

## Chapter 3

# Computation of flows for on-demand irrigation systems

One of the most important problems for an on-demand irrigation system designer is the calculation of the discharges flowing into the network. Such discharges strongly vary over time depending on the cropping pattern, meteorological conditions, on-farm irrigation efficiency and farmers' behaviour.

In this paper, each group of hydrants operating at a given instant is called “hydrants’ configuration”. Each hydrant’s configuration produces a discharge configuration (or flow regime) into the network. The term “node” includes both hydrants and junctions of two pipes, whereas the term “section” is used to describe the pipe connecting any two nodes.

The design capacity is usually determined considering short-term peak demand and considering an average cropping pattern for the whole system. But, the individual cropping pattern may differ from the designed one, and the irrigation system may be either undersized or oversized.

In view of the difficulty of this problem, empirical methods have been used. For example, the US Bureau of Reclamation (1967) recommends solving each case on individual basis and gives only general indications like: the maximum demand may generally be estimated at 125-150% of the average demand. Systems operating for a 12- month season may require a capacity large enough to carry from 10 to 15% of the total annual demand in the peak month. Those operating for a 7-month season may require a capacity large enough to carry from 20 to 25% of the total annual demand during the peak month. However, the actual maximum demand should be determined by detailed analysis of individual projects.

The advent of on-demand, large-scale irrigation systems in the early 1960s in France fostered the development of statistical models to compute the design flows. Examples of such models are the first and the second Clément formula (1966). But only the first demand formula has been widely used because of its simplicity.

Although these models are theoretically sound, the assumptions governing the determination of their parameters do not take into account the actual functioning of an irrigation system. In view of these limitations a number of researchers tackled the problem by simulating irrigation strategies. As an example, Maidment and Hutchinson (1983) modeled the demand pattern over a large irrigation area taking into account the size of irrigated area, the soil type, the cropping pattern, the irrigation strategy and the weather variation. However, they had to average out the demand hydrograph over time to avoid unrealistic very high water demand one day and very low the next.

Recently other approaches have been developed combining simulation of irrigation strategies, based on the soil water balance, and statistical models (Abdellaoui, 1986; Walker *et al.*, 1995; Teixeira *et al.*, 1995). The result of these methods is a single distribution of one design flow for

each pipe section of the network. They will be referred to in the following as One Flow Regime Models (OFRM).

OFRM do not actually take into account the hydraulic functioning in an on-demand collective irrigation network. Indeed, in such systems there is occurrence of several flow regimes according to the spatial distribution of the hydrants that are simultaneously in operation. Therefore, improving the design and the performance of an irrigation system operating on-demand requires consideration of these flow regimes in the design process. The new approach, called Several Flow Regimes Models (SFRM), is based on this concept. In this chapter OFRM will be reviewed before presenting the process of generating flow regimes for the SFRM.

## ONE FLOW REGIME MODELS (OFRM)

### Statistical models: the Clément models

One study that deals explicitly with calculation of pressurized irrigation systems capacity for on-demand operation is the work of Clément (1966). Two different models were proposed. One (called the first Clément model) is based on a probabilistic approach where, within a population of  $R$  hydrants, the number of hydrants being open simultaneously is considered to follow a binomial distribution. The other (called the second Clément model) is based on simulating the irrigation process as a birth and death process in which, at a given state  $j$  ( $j$  hydrants open), the average rate of birth is proportional to  $(R-j)$  and the average rate of death is proportional to  $j$ . The Clément models, although based on a theory, were extensively used for designing sprinkler irrigation systems in France, Italy, Morocco and Tunisia.

### The first Clément model

#### Background equations

In on-demand irrigation systems, the nominal discharge of the hydrants ( $d$ ) is selected much higher than the duty,  $D$  (the duty is the flow based on peak period water requirement on a 24-hour basis:  $D = q_s A_p$ , where  $q_s$  is the continuous specific discharge and  $A_p$  is the area of the plot irrigated by the hydrant). It allows farmers to irrigate for a duration lower than 24 hours. This condition implies that the event to find all the hydrants simultaneously operating has very low probability. Thus, it is not reasonable to calculate the irrigation network by adding the discharges delivered at all the hydrants simultaneously. Consequently, probabilistic approaches for computing the discharges into the sections of an on-demand collective network have been widely used in the past and are still used actually.

The most utilized is the probabilistic approach proposed by Clément (1966) and it is summarized hereafter.

