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Methodological issues in the estimation of the prevalence of undernourishment based on dietary energy consumption data: A review and clarification

By Loganaden Naiken

Abstract:

Sukhatme had in the early 1960’s originally formulated the estimate of the proportion of undernourished in a population (PU) within a bivariate distribution framework where dietary energy consumption (DEC) and dietary energy requirement (DER) are considered as random variables. However, in the absence of data on DEC and DER of individuals expressed in the form of bivariate distribution, Sukhatme had suggested a formula that considers the part of the distribution of DEC below a cut-off point representing the lower limit of the distribution of DER as an estimate of PU. However, this univariate approach has been criticised as yielding an underestimate of the magnitude of the prevalence undernourishment in a population. In response to this critic, Sukhatme has attempted to justify the approach by invoking the theory of intra-individual changes in DER. As this theory has led to a controversy rather than a clarification of the univariate approach, doubts regarding its validity still prevail. Following a review of these developments including the concept of DER, this article shows that the formulation of PU within the bivariate distribution framework is inappropriate. Subsequently, the relevance of the univariate approach is clarified. Finally, the article addresses certain issues relating to practical estimation of the prevalence measures based on household rather than individual data pertaining to DEC.

Keywords: adequately nourishment; dietary energy requirement (DER); undernourishment; dietary energy consumption (DEC); depth of undernourishment; index of food security; over-nourishment.

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1. Background and Introduction

FAO has been traditionally estimating the prevalence of undernourishment in a population (PU) on the basis of observed food energy consumption or intake data, i.e. dietary energy consumption (DEC), and normatively specified dietary energy reference intake, i.e. dietary energy requirement (DER). In this context, both DEC and DER are expressed in terms of number of kilocalories (kcal) per individual/day and adequate nourishment is defined as the state whereby an individual’s observed DEC matches his or her DER, i.e. DEC=DER. Consequently undernourishment and over-nourishment refer to DEC<DER and DEC>DER respectively. Thus the prevalence of undernourishment, refers to the proportion of individuals with DEC<DER. This measure, which is considered to be as an indicator of food insecurity, has been adopted by the UN as one of the indicators for tracking the Millennium Development Goals.

The approach of measuring undernourishment on the basis of food expressed in terms of dietary energy (kcal) alone is justified from two perspectives. Firstly, a certain amount of dietary energy is essential for the maintenance of the body-weight and work performance of an individual. Secondly, an increased amount of dietary energy, if derived from normal staple foods, brings with it more protein and other nutrients as well, while raising the latter, without ensuring a certain amount of dietary energy, is unlikely to be of much benefit in terms of meeting food needs. In other words, dietary energy is an appropriate measure of the amount of food consumed by individuals. The distinction between the measure of undernourishment based on food and those based on anthropometric indices such as weight-for-age, weight-for-height, height-for-age and body mass index (BMI) is that while the former reflects the phenomenon food deprivation or hunger per se, the latter reflect the physical consequences of both food deprivation and adverse health and environmental conditions.

The estimation of PU faces problems on the side of both DEC and DER. On the one hand, nationally representative data sets pertaining to observations of DEC at the individual level are practically nonexistent. On the other hand, DER is not an observed fact but a normatively derived measure that is subject to random variation. However, information on DEC corresponding to a sample of households from the population is derivable from the food consumption data collected in national consumption/expenditure surveys. Hence, assuming the availability of such data and information pertaining to the mean and standard deviation of DER, Sukhatme, while being the Director of the FAO Statistics Division, had attempted to estimate PU in connection with a paper presented at a joint meeting of the Royal Statistical Society and the Nutrition Societies of London (Sukhatme, 1961). The methodology, which implies the definition undernourishment as the state whereby an individual’s DEC is below a minimum DER level has been adopted by FAO in preparing its’s estimates of the prevalence of undernourishment in the world. However, some aspects of the methodology, mainly
concerning the consideration of the random component of the variation in DER and the definition of undernourishment are still subject to debate and controversy. Certain issues arise also in deriving the estimate based on household rather than individual consumption data from sample surveys. All these issues raise doubts about the validity of the methodology adopted by FAO. Therefore the aim of this paper is to clarify these issues and consequently justify the FAO approach.

The paper is divided into six main sections in addition to the present one. Section 2 reviews and clarifies the concept of DER and its variation. Section 3 refers to the statistical posed in attempting to estimate the prevalent of undernourishment. Section 4 introduces the probability distribution framework for estimating of $PU$, thus providing the opportunity to present the bivariate distribution formula that Sukhatme initially indicated as a mathematical expression for $PU$ as well as the formula that he actually proposed and applied in the early 1960’s (henceforth for convenience referred to as the “univariate distribution formula”). Section 5 discusses the criticism of the univariate distribution formula originally raised by Lörstad (1974) and includes a discussion of the theory of intra-individual variation in DER that Sukhatme later proposed in response to this criticism as well as the resulting controversy and belief in the superiority of the bivariate distribution formula. Section 6 clarifies the problem in the formulation of $PU$ within the bivariate distribution framework, and subsequently explains the relevance of the univariate distribution framework which enables the derivation of not only $PU$ but also the prevalence of adequate nourishment ($PA$) and over-nourishment ($PO$). Subsequently two additional measures relating to the nutritional situation status are presented. Section 7 discusses the approach regarding the estimation based on household DEC data from sample surveys data and in this connection clarifies an issue raised by an alternative approach proposed by researchers at the International Food Policy Research Institute (IFPRI). Section 8 concludes the paper and refers to the way forward.

### 2. The concept of DER and its variation

As DER is not an observed fact and the methodological issues being clarified in this article mainly concern the treatment of it’s random variation in the estimation of $PU$, it is appropriate to begin with a review and clarification of this concept and its variation.

At the international level DER is derived according to principles established by expert groups on nutritional requirements. It actually refers to the dietary energy expenditure (DEE) needed by individuals in order to remain healthy and physically active. However, as according to the law of thermodynamics energy output must equal energy input, the past practice of the international expert groups on nutritional requirements was to base the DER on studies of DEC (energy input) corresponding to reference groups composed of apparently well
nourished individuals having fixed sex, age, and physical activity and body-weight (FAO, 1957 and FAO, 1973). However, because of the presence of random day-to-day variation in DEC, the daily DECs were averaged over a number days (a week) to reflect usual DEC and hence DER per individual/day.

