	15	110
Industry	30	30	15	5	80
Households	20	10	0	15	45
Government	10	20	5	2	37
total	110	80	45	37	272

Matrix C

	Agricult	Industry	Households	Government
Agriculture	0.455	0.250	0.556	0.405
Industry	0.273	0.375	0.333	0.135
Households	0.182	0.125	-	0.405
Government	0.091	0.250	0.111	0.054
total	1.000	1.000	1.000	1.000

The expenditure from this aggregate and, in particular, the variations, are considered "policy instruments". Variations in expenditure such as transfers to households and to companies from the government or from the ROW, or variations in public demand for specific goods and services, change the receipts in endogenous accounts. By changing the receipts in endogenous accounts, it is expected that there will be changes in the level of endogenous economic variables, and in particular, variables which are targeted by policies[6].

Exogenous accounts also receive payments from endogenous accounts. These payments are considered as ***Leakages,*** as they exit the endogenous part of the economic system and do not contribute to the multiplicative process described in section 5.1 above**.** We will call these payments L_j. For example, in matrix S, L_1=10, L_2=20, L_3=5, L_4=2.

[6] It must be noted that expenditure from exogenous accounts to exogenous accounts themselves, such as the transfer of 2 mu from the government to the government itself in the example, is not a policy instrument, since it has no impact on the endogenous part of the system

Using the above notation, a generic SAM can be split into endogenous and exogenous parts as shown in the figure below:

$$S = \left[\begin{array}{cccc:c|c} e_{11} & e_{12} & \dots & e_{1k} & d_1 & s_{1\bullet} \\ e_{21} & e_{22} & \dots & e_{2k} & d_2 & s_{2\bullet} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ e_{k1} & e_{k2} & \dots & e_{kk} & d_k & s_{k\bullet} \\ \hdashline l_1 & l_2 & \dots & l_k & l_n & s_{n\bullet} \\ \hline s_{\bullet 1} & s_{\bullet 2} & \dots & s_{\bullet k} & s_{\bullet k} & T \end{array}\right]$$

Based on the part of the SAM shown below, receipts from each endogenous account can therefore be expressed as the sum of endogenous components and an exogenous component:

$$s_{i\bullet} = \left[e_{i1} + e_{i2} + \ldots + e_{ik}\right] + d_i \quad \text{or}$$

$$s_{i\bullet} = \sum_{j=1}^{k} e_{ij} + d_i$$

(row total of the account *i*) = (endogenous components) + (exogenous comp.)

5.3. The matrix of endogenous coefficients and the vector of leakages

Considering the partition of matrix S, the following can be expressed:

a) Matrix A of endogenous accounts coefficients; (dimension k x k);
b) Vector L of leakages of (dimension l x k).

$$A = \begin{bmatrix} a_{11} = \dfrac{e_{11}}{s_{\bullet 1}} & a_{12} = \dfrac{e_{12}}{s_{\bullet 2}} & \dots & a_{1k} = \dfrac{e_{1k}}{s_{\bullet k}} \\ a_{21} = \dfrac{e_{21}}{s_{\bullet 1}} & a_{22} = \dfrac{e_{22}}{s_{\bullet 2}} & \dots & a_{2k} = \dfrac{e_{2k}}{s_{\bullet k}} \\ \dots & \dots & \dots & \dots \\ a_{k1} = \dfrac{e_{k1}}{s_{\bullet 1}} & a_{k2} = \dfrac{e_{k2}}{s_{\bullet 2}} & \dots & a_{kk} = \dfrac{e_{kk}}{s_{\bullet k}} \end{bmatrix}$$

$$L = \begin{bmatrix} L_1 = \dfrac{l_1}{s_{\bullet 1}} & L_2 = \dfrac{l_2}{s_{\bullet 2}} & \ldots\ldots & L_k = \dfrac{l_k}{s_{\bullet k}} \end{bmatrix}$$