Let  $q_s$  be the specific continuous discharge, 24 hours per day ( $l s^{-1} ha^{-1}$ ),  $A$  is the irrigated area (ha),  $R$  the total number of hydrants,  $d$  the nominal discharge of each hydrant ( $l s^{-1}$ ),  $T$  the duration of the peak period (h),  $T'$  the operating time of the network (h) during the period  $T$ ,  $r$  the coefficient of utilization of the network (defined as the ratio  $T'/T$ ). The average operation time  $t'$  of each hydrant during the peak period (h) is then

$$t' = \left( \frac{q_s A T}{R} \right) / d \quad (1)$$

The elementary probability,  $p$ , of operation of each hydrant is defined as

$$p = \frac{t' - t'}{T' - rT} = \frac{q_s AT}{Rd - rT} \quad (2)$$

thus,

$$p = \frac{q_s A}{r R d} \quad (3)$$

Therefore, for a population of  $R$  homogeneous hydrants, the probability of finding one hydrant open is  $p$ , while  $(1 - p)$  is the probability to find it closed.

The number of operating hydrants is considered a random variable having a binomial distribution with mean

$$\mu = R p \quad (4)$$

and variance

$$\sigma^2 = R p (1 - p) \quad (5)$$

Therefore, the cumulative probability,  $P_q$ , that among the  $R$  hydrants there will be a maximum of  $N$  hydrants simultaneously operating is:

$$P_q = \sum_{K=0}^N C_R^K p^K (1 - p)^{(R-K)} \quad (6)$$

where:

$$C_R^K = \frac{R!}{K!(R - K)!} \quad (7)$$

is the number of combinations of  $R$  hydrants taken  $K$  at a time. When  $R$  is sufficiently large ( $R > 10$ ) and  $p > 0.2-0.3$ , the binomial distribution approximates the Laplace-Gauss normal distribution whose cumulative probability ( $P_q$ ) for having a maximum of  $x$  hydrants simultaneously operating (with  $-\infty < x < N$ ) is:

$$P_q = \frac{1}{\sqrt{2p}} \int_{-\infty}^{U(P_q)} e^{-\frac{u^2}{2}} du \quad (8)$$

where  $U(P_q)$  is the standard normal variable corresponding to the probability  $P_q$ , and  $u$  is the standard normal deviate given by:

$$u = \frac{x - R p}{\sqrt{R p (1 - p)}} \quad (9)$$

The integral (8) is solved developing in series the exponential function  $e^{-\frac{u^2}{2}}$ . The solutions of this integral have been tabulated (see Table 1) and so, according to a prefixed value  $P_q$ , it is possible to determine the corresponding value  $U(P_q)$ .

Knowing  $U(P_q)$ , it is possible to calculate the number of hydrants simultaneously operating,  $N$ , through the relationship (9). In fact, for  $u = U(P_q)$  we have:

$$N = R p + U(P_q) \sqrt{R p (1 - p)} \quad (10)$$

that is the first formula of Clément.

Considering hydrants with the same discharge, the total discharge downstream a generic section  $k$  is given by:

$$Q_k = R p d + U(P_q) \sqrt{R p (1-p) d^2} \quad (11)$$

and, for different discharges of hydrants ( $d_i$ ), where  $I$  is the hydrant class number

$$Q_k = \sum_i R_i p_i d_i + U(P_q) \sqrt{\sum_i R_i p_i (1-p_i) d_i^2} \quad (12)$$

The first Clément model is based on three major hypotheses that limit its applicability (CTGREF, 1974; CTGREF, 1977; Lamaddalena and Ciollaro, 1993).

- The first hypothesis concerns the parameter  $r$ . It is defined by Clément as coefficient of utilization of the system in the sense that, during the design phase, the duration of the day for irrigation, within the peak period, is considered shorter than 24 hours. This parameter, defined at the network level in an irrigation system operating on-demand, should have a value equal to one because these systems may have to work 24 hours per day. In practice, the parameter  $r$  should correspond to the operating time of each hydrant and, therefore, it is not correct to use it for the global design of the system. Nevertheless, from a conceptual point of view, it may be considered as a parameter which helps adjusting the theoretical formulation to a homogeneous population of discharges, withdrawn in the field, appropriately chosen through a statistical approach. It must be pointed out that the Clément model, like all other models, only offers a schematic representation of an actual network. Therefore, it must be adjusted or calibrated by introducing field data relative to existing networks. In particular, values of the parameter  $r$  should be, whenever possible, selected for homogeneous regions and for particular crops. An example of the field Clément model calibration, for an Italian irrigation network, is reported in Annex 1.
- The second hypothesis concerns the elementary probability of opening each hydrant. It refers to an estimation of the average operating time of each hydrant. But, the probability to find a hydrant working at a given time  $t$  depends on its state at the previous time  $t-1$ . In order to justify the binomial law, this probability should characterize a series of events such that when the farmer opens his hydrant at a time  $t$ , he would close it after a laps of time  $dt$ , and he decides to re-open or to leave it closed at a successive time  $t+dt$ , and so on. This is not real because a farmer opens his hydrant and leaves it in the same state for a large number of laps of time  $dt$ . Moreover, the elementary probability varies during the day according to the farmer's behaviour.
- The third hypothesis considers the independence of the hydrants and their random operation during the peak period. This hypothesis might seem justified because the farmers should behave autonomously and not according to the operation of the neighbour farmers. Nevertheless, the rhythm of nights and days and the similitude of the crops within an irrigation district condition the farmer's behaviour, so that this hypothesis is not fully reliable.