However, even after averaging the daily energy intakes of each individual in the reference group over a number of days to reflect usual DEC, significant differences remained between the usual DECs of the individuals in the reference group. The variation in DER refers to these inter-individual differences. Widdowson (1947) had hypothetically attributed this variation among similar individuals to differences in the efficiency of energy utilisation, i.e. some individuals metabolise energy more or less efficiently than others and hence need less or more energy as compared to others. However, the specified DER actually referred to the overall average DEC in the reference group with the dispersion around this average referring to the inter-individual differences.

Sex, age, body-weight and physical activity are factors that systematically affect the level of DER. Therefore, the practice of basing DER on the usual DEC of well-nourished individuals having fixed sex, age, body-weight and physical activity reflects an attempt to exclude the effects of these known systematic factors on the DER. As the effects of the systematic factors have been removed, it follows that the remaining inter-individual variation may be considered to be of random nature. Hence, if we assume the DER in a population, composed of individuals of the same sex, age body-weight and physical activity, to be normally distributed we may be express it as follows:

\[ R = \mu_R + C \]

\[ C \approx N(0, \sigma^2_C) \] ........................ (1)

where \( \mu_R \) is the specified average DER and \( C \) is a random term which, according to Widdowson, represents the effect of the unknown efficiency of energy utilisation.

The sex-age specific average DERs recommended in the past by the international expert groups actually started with the specification of \( \mu_R \) for individuals of the “reference man” and “reference woman” types. The reference man and woman referred to an adult male and female respectively in age group 20 – 29 living in a climate with mean annual temperature of 10\(^\circ\) C with fixed body-weight (65 kg. for males and 55 kg. for females) and performing moderate physical activity. By adjusting the averages for the “reference man” and “reference woman” to account for different states and situations such as growth in children, pregnancy and lactation for women in the reproductive age groups, age, climate etc, the averages for individuals in the other sex-age groups were obtained.

However, the practice of basing the DER on the usual DEC of reference groups composed of apparently well nourished individuals and specifying the averages for reference man and woman was reviewed by the FAO/WHO/UNU Expert Consultation on Energy and Protein
Requirement that met in 1981 (WHO, 1985). The Expert Consultation felt that this approach implied the assumption that the individuals’ respective body-weights and physical activity levels were consistent with good health and productive activity but this might not have been so in reality. Moreover, basing the DER on the concepts of reference man and woman having fixed body-weight and physical activity (moderate) was considered to be too restrictive in a world where there is a wide range of body-size as well as physical activity level that are consistent with good health and productive physical activity among individuals in a given sex-age class.

In view of the above, the Expert Consultation introduced major changes in the definition and derivation of DER. The DER was thus defined as the “energy intake level that will balance energy expenditure when individuals have a body-size and physical activity level that are consistent with good health and that will allow for the maintenance of economically necessary and socially desirable physical activity”. This definition clearly implied that “body-size and physical activity level that are consistent with good health and that will allow for the maintenance of economically necessary and socially desirable physical activity” became the basis for the derivation of DER. Moreover, it meant that the DER for individuals in a given sex-age class should be based on normatively specified body-weight and physical activity rather than the actual ones. Hence the Expert Consultation discontinued the approach based on the DEC of well nourished individuals and, instead, expressed DER in terms of the DEE needed to maintain normatively specified body-weight and physical activity. In line with this new approach, the Expert Consultation formalized the estimation of the DER by sex-age classes for adolescents and adults in terms of the DEE needed for maintaining normatively specified body-weight and a multiplying factor to account for the normatively specified physical activity level (referred to as the PAL index). However, for children the Expert Consultation was unable to make similar recommendations and hence provided sex-age specific average DERs based on the DEC of healthy and active children in the developed countries.

The FAO/WHO/UNU Expert Consultation on Human Energy Requirements that met in 2001 (FAO, 2004) consolidated the normatively derived DEE approach and extended it to children also. For the purpose of practical application, the Expert Consultation provided a set of sex-age specific regression equations linking DEE with body-weight (expressed in terms of kg) for the derivation of the estimates of the DEE needed for body weight maintenance for all sex-age groups with the exception of infants and children. For the latter, the regression equations provided for the derivation of the total energy expenditure (TEE) needed. The body-weight norms refer to the range of acceptable weight-for-age (for children) and the range of acceptable weight for attained height (for adolescents and adults) adopted in deriving anthropometric indices of undernutrition and overnutrition. As regards the energy needed for physical activity performance, the approach for adults differed from that for adolescents. In the case of adults, three PAL indices reflecting sedentary, moderate and vigorous lifestyles are given to reflect the range of variation in the physical activity norm.
while in the case of adolescents, the suggested approach was to reduce and increase the DEE based on the body-weight norms by 15% to reflect sedentary and vigorous lifestyles respectively.

Thus, the FAO/WHO/UNU Expert Consultation that met in 2001 provided the basis for the specification of the range of DER rather than simply the average for groups of individuals in the different sex-age classes. Nevertheless, the DER in a population composed of individuals in a given sex-age class can still be represented by expression (1) but with the random term \( C \) now representing the combined effect of the unknown ideal body-weight and physical activity.

It should be noted that according to the new approach the inability to specify the DER of an individual of given sex and age is due to the fact that the ideal body-weight or physical activity of an individual is unknown and there is no objective way to assign a particular combination of body-weight and physical activity within the normatively specified ranges to an individual. Another point is that although data on actual body-weight and physical activity of individuals may be available it would be mistake to use them in attempting to specify the DEE corresponding to individuals. This is because the DEE based on an individual’s actual body-weight and physical activity level is a reflection of the individual’s usual DEC rather than the DER as a norm or reference value for DEC.