Which result from a partition of coefficients c_{ij} of matrix C, completely analogous to that of matrix S:

$$C = \left[\begin{array}{cccc:c}
\frac{e_{11}}{s_{\bullet 1}} & \frac{e_{12}}{s_{\bullet 2}} & \dots & \frac{e_{1k}}{s_{\bullet k}} & \frac{d_1}{s_{\bullet n}} \\
\frac{e_{21}}{s_{\bullet 1}} & \frac{e_{22}}{s_{\bullet 2}} & \dots & \frac{e_{2k}}{s_{\bullet k}} & \frac{d_2}{s_{\bullet n}} \\
\dots & \dots & \dots & \dots & \dots \\
\frac{e_{k1}}{s_{\bullet 1}} & \frac{e_{k2}}{s_{\bullet 2}} & \dots & \frac{e_{kk}}{s_{\bullet k}} & \frac{d_k}{s_{\bullet n}} \\
\hdashline
\frac{l_1}{s_{\bullet 1}} & \frac{l_2}{s_{\bullet 2}} & \dots & \frac{l_k}{s_{\bullet k}} & \frac{l_n}{s_{\bullet n}} \\
\hline
\frac{s_{\bullet 1}}{s_{\bullet 1}} & \frac{s_{\bullet 2}}{s_{\bullet 2}} & \dots & \frac{s_{\bullet k}}{s_{\bullet k}} & \frac{s_{\bullet n}}{s_{\bullet n}}
\end{array}\right]$$

The matrix A and vector L derived from the simple SAM from the example in Figure 4 are indicated below:

Figure 4: Matrix A and vector L. Example

	0.455	0.250	0.556
A =	0.273	0.375	0.333
	0.182	0.125	0.000
L =	0.091	0.250	0.111

The total receipts in each account *i*, can also be expressed in terms of coefficients of matrix A, bearing in mind that the total receipts in account *i* is equal, by definition, to the sum of expenditure in each account to this account *i*. In mathematical terms this can be expressed as follows:

$$s_{i\bullet} = \left[a_{i1}s_{\bullet 1} + a_{i2}s_{\bullet 2} + \dots + a_{ik}s_{\bullet k}\right] + d_i \qquad (4)$$

or:

$$s_{i\bullet} = \sum_{j=1}^{k} a_{ij}s_{\bullet j} + d_i \qquad (i=1\dots k)$$

(row total of the account *i*) = (endogenous components) + (exogenous comp.)

For instance, receipts in account 1 are equal to the sum of the percentage of expenditure in each account to account 1 multiplied by the total expenditure in each account.

110	= (0,455	x 110)	+ (0,250	x 80)	+ (0,556	x 45)	+ 15
Total Receipts account 1	%expenditure Account 1 to account 1	Total expend. account 1	% expenditure account 2 to account 1	Total expend. account 2	% expenditure account 3 to account 1	Total expend. account 3	Exogenous demand to account 1

This also holds for the total of the exogenous account:

$$s_{n\bullet} = \sum_{j=1}^{k} a_{nj} s_{\bullet j} + d_n \qquad (5)$$

For example, receipts in the exogenous account (government) of the SAM in Figure 2 result from the following calculation:

37	= (0,091	x 110)	+ (0,250	x 80)	+ (0,111	x 45)	+ 2
Total receipts exogenous account	% expenditure account 1 to exogenous account	Total expendit. account 1	% expenditure account 2 to exogenous account	Total expendit. account 2	% expenditure account 3 to exogenous account	Total expend it. accoun t 3	Exogenous expendit. to exogenous account

It must be noted that, all relationships shown above until this point have been considered accounting identities, verified by any SAM (provided that the row totals equal column totals). For example, in a SAM with exogenous components different to those in matrix S in Figure 3, row and column totals and/or payments and receipts in each account will be different. As a result, the coefficients will also be different. Nevertheless, the relationships shown in the box will always be verified.