The importance of the  $r$  coefficient is stressed also in Figure 10. In this figure two parameters are defined: the elasticity of the network,  $e_r$  (Clément and Galand, 1979):

$$e_r = \frac{Q_{Cl}}{q_s A} \quad (13)$$

and the average elasticity of the hydrants (called also farmers "degree of freedom"),  $e_h$ ,

**TABLE 1**  
Standard normal cumulative distribution function

$P_q$	$U(P_q)$
0.90	1.285
0.91	1.345
0.92	1.405
0.93	1.475
0.94	1.555
0.95	1.645
0.96	1.755
0.97	1.885
0.98	2.055
0.99	2.324

$$e_h = \frac{R d}{(q_s A)} \quad (14)$$

The ratio  $e_R$  is a measure of the over-capacity of the network and is a characteristic of on-demand operation. The ratio  $e_h$  defines the freedom afforded to farmers to organize their irrigation.

The values of  $e_R$  refer to a network designed to supply equal flows at all hydrants. When the hydrant design flows are unequal, the values of the ratio are slightly greater. Nevertheless, whether the hydrants are homogeneous or not, taking into account the probability of the demand being spread results in a network peak design flow which is very much smaller than that which would be obtained by summing the flows at all hydrants.

The degree of freedom that is to be afforded to farmers should be selected according to criteria such as size and dispersion of plots, availability of labor, type of on-farm equipment, frequency of irrigation. Hydrants with capacities of one and a half to twice the value of the duty correspond to the lowest feasible degree of freedom. With smaller values, the probability of an hydrant being open becomes too great for the demand model to apply. Conversely, hydrant capacities should not exceed six to eight times the value of the duty. This corresponds to a very high degree of freedom.

Figure 10 illustrates the variation of the elasticity of the network,  $e_R$ , versus the total number of hydrants,  $R$ , for different values of the elasticity of the hydrants,  $e_h$ . The curves have been drawn for  $U(P_q)=1.645$ . Considering a value of elasticity at the hydrant  $e_h = 4.5$ , for a network having  $R=100$  hydrants, the elasticity of the network  $e_R$  varies from about 1.43 (corresponding to  $r = 0.9$ ) to about 2.03 (corresponding to  $r = 0.6$ ). It means that the upstream discharge in an on-demand network (from Eq. 13):

$$Q_{cl} = e_R q_s A \quad (15)$$

may increase about 45% if a coefficient  $r=0.6$  is chosen instead of a coefficient  $r=0.9$ .

Furthermore, from Figure 10 it can be seen that, for on-demand systems, the ratio of the peak flow in the network to the assumed continuous flow (elasticity of the network) increases as the number of hydrants decreases. With hydrant capacities two to four times greater than the duty, by selecting  $r=0.9$  the peak flow in a network having 100 hydrants is only 27 to 40 percent greater than the continuous flow; while by selecting  $r=0.6$  the peak flow in a network having 100 hydrants is 79 to 98 percent greater than the continuous flow. It means that the coefficient  $r$  has much more influence on the design capacity of the network respect to the elasticity of the hydrants. Therefore, in order to give more freedom to farmers, it is more appropriate to select higher hydrant elasticity.