The consequence of the de-linkage of the DER from the actual body-weight and physical activity of individuals is that, except for the effect of the actual average height factor used to determine the range of the normatively specified body-weight, the range of DER for individuals by sex-age groups is not expected to differ from country to country. In other words, the average height of the individuals in each sex-age group is the only country specific information taken into account in specifying the range of DER by sex-age groups for a population.
3. **The Statistical Problem in attempting to estimate PU**

Because of the differences in the DER of individuals, undernourishment has been heuristically defined as the state whereby an individual’s observed DEC during certain reference period (measured as the average DEC over the period) is below the individual’s own DER. Hence the prevalence of undernourishment refers to the proportion of individuals in a population with DEC below the individuals’ respective DERs. However, while the DEC of an individual may – at least in principle – be known, this is not so in the case of the DER. As indicated in the previous section, DER is not specified as a fixed point for an individual but by sex-age groups as range for the individuals in each group. Moreover, the data pertaining to the DEC of individuals (or households) normally emanate from a probability sample taken from the population rather than a census or complete enumeration. In view of these, the classification of individuals in the undernourished, adequately nourished and over-nourished categories can be made in the probability rather than absolute sense. This in turn means that the evaluation of PU needs to be conceived within a probability distribution framework.

4. **Sukhatme’s Probability Distribution Framework for Evaluating PU**

As the specified range of DER refers to individuals classified by sex and age, we shall assume here that that (a) the population is composed of individuals who are homogenous or similar with regard to sex and age and (b) data pertaining to DEC have been collected at the individual level. In Section 6, which considers the estimation of PU based on DEC data pertaining to households rather than individuals, we shall address the issue of dealing with a population composed of individuals in different sex-age classes.

As PU has been heuristically defined as being the proportion of individuals with DEC below their respective DERs, Sukhatme (1961) had initially expressed it as the probability for the DEC of the individuals in the population to be below their respective DERs and hence he had formulated it within a continuous bivariate distribution framework as follows:

$$PU = \int_{x<r} f_{XR}(x,r) \, dx \, dr$$

where \((X,R)\) is a bivariate random variable representing DEC and DER respectively, \(f_{XR}(x,r)\) is the joint density function and \(x<r\) refers to the event of an individual’s DEC being below the individual’s DER.

The above means that PU was equated to \(\text{Prob}(X<R)\). However, Sukhatme himself had noted that, since DER is not specified at the individual level, the information pertaining to \(X\) and \(R\) cannot be expressed in the form of a bivariate distribution and hence the heuristic definition
of undernourishment as the state whereby an individual’s DEC is below his or her own DER cannot be applied. Therefore, as at that time the specified DER referred to the mean rather than the range, he exploited the fact that some information pertaining to the standard deviation was available and proposed as a practical device the following formula involving the distribution of $X$ and a cut-off point representing the minimum DER:

$$PU= \int_{x < (\mu_R - 3\sigma_R)} f_X(x) \; dx = \text{Prob}\{X < (\mu_R - 3\sigma_R)\} = F_X(\mu_R - 3\sigma_R) \tag{6}$$

where $f_X(x)$ is the density function of $X$, $F_X(x)$ is the distribution function of $X$ and the cut-off point, $\mu_R - 3\sigma$, is an estimate of the lower limit of the range of $R$, i.e. the minimum DER, under the assumption that the distribution of $R$ is normal with mean $\mu_R$ and standard deviation $\sigma_R$.\(^2\)

In arriving at the above univariate distribution formula for $PU$, Sukhatme had argued that, in a healthy and active population with everyone adequately nourished, i.e. a population where $f_X(x) = f_R(r)$ and hence $\text{Prob}(X=R)=1$, one would expect no more than 1 per cent of the population to have a DEC that is below the level corresponding to $\mu_R - 3\sigma_R$. Therefore, in a population where undernourishment exists and hence $f_X(x) \neq f_R(r)$, $F_X(\mu_R - 3\sigma_R)$ represents an estimate of $PU$. In a later paper, Sukhatme (1973) had indicated that $\mu_R - 2\sigma_R$ would be more appropriate than $\mu_R - 3\sigma_R$ as the estimate of the lower limit of the range of $R$. However, as in the new approach $R$ is specified in terms of a range rather than $\mu_R$, it is more appropriate to refer to the lower limit as $r_L$ and hence express (6) as

$$PU= \int_{r_L}^{\infty} f_X(x) \; dx = \text{Prob}(X < r_L) = F_X(r_L) \tag{7}$$

Thus, according to the above formulation, undernourishment is defined as the state whereby an individual’s DEC is below a threshold representing the minimum DER rather than the unknown actual DER of the individual. This definition however ensures that the individual is practically in all probability undernourished.

5. **Issues Concerning the Formulation of $PU$ as $\text{Prob}(X < r_L)$ rather than $\text{Prob}(X < R)$**

FAO has in fact adopted the univariate distribution formula in preparing its estimates of $PU$. Nevertheless, this formula is still subject to debate and criticism. Part of the problem is that Sukhatme’s justification of the formula was not clear. However, the main criticism is that, by considering only those with DEC below the lower limit of the range of DER as undernourished, the univariate distribution formula ignores the likelihood of an individual’s DEC being below the individual’s own DER despite being within the range of DER. Hence it is argued that the formula is bound to underestimate $PU$. Lörstad (1974), a statistician

\(^2\) Note that in Sukhatme’s study individuals of different sex and age were converted into “reference man” equivalents and hence $X$ and $R$ actually referred to DEC and DER in a population composed of “reference man”.

working in the Food and Nutrition Division of FAO in the early 1970’s was the first to bring up and investigate into the issue. Sukhatme (1982) had responded by invoking his controversial theory of intra-individual changes in DER in order to justify the $r_L$ threshold. This section discusses Sukhatme’s theory, the subsequent controversy and the persistent belief in the superiority of the bivariate distribution formula.

5.1. Sukhatme’s theory of intra-individual changes in DER

It should be pointed out from the outset that Sukhatme’s theory of intra-individual changes in DER was conceived in the context of the past approach of basing the DER on data corresponding daily series of DEC of healthy and active individuals of the reference type. As indicated in section 2, the approach was to average daily DEC over a number of days in order to reflect habitual DEC. This approach, which implied ignoring the within individual day-to-day variation and hence considering individual DER as a being a fixed point (i.e. the mean over the day-to-day variation) was contested by Sukhatme (1982). His argument was that it is the intra-individual day-to-day changes that should be considered in explaining the variation in the DER of individuals of given sex, age, body-weight and physical activity.