5.4. From accounting identities to model equations

A model based on SAM basically answers the question, what is the level of income (and expenditure) in each endogenous account generated by a certain level of exogenous expenditure and, also, what level of "leaks" is associated with a certain level of income (and expenditure) in each endogenous account.

The accounting identities illustrated in section (4) form the basis of the equations which link the exogenous expenditure and the level of endogenous income and expenditure.

In order for these identities to be used as a basis for economic model equations it is necessary that:

a) expenditure equals income in endogenous accounts $s_{\bullet h} = s_{h\bullet}$. This total income (and expenditure) will be called X_i *(i=l,... k)* or vector *X;*

b) total expenditure and income in endogenous accounts are considered endogenous variables X_i *(i=l,...k).*

c) coefficients a_{ij} be considered fixed. That means that the average expenditure coefficients, from each account, must be calculated based on the SAM as parameters which also reflect marginal expenditure changes;

d) assume that relationships between endogenous variables and exogenous variables are linear (hypothesis of lack of substitution between different inputs and factors for all productive sectors and between different final goods for all institutions);

e) assume that each item of exogenous expenditure is met by a supply of goods and services from the economic system, in other words, that the economic system will not encounter constraints in terms of productive capacity (hypothesis of surplus productive capacity);

f) assume that prices of goods and services used to express values in the SAM do not change because of the impact of changes in exogenous demand (hypothesis of fixed prices).

Based on the previous hypotheses, the economic model can therefore be represented by a system of simultaneous linear equations:

$$X_1 = [a_{11}X_1 + a_{12}X_2 + ... + a_{1k}X_k] + d_1$$
$$X_2 = [a_{21}X_1 + a_{22}X_2 + ... + a_{2k}X_k] + d_2$$
……………………………………
$$X_k = [a_{k1}X_1 + a_{k2}X_2 + ... + a_{kk}X_k] + d_k$$

where in more compact notation:

$$X_i = \sum_{j=1}^{k} a_{ij}X_j + d_i \quad (i,j = 1,\ldots.k)$$

The solution from the model will therefore provide, for example, values for the demand of goods, levels of activity, factor remuneration, household income etc., for a given level of payments in the exogenous accounts, for example, government consumption, or purchases from abroad, transfers to households etc., paid into endogenous accounts.

The system presents k unknowns X_i (*i=l,..., k) and* k linear equations in the variables. If no single equation is a linear combination of the others, the system should be solvable.

An additional relationship, which enables "leaks" to be calculated at any level of income in endogenous accounts, is derived from the relationship (5):

$$X_n = \sum_{j=1}^{k} a_{nj}X_j + d_n$$

5.5. The model's solution

The algebraic solution for the stated exogenous expenditure can be found using different linear system solution methods.

It can be solved using matrix algebra as proposed below.

In matrix notation, the equation system can be written as follows:

$$\underset{(k\,x1)}{X} = \underset{(k\,x\,k)}{A}\;\underset{(k\,x1)}{X} + \underset{(k\,x1)}{D}$$

where, the dimensions of the matrix (meaning already illustrated above) are specified in brackets. That produces:

$$X - A X = + D$$

$$(I - A) X = + D$$

$$(I - A)^{-1}(I - A) X = (I - A)^{-1} D$$

$$X = (I - A)^{-1} D$$

The solution of the system is therefore the product of the inverse matrix $M = (I - A)^{-1}$ and of the vector of exogenous expenditure *D*.

The matrix $M = (I - A)^{-1}$ is known as the ***multipliers' matrix***, because, analytically, it enables the effects of exogenous expenditure to be transmitted to the economic system through a process of "multiplying" impacts which follow an iterative circuit of production, distribution and use of income.

Meanwhile, a level of leakages, namely from exogenous accounts, is associated with each level of endogenous account:

Example

Based on the simple SAM in Figure 4, reported in panel A of Figure 5, we would like to calculate the impact on household income (policy objective) of a public purchasing policy of agricultural output, through a 1 mu increase of exogenous agricultural demand (policy instrument).

To identify the impact of the envisaged policy measure X (pol), on household income, vector X must be calculated again, for which the third component is household receipts (or expenditure).

The table in Figure 5, panel B, shows the exogenous components vector in the base situation, the change induced by the policy measure and the new exogenous components vector.

Figure 5: Impact analysis of policy measures

Panel A: the SAM

	Agricult	Industry	Households	Government	total
Agricult	50	20	25	15	110
Industry	30	30	15	5	80
Households	20	10	0	15	45
Government	10	20	5	2	37
total	110	80	45	37	272

Panel B:Change in exogenous demand

	D WoP	D(delta) Change	D WiP
Exogenous agricultural edmand	15	1	16
Exogenous industrial demand	5	0	5
Exogenous income of households	15	0	15

Panel C: Unknown value

	X(r) WoP	X(delta) change	X(pol) WiP
Total agricultural demand	110	?	?
Total industrial demand	80	?	?
Total households income	45	?	?

Panel D: Policy impact

	X(r) WoP	X(delta) change	X(pol) WiP
Total agricultural demand	110.0	3.818	113.82
Total industrial demand	80.0	2.182	82.18
Total households income	45.0	0.967	45.97

New vector X (pol) in panel D in the Figure 5 was calculated by solving the system of equations:

$$X_{pol} = (I - A)^{-1} D_{pol} ;$$

This requires that the following route be followed:

a) calculate the matrix (I-A) by taking the difference between matrix I (identity matrix) and coefficients' matrix A :

$$I = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$A = \begin{matrix} 0.455 & 0.250 & 0.556 \\ 0.273 & 0.375 & 0.333 \\ 0.182 & 0.125 & 0.000 \end{matrix}$$

I - A =	0.545	-0.250	-0.556
	-0.273	0.625	-0.333
	-0.182	-0.125	1.000

b) calculate the inverse matrix $M = (I - A)^{-1}$. 7

$(I-A)^{-1}$ =	3.818	2.091	2.818
	2.182	2.909	2.182
	0.967	0.744	1.785

c) calculate the product $X_{pol} = (I - A)^{-1} D_{pol}$ which produces[8]:

X(pol) =	113.82
	82.18
	45.97

Changes in vector X are analysed in panel D in Figure 5.

It must be noted that the change in vector X, $\Delta X = (X_{pol} - X_{WoP})$, i.e.

$\Delta X_1 = 3{,}818$; $\Delta X_2 = 2{,}182$; $\Delta X_3 = 0{,}967$ corresponds exactly to the first column in the multiplier matrix.

This is not by chance. The multipliers matrix actually reports, column by column, the impact of unitary changes in exogenous receipts of each account on the totals of all other accounts.

The multipliers matrix can therefore be read, column by column, as follows:

The matrix element m_{11} shows the change in monetary units of the **total receipts** in the first account due to a unitary change in **exogenous receipts** in the same account;

The matrix element m_{21} shows the change in monetary units of the **total receipts** in the second account due to a unitary change in **exogenous receipts** in the first account;

The matrix element m_{12} shows the change in monetary units of the **total receipts** in the first account due to a unitary change in **exogenous receipts** in the second account;

The matrix element m_{34} shows the change in monetary units of the **total receipts** in the fourth account due to a unitary change in **exogenous receipts** in the third account;

[7] The conditions for having an inverse matrix and methods for calculating it are set out in the appendix to this module.
[8] The methods for calculating matrix products are set out in the appendix to this module.

In general, the matrix element m_{ij} shows the change in monetary units of the **total receipts** in account *i* due to a unitary change in **exogenous receipts** in account *j*.

We can see immediately, based on the inverse matrix *M* in the example, that a unitary exogenous increase in demand for industrial products (*j*=2) would produce an increase in household income (*i*=3) of 0.744 monetary units.

Note that a change in vector X also generates a change in vector of leakages.

6. References

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