He had examined the above issue by analyzing a data set consisting of a series of daily energy intake (DEC) and DEE (as measured by a calorimeter) reported by Edholm et al (1970) for a reference group composed of a number of healthy army recruits maintaining bodyweight and engaged in similar physical activities. This data set enabled an analysis of the daily DEC and DEE of the well nourished separately and in terms of the difference between DEC and DEE. Through a variance analysis carried out on the data, he had noted that when averaging both the daily DECs and daily DEEs over a period of five consecutive days the period-to-period differences between the two averages for the same subject did not disappear, but persisted. He took this as a suggestion that the day-to-day variation in the DEC of the individuals is related to the day-to-day variation in energy expenditure in such a way that the law of thermodynamics (i.e. DEC = DEE) is respected every day in the probability sense. Hence he concluded that the day-to-day changes in the DEC of the individuals of reference type “arises from intra-individual variation which is stochastic stationary in character, thereby meaning that requirement is dynamic and self-regulated and not static as assumed in nutrition literature”. He interpreted this intra-individual variation as being a reflection of the capacity of an individual to regulate his or her energy balance over a range of DEC (the range of homeostasis) by varying his or her efficiency of energy utilization in the same manner as shown in an earlier study pertaining to nitrogen by Sukhatme and Margen (1978). This meant that what Widdowson had interpreted as being due to inter-individual differences actually arises from intra-individual changes.

Thus Sukhatme had interpreted the variation in DER in the dynamic rather than static sense and thus resulting in the consideration of $\mu_R - 2\sigma_R$ and $\mu_R - 2\sigma_R$ as being the limits of the
range of variation. Hence an individual should be considered as probably adequately nourished as long as his or her DEC lies within these limits. Therefore the reason for excluding the individuals with DEC falling within the range of variation of DER in the estimation of $PU$ is that they are likely to be adequately nourished.

Sukhatme had estimated the coefficient of variation (CV) of the day-to-day changes in the intakes of the individuals within the reference group to be about 0.2. He therefore stated that “man’s requirement for any day or period is not fixed but dynamic, adapting itself to DEC over a fixed range from 60 to 140 per cent of the average DER”.

5.2. The controversy surrounding Sukhatme’s theory of intra-individual changes in DER

Sukhatme’s interpretation of the variation in DER from the perspective of the day-to-day changes in the DEC of the individuals in the reference group had led to a major controversy in the literature (James et al, 1989 and Osmani, 1992) specially as the conventional practice in food and nutritional assessments has been to ignore the day-to-day variation in DEC and hence adopt the usual or habitual concept. However, it is his interpretation of the variation in DER as being a reflection of the capacity of an individual to modify his or her efficiency of energy utilization in response to changes in DEC over a wide range (without detriment to health or function) that has been the subject of debate and criticism. The debate was not on whether such a variation exist or not but whether the range of variation could be as wide as that postulated by Sukhatme. Actually, such intra-individual variation is considered to be of negligible importance (James and Schofield, 1990).

In fact no reference to Sukhatme’s analysis and conclusion has been made in the reports of either the 1981 or 2001 FAO/WHO/UNU Expert Consultations on nutritional requirement. This is probably because the approach of basing the DER on daily series of DEC in reference groups composed of apparently well nourished individuals has been discontinued. As indicated in section 2, beginning with 1981 FAO/WHO/UNU Expert Consultations on nutritional requirements the DER is derived directly by sex-age groups on the basis of norms for body-weight and physical activity.

5.3. The persistent belief in the superiority of the bivariate distribution formula

The negligible importance attached to Sukhatme’s of interpretation of the variation in DER in the context of the day-to-day changes in the DEC of well nourished individuals and the persistent belief that the $\text{Prob}(X<r_L)$ formulation as adopted by FAO leads to an underestimation of $PU$ have led other scholars (who have attempted to address the problem) to favour the evaluation of $\text{Prob}(X<R)$ as initially formulated by Sukhatme (Osmani, 1992; Kakwani, 1992; Anand and Harris, 1992, Svedberg, 2003 and Anriquez et al, 2012). Attempts have also been made to apply this formulation (Lörstad, 1974, Kakwani, 1992 and Svedberg, 2003). In this connection, the problem of absence of bivariate data pertaining to $X$
and $R$ has been circumvented by assuming that the joint distribution of $X$ and $R$ or the conditional distribution of $R$ given $X=x$ is normal or lognormal and considering the distributions of $X$ and $R$ to be the marginal distributions. The unknown correlation parameter was given an imputed value since simulations undertaken have shown that $PU$ was rather insensitive to this parameter.

However, the estimates of $PU$ resulting from the above mentioned experiments have proved to be too high to be realistic. For example Lörstad (1974) has shown on the basis of distribution data on DEC for Burma reported in Sukhatme’s original article (Sukhatme, 1961) that the estimate, while being practically insensitive to correlation, was consistently very large as compared to that obtained through the univariate distribution formula. It varied between 66% and 72% for correlation coefficients ranging from 0 to 0.8 while the estimate obtained from the application of the univariate formula was only 33%.


The persistent belief in the superiority of the bivariate distribution formula (5) which invariably leads to an estimate of $PU$ that is much higher than that resulting from the univariate formula (7) adopted by FAO is obviously a matter of concern. Therefore this section will first address the problem with the formulation of $PU$ within the bivariate distribution framework and then justify the relevance of the univariate distribution framework. Subsequently, the issue of inter-individual versus intra-individual variation of DER will be clarified and the derivation of the related measures of depth of undernourishment and correlation between DEC and DER within the univariate distribution framework will be explained.

6.1 The problem with the bivariate distribution framework

In the bivariate distribution framework $Prob(X<R) + Prob(X>R) = 1$ and hence $Prob(X=R) = 0$. Thus the bivariate distribution formula for $PU$ implies the denial of the existence of the probability for a population to be adequately nourished and consequently an individual is considered to be always either undernourished or over-nourished. This in turn means that both $PU$ and the prevalence of over-nourishment ($PO$) will be overstated since each is bound to include part of the population that should be considered as likely to be adequately nourished.

The above problem is not due to the adoption of a continuous distribution framework as indicated by some scholars (Cafiero and Gennari, 2011) but, as explained below, the fact that the bivariate distribution is not the appropriate probability framework for considering the estimation of $PU$. 
Firstly, in the bivariate distribution framework the DER of an individual is treated as being a known fixed point just as in the case of the observed DEC. However, the fact of matter is that the DER is not specified at the individual level but at the population level as a range for the individuals in the population. This obviously means that the DER of an individual needs to be treated as being a range of possible points rather than a known fixed point (see section 6.3 for an elaboration of this argument).

Secondly, the consideration of a bivariate probability distribution framework is relevant in the context of an assessment of the relationship between two different variables, e.g. height and weight. As the two variables are different, it is obvious that the probability of their being equal does not exist or is irrelevant. However, the situation is not the same in the present context. The estimation of \( PU \) implies the comparison of observed DEC with DER rather than the assessment of the relationship between them.

Thirdly and more importantly, the fact that adequate nourishment is defined by the event DEC=DER means that DER refers to the DEC of the adequately nourished individuals. Hence DEC and DER in essence refer to the same variable, i.e. DEC, with the only distinction being that while the former refers to an observed fact the latter is a counterfactual conditioned by the event of the individual being adequately nourished. In fact, as will be indicated in the next subsection, the possible values of DER will be replicated in the possible values of DEC and thus leading to the probability of DEC being equal to DER. Hence DEC and DER cannot be treated as being a bivariate random variable and consequently \( PU \) cannot be formulated within a bivariate distribution framework.

6.2 The justification of the univariate distribution framework

The fact that DER refers to adequate DEC means that the possible values of \( X \) are bound to include the possible values \( R \). In other words the possible values of \( R \) will be replicated in the possible values of \( X \). Consequently, the joint distribution of \( X \) and \( R \) reduces to the probability distribution corresponding to the replicates of the values of \( R \) in \( X \), i.e. the event \( x=r \). This in turn means that the joint distribution reduces to a distribution of \( X \) that is identical to the fixed distribution of \( R \), i.e. \( f_X(x) = f_R(r) \), so that \( x=r \) for all \( x \) and hence \( \text{Prob}(X=R) = 1 \).

Thus, the joint distribution defines the condition for a population to be considered as adequately nourished in the probability sense. In view of this a credible probability distribution framework for evaluating the nutritional status at the population level must consider \( PU \) and \( PO \) as being the result of a departure from this condition of perfect nourishment.

Given that \( f_R(r) \) is fixed and the condition for a population to be adequately nourished in probability terms is \( f_X(x) = f_R(r) \), it follows that a departure from this perfect state through the occurrence of mismatching events, i.e. \( x<r \) and \( x>r \), must lead to a flattening of the \( f_X(x) \) curve and an extension of its range outside that of \( f_R(r) \) as shown in Figure 1 below.
Figure 1: The probability distribution framework for estimating \( PU, PA \) and \( PO \)

Note that \( \mu_X \) and \( \mu_R \) are shown as being equal for the purpose of graphical representation as well as demonstrating the fact that undernourishment and overnourishment may exist even when the means of the two distributions are equal. The position of \( r_L \) and \( r_U \) on the x-axis is a reflection of the fact that in practice they represent approximations rather than the true limits of the range of \( R \).

As the part of \( f_X(x) \) over the range of \( R \), i.e. \( f_X(r) \), refers to the probability distribution associated with the event \( x=r \), it follows that the parts below and above the range of \( R \) must refer to the probability distribution associated with events \( x<r \) and \( x>r \) respectively. Hence the prevalence of adequate nourishment in the population \( (PA) \) should be expressed as \( \text{Prob}(x=r) \) and hence derived as an integral of \( f_X(x) \) over the range of \( R \). Consequently, the prevalence of undernourishment \( (PU) \) and the prevalence of overnourishment \( (PO) \) should be expressed as \( \text{Prob}(x<r) \) and \( \text{Prob}(x>r) \) respectively with the former evaluated as the integral of \( f_X(x) \) up to the lower limit of the range of \( R \) and the latter as the integral of \( f_X(x) \) above the upper limit of the range of \( R \).

In practice the lower and upper limits of the range of \( R \), i.e. \( r_L \) and \( r_U \) respectively, are not derived precisely. Hence one has to consider the estimates of \( PU \), \( PA \) and \( PO \) as being approximative rather than precise. We may therefore formulate them as follows:

\[
PU = \text{Prob}(x<r) \approx \int_{-\infty}^{r_L} f_X(x) \, dx = \text{Prob}(X<r_L)
\]

\[
PA = \text{Prob}(x=r) \approx \int_{r_L}^{r_U} f_X(x) \, dx = \text{Prob}(r_L<X<r_U)
\]

\[
PO = \text{Prob}(x>r) \approx \int_{r_U}^{\infty} f_X(x) \, dx = \text{Prob}(X>r_U)
\]
If $R$ is assumed to be normally distributed it follows that $r_L$ and $r_U$ could be approximated by $\mu_R - 2\sigma_R$ and $\mu_R + 2\sigma_R$ respectively. However, the fact that in the present case $R$ is specified in terms of a range rather than $\mu_R$ and $\sigma_R$ means that we can directly derive approximations of these limits.

6.3 Assessment at the individual level and the inter-individual versus intra-individual variation in DER issue

Although the role of cut-off points $r_L$ and $r_U$ in defining the range within which DEC should be considered as probably adequate has been justified in the context of an inference made at the population level, they are equally applicable in the context of an inference at the individual level, i.e. in deciding whether an individual’s observed DEC should be considered as being inadequate, adequate or excessive.

At the individual level, as indicated earlier, the possible values of $R$, i.e. the values ranging from $r_L$ to $r_U$, need to be interpreted as referring to the possible values of the unknown individual DER. This implies the treatment of the random variation in DER as being intra-individual. However, this intra-individual variation does not imply that an individual’s DER can change from one point to another within the range of $R$ as postulated by Sukhatme but rather that the unknown individual DER is likely to be located at any point within the range. In other words, the different values of $R$ have equal probability of being the individual’s DER. Hence an individual’s DEC should be considered as probably adequate if it falls anywhere between $r_L$ and $r_U$. Thus an individual can be considered as undernourished only if his or her DEC is below $r_L$ and over-nourished only if it is above $r_U$. However, while the condition of adequate nourishment for those with DEC between $r_L$ and $r_U$ is only probable it is practically certain with regard to undernourishment and over-nourishment for those with DEC below $r_L$ and above $r_U$ respectively since by definition an individual’s DER cannot be below $r_L$ and above $r_U$.

Thus given unordered DEC data corresponding to a sample of individuals from the population and the limits of $R$, i.e. $r_L$ and $r_U$, $P_U$, $P_A$ and $P_O$ can also be derived by classifying the individuals in the three categories of nutritional status and dividing the totals in each category by the number of individuals in the sample as formulated below.

$$PU = \frac{\sum (x_i < r_L)}{n}$$

$$PA = \frac{\sum (r_L < x_i < r_U)}{n}$$

$$PU = \frac{\sum (x_i > r_U)}{n}$$
where \( x_i \) refers to the DEC corresponding to individual \( i = 1 \ldots n \) in a sample of \( n \) individuals, the events \((x_i < r_L), (r_L < x_i < r_U)\) and \((x_i > r_U)\) are registered as 1 and the summation is over all \( n \) individuals.

It follows from the above discussion that the issue of whether the random variation in DER needs to be treated as being inter-individual or intra-individual depends on whether the inference on nutritional status is being considered at the population or individual level. At the population level, where the inference is based on the distributions of DEC and DER, the possible values of \( R \) are assumed to represent the DERs of different individuals in the population and hence the variation is treated as resulting from inter-individual differences. However, at the individual level, where the inference is based on the DEC of single individuals, the fact that an individual’s DER is unknown dictates the consideration of the possible values of \( R \) as being the possible values of the individual’s DER. Hence the variation is treated as being intra-individual.

6.4 The average depth of undernourishment

The depth of undernourishment refers to the food deficit or food gap of the undernourished. The food deficit is defined as the difference between the DEC and DER of an undernourished individual. As an individual’s DER is unknown it follows that the depth of undernourishment cannot be calculated at the individual level.\(^3\) However, the fact that the average DER of the population can be estimated means the depth of undernourishment can derived at the population level by considering the difference between the average DEC of the undernourished sub-population and the average DER of the population, i.e. \( \mu_R \).

As, according to the new approach, the specified DER refers to the range of \( R \) rather than \( \mu_R \), we may consider it as being the average DEC of the adequately nourished in the probability sense, i.e. the mean of \( f_X(x) \) given \( r_L < X < r_U \), by virtue of the fact that practically all the values of \( R \) are replicated in this sub-population. Hence, we may formulate the average depth of undernourishment as follows:

\[
DU = \mu_X (\text{adequately nourished}) - \mu_X (\text{undernourished})
\]  

(15)

where \( \mu_X (\text{undernourished}) \) and \( \mu_X (\text{adequately nourished}) \) are formulated as follows:

\[
\mu_X (\text{undernourished}) = \int_{r_L}^{r_U} \frac{x \, dx}{\int f_X(x) \, dx}
\]  

(16)

\(^3\) Note that, although \( r_L \) has been used as the cut-off point for classifying an individual’s defining undernourishment, it does not refer to the DER of the undernourished and hence cannot be used to derive the depth of undernourishment. Moreover, as the possible values of \( R \) have equal probability of being an individual’s DER, the average DER, i.e. \( \mu_R \) cannot be adopted since it does not represent the most probable estimate.
Needless to say, the above refers to the case where the observed sample data on DEC in the population are expressed in terms of a continuous frequency function, i.e. $f_X(x)$. Otherwise, $\mu_X$ (undernourished) and $\mu_X$ (adequately nourished) may be estimated on the basis of sample data directly classified into the undernourished and adequately nourished categories as indicated in the previous subsection.

### 6.5 The correlation between DEC and DER as an indicator of food security

The fact that, as indicated earlier, the joint distribution of $X$ and $R$ reduces to a distribution of $X$ that is identical to the distribution of $R$ means that it is equivalent to the distribution of $R$. This in turn mean that the covariance of $X$ and $R$ reduces to the variance of $R$. It therefore follows that that the coefficient of correlation $\rho$, is of the intra-class type (ICC) and is derived as follows:

$$
\rho = \frac{\text{Cov}(X,R)}{\sigma_R \sigma_X} \\
= \frac{\sigma^2_R}{\sigma_R \sigma_X} \\
= \frac{\sigma_R}{\sigma_X}
$$

Thus, as the distribution of $R$ and hence $\sigma_R$ is fixed, $\rho$ provides a measure of degree to which $X$ is similar to $R$. As the latter refers to DEC in an adequately nourished and hence food secure population, it follows that the $\rho$ indicates the degree to which the actual distribution of DEC in a population is similar to or reflects the situation in a food secure population. As such it may be considered as an indicator of food security. The closer is $\rho$ to 1 the greater is the food security.

The variance $\sigma^2_R$ is not specified but, as it refers to the region of $X$ overlapping the range of $R$, we may estimate it by computing the variance of $X$ corresponding to adequately nourished sub-population as follows:

$$
\sigma^2_R = \frac{r_U}{\int_{r_U}^{r_L} f_X(x)dx / \int_{r_L}^{r_U} f_X(x)dx} \int_{r_U}^{r_L} (x - \mu_X(\text{adequately nourished}))^2 f_X(x)dx
$$

(18)

where $\mu_X$ is estimated as per formula (16).

Thus given unordered sample data on individual DEC, $r_L$ and $r_U$, the ICC can be easily calculated on the basis of $\sigma^2_X$ and $\sigma^2_R$, estimated directly on the basis of the DEC
corresponding to the whole sample and the part classified into the adequately nourished category respectively.

7. Estimation Based on Household DEC Data from Sample Surveys

It follows from the previous discussion that the estimation of $PU$, $PA$, $PO$, $DU$ and $\rho$ require sample data on the DEC of individuals classified by the sex-age classes and estimates of $r_L$ and $r_U$ corresponding to each sex-age class. Given these, the individuals can be classified in the three categories of nutritional status and hence the sex-age specific estimates of the relevant measures could be derived and the results combined to arrive at the estimates for the total population. In this way, the latter estimates would take into account the effect of sex-age differentials in nutritional status.

However nationally representative sample surveys, where the individual is the unit of data collection for food consumption, are practically non-existent. The only sources of nationally representative data are the household budget or consumption and expenditure surveys that provide information on DEC pertaining to households. This means that it is not possible to derive the relevant estimates by sex-age classes and thereby taking into account the sex-age differentials in nutritional status. Nevertheless, the assessment of nutritional status of a population on the basis household DEC data is useful since it is by and large at the household or family level that food is acquired, shared and consumed by individuals. This section describes the approach in using the household level data from national household consumption surveys for the purpose of assessing the nutritional status of a population.

7.1. The approach involving the expression of household DEC on per individual basis

In using the household DEC data for making inference on the nutritional status of individuals, the household DEC may be individualized by expressing it on per individual basis (through the division of the household total by the number of individuals in the household). This in effect implies the representation of the household members’ DEC by by the household per average. A necessary assumption made in this approach is that the intra-household variation of DEC reflects the intra-household sex-age differential in DER and hence the individuals in the household have the same nutritional status. This means that any difference in nutritional status due to sex or age that may exist is disregarded.

The above approach actually implies the modification of the consumption unit from the household to the average individual implied by the expression of aggregates on per capita basis. Consequently, the population is assumed to be composed of equivalent average individuals. Hence the cut-off points, $r_L$ and $r_U$, should also refer to this unit. As the cut-off points are specified by sex-age groups, it follows that they should be derived as the average of the sex-age specific $r_L$ and $r_U$ with the proportion of the population in the different sex-age
groups as weight. The derivation of the two weighted averages, referred to as the minimum dietary energy requirement (MDER) and maximum dietary energy requirement (XDER) respectively, is described below.

7.1.1. Derivation of MDER and XDER

As indicated in section 2, for practical application, the FAO/WHO/UNU Expert Consultation on Human Energy Requirement (FAO, 2004) provided a set of equations for the estimation of the energy expenditure for body maintenance based on body-weight norms (expressed in kg) for all sex-age groups. As regards physical activity three alternative PAL indices reflecting sedentary, moderate and vigorous lifestyles to account for physical activity are given. Therefore, the 5th and 95th percentiles of the distribution of weight for attained height in the WHO reference population have been taken as the body-weights to derive the energy expenditure for body maintenance corresponding to \( r_L \) and \( r_U \) respectively. In the case of children, the 5th and 95th percentiles referred to the distribution of weight for age in the WHO reference population. As regards physical activity for the adult’s sex-age group(s), the sedentary lifestyle is adopted to specify \( r_L \) and the vigorous lifestyle for XDER in the case of \( r_U \). For adolescents the energy expenditure derived on the basis of body-weight is reduced by 15% to arrive at \( r_L \) and increased by 15% in the case of \( r_U \).

Having derived the sex-age specific \( r_L \) and \( r_U \), MDER and MDER are obtained as follows:

\[
MDER = \sum_{j} r_{Lj} p_j
\]

\[
XDER = \sum_{j} r_{Uj} p_j
\]

where \( r_{Lj} \) and \( r_{Uj} \) refer to \( r_L \) and \( r_U \) corresponding to the \( j \)th sex-age class, \( p_j \) refers to the proportion of the population in the \( j \)th sex-age class and the summation is over all the relevant sex-age classes.

One may use the sex-age distribution of the individuals in all the households sampled to derive \( p_j \). However, as household members’ characteristics in terms of sex and age are not sampling criteria in the household consumption surveys, they may not be reliable for inference on the distribution in the population. Hence, it is preferable to derive \( p_j \) from the usual sources of demographic statistics.
Thus, given MDER and XDER and the household per individual DEC data corresponding to a sample of households, the later could be classified into the three nutritional categories and hence $PU$, $PA$ and $PO$ derived by dividing the absolute numbers in the three categories by the total number of households in the sample. The means and variances needed for estimating $DU$ and $\rho$ could also be derived directly on the basis of the DEC of the households classified in the relevant categories.

7.1.2 Estimation based on the parameters of the distribution of DEC

FAO has however adopted an approach that implies an inference at the population level on the basis of the parameters of the underlying distribution of DEC rather than through the classification the individual observations in the sample in the three categories of nutritional status. This approach involves the following steps:

- Constructing the relative frequency distribution of household DEC per individual on the basis of the sample data so that the random variable $X$ is represented by household DEC per individual.
- Fitting a continuous theoretical distribution to the derived empirical distribution of $X$
- Given the estimated parameters of the fitted distribution, the estimates of the different measures of nutritional status can be derived as per the formulae presented in Section 6.2 but with $r_L$ and $r_U$ replaced by MDER and XDER respectively

The above “parametric” approach has the following advantages:

- The preparation of projections of $PU$ (and, needless to say, as well as $PO$ and $PA$) is facilitated by the expression of the distribution of DEC in terms of its parameters;
- The household DEC derived from the household food consumption data collected in many surveys are often subject to a number of issues that tend to exaggerate the variance and consequently inflate the prevalence of undernourishment for the chosen reference period of the estimates. In such cases the parametric approach enables the introduction of corrections by adjusting the relevant parameter(s) of the fitted distribution.
- For a number of countries the relevant survey data are not available. On the other hand, information pertaining to the national per capita DEC derived through the FAO food balance sheets is available for practically all countries. As the latter represent the mean of the distribution of DEC in the population, the parametric approach facilitates the use of these means in the global estimation process. This is because the other needed parameter(s) are derivable through ad hoc procedures (FAO, WFP and IFAD, 2012). FAO studies based on household survey data pertaining to DEC have been published separately (FAO, 2008 and FAO, 2012)
As regards the theoretical distribution to be fitted, tests undertaken by FAO in the past on the basis a few available survey data sets have shown that the lognormal distribution provides a reasonably good fit to the observed distribution. In view of this, FAO had previously adopted the lognormal distribution so that the only the mean and the variance need to be estimated from the sample data. However, recently the lognormal distribution was replaced by the skew-normal distribution in order to allow for the possibility of changes in the skewness (in particular the lower skewness as implied in the lognormal distribution). This is also mandated by the fact that over the years the mean DEC has increased sizably for most countries. Consequently, unless one would find a very low coefficient of variation, the lognormal with high mean DEC would imply excessive mass under the upper tail of the distribution (FAO, WFP and IFAD, 2012).

7.2 Estimation through the application of household specific cut-off points

The above approach of modifying the consumption unit from the household to the average individual and applying a cut off based on the sex-age composition of the population has been criticized for ignoring the effect differences in the sex-age composition of households in the estimation process. Hence, in an International Food Policy Research Institute (IFPRI) report, Smith, Alderman and Aduayom (2006) had proposed an approach for estimating $PU$ that involves the following steps:

- Specifying $r_L$ corresponding to each household member by taking into account sex and age
- Summing the $r_L$ of the individual household members to arrive at the total for the household
- Comparing each household’s DEC with its $r_L$ and identifying the households with DEC below their respective $r_L$ and
- Summing the individuals in all the households where household DEC is below the household’s $r_L$ and dividing the sum by the number of individuals in all households.

The IFPRI report presented estimates of $PU$ using the above approach on the basis of household DEC data derived from surveys carried out in 12 countries in Sub-Saharan Africa. There are a number of issues relating to the reliability of the household level DEC data from these surveys as well as the specification of $r_L$ and hence the resulting estimates of $PU$ (Sibrian, Naiken and Mernies, 2007). However, the focus here is on the newly proposed approach as compared to the approach previously discussed rather than the reliability of the data used and the cut-off point chosen.
As the approach applies household specific cut-off points rather than a single cut-off point based on the sex-age composition of the population (i.e. MDER), it is claimed to take into account the effect of differences in the sex-age composition of the households and hence represents an improvement in the method of estimation. However, this is an illusion rather than a reality. Although the approach takes into account the sex and age of the household members in classifying the household DEC as being in the inadequate category, the fact that all the members of the thus classified households are considered as undernourished clearly imply that the intra-household sex-age differences in nutritional status is ignored, just as it is in the case where household DEC is expressed on per individual basis. For this reason, as shown in the IFPRI report itself (Smith, Alderman and Aduayom, 2005), the results for PU are practically the same as those resulting from the application of the approach based on household DEC expressed on per individual basis and the application of a single cut-off point if the same household survey data set and the same figures for the sex-age specific \( r_L \) are adopted. The fact of the matter is that, as the DEC data refer to households rather than individuals, it is impossible to consider the effect of differences due to sex and age on PU.

Furthermore, although the approach leads to practically the same results, it’s validity from a purely theoretical point of view is questionable. This is because it implies an inference on the prevalence of undernourishment at the household level based on the sex and age characteristics of the individual household members which are not sampling criteria for the DEC data collected in the survey and therefore are not reliable for inferential purposes.

It follows from the above discussion that the proposed approach does not really represent an improvement as compared to the approach based on household DEC expressed on per individual basis.

8. Concluding Remarks and Way Forward

The issues or concerns relating to the methodological framework for estimating the prevalence of undernourishment has been plagued by misunderstandings and confusion partly stemming the fact that Sukhatme had originally considered the \( \text{Prob}(X<r_L) \) formulation for PU, as being a practical device in the absence of bivariate data for the evaluation of \( \text{Prob}(X<R) \). This had given the impression that the bivariate distribution formula is the appropriate one. His subsequent justification of the \( \text{Prob}(X<r_L) \) formulation by invoking the theory of intra-individual changes in DER in effect meant that his original formulation within the bivariate probability distribution framework was irrelevant. However, the fact that the theory of intra-individual changes in DER itself had led to a controversy rather than an explanation of the relevance of the univariate distribution formula, had kept alive the myth of
the bivariate distribution formula and thus encouraging some scholars to attempt its evaluation by resorting to models and assumptions.

The paper has clarified the problem with the formulation of $PU$ within the bivariate distribution framework and shown the relevance of univariate distribution formula by highlighting the need for a probability distribution framework that recognises the existence of a probability for adequate nourishment in addition to undernourishment and over-nourishment. The article has also clarified the related measure of the depth of undernourishment and the fact that the correlation coefficient in the univariate distribution framework measures the degree to which $X$ is similar to $R$ and hence is useful as an index of food security.

For the purpose of the trichotomous division of the population and the derivation of the related measures, it is necessary to obtain reliable estimates of the lower and upper limits of the range of DER by sex-age classes. However, the problem is that the guidelines for the application of the principles recommended by the international expert groups on nutritional requirements have so far tended to focus on the average DER {James and Schofield (1990) and FAO (2004)} which is not useful as cut-off point for estimating $PU$ or $PO$. In other words, there are no formal guidelines regarding the derivation of the two cut-off points needed for arriving at these estimates.

In connection with its efforts to estimate $PU$, FAO had relied on the advice of informal technical groups convened by the Statistics Division for the specific purpose of defining the lower limit of the range of DER in the population. The technical group convened in 2005 for considering the approach in the light of the principles set by the 2001 FAO/WHO/UNU Expert Consultation on Human Energy Requirements had in fact made recommendations regarding not only the lower limit but also the upper limit. This has paved the way for the eventual estimation of the prevalence of adequate nourishment and over-nourishment also. However, it is important for the relevant international organizations to set up an international group of experts to consider and make firm recommendations regarding these limits and subsequently issue appropriate guidelines to countries for the specific purpose of estimating the relevant measures of nutritional status.

The paper has also addressed the issue arising from the use of household rather than individual DEC data from national sample surveys and the approach taken when the household level data collected reflect reliable measures of the usual consumption of households. In this connection, it has also clarified that the approach suggested by Smith, et al is actually not an improvement as compared to the approach involving the expression of household DEC data on per individual basis.
References


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