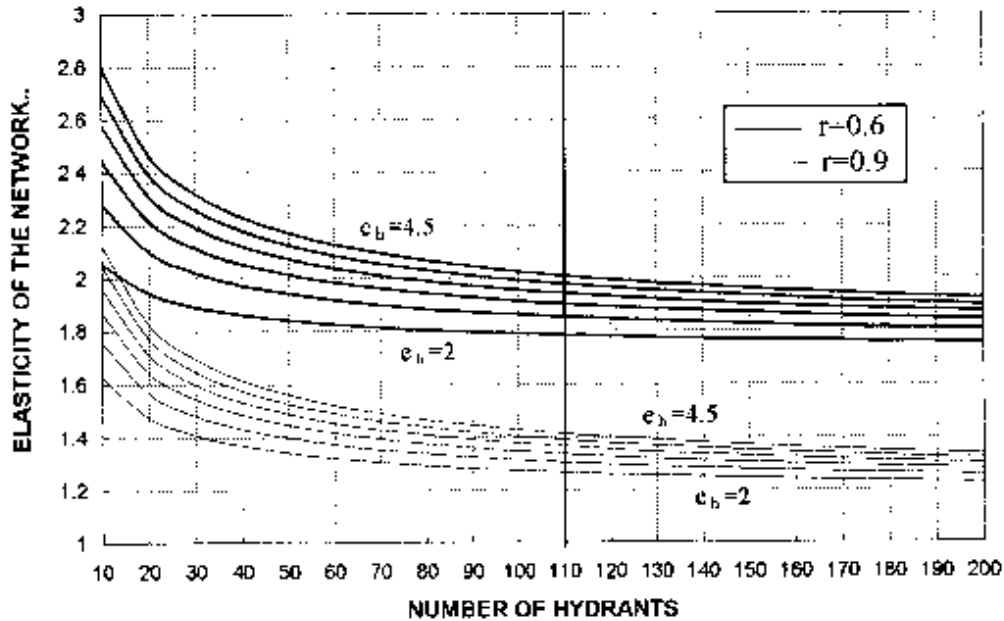
The values selected for the parameter  $r$  normally lie between 16/24 ( $r=0.67$ ) and 22/24 ( $r=0.93$ ). The performance analysis of existing networks is the most reliable approach for selecting the coefficient  $r$  best suited to a given irrigation context.

The parameter  $U(P_q)$  defines the "quality of operation" of the network; it normally has a values ranging from 0.99 to 0.95. It is hardly possible to go below a value of 0.95. A significant reduction of this parameter beyond these values can lead to the occurrence of unacceptable failures to satisfy the demand in certain parts of the network (Galand *et al.*, 1975).

In view of the hypotheses made when formulating the on-demand model it is recommended that a deterministic approach be adopted at the extremities of the network by cumulating the flows at the hydrants when their number falls below a certain value which, in practice, lies between four and ten.

**FIGURE 10**

Variation of the elasticity of the network,  $e_R$ , versus the total number of hydrants for different values of  $r$  and  $e_h$ , for  $U(P_q) = 1.645$ .



In certain cases it may happen that the calculated discharge of a section serving five or six hydrants is less than that of the downstream section serving four hydrants whose flows have been summed. In this case the discharge in the upstream section will be equal the discharge in the downstream section.

#### *Determination of the specific continuous discharge*

In order to apply the above methodology it is necessary to know the value of the specific continuous discharge,  $q_s$  ( $l\ s^{-1}\ ha^{-1}$ ) in the network downstream of the section under consideration. Its value can readily be determined when:

- The cropping pattern is identical throughout the area. If this is so the specific continuous discharge,  $q_s$  ( $l\ s^{-1}\ ha^{-1}$ ), estimated by giving due weight to each of the crops, holds good for every farm and all branches of the network under consideration.
- The cropping intensity is identical throughout the area. When this is so, the ratio between the net irrigated area and the gross area also holds good for every holding and all parts of the network under study.

A number of computer packages are available for such a computation (CROPWAT, ISAREG, etc.), as well as an extensive literature. Therefore, the reader is referred to them for its calculation.

#### *Discharge at the hydrants*

Although a farmer supplied by an on-demand system is free to use his hydrant at any time, a physical constraint is nevertheless imposed as regards the maximum flow he can draw. This is achieved by fitting the hydrant with a flow regulator (flow limiter). The discharge attributed to each hydrant is defined according to the size and crop water requirements of the plot. It is

always greater than the duty so as to give the farmer a certain degree of freedom in the management of the irrigation.

The ratio between the discharge attributed to each hydrant and the duty is a measure of the "degree of freedom" which a farmer has to manage irrigation. The wide variety of agronomic situations is reflected by the wide range of the value of the degree of freedom found in practice (FAO-44, 1990):

- High degree of freedom: family holdings with limited labour, low crop water requirements, small or scattered plots, low investment level in on-farm equipment;
- Low degree of freedom: large size plots, large scale farming, abundant labour, high investment level in on-farm equipment.

Since the maximum flow at hydrants is fixed by flow regulators it is usual to opt for a standard range of flows. Such ranges vary from country to country.

In southeastern France, for instance, a range of six hydrants has been standardized, corresponding to the following discharges:

Class of hydrant	0	1	2	3	4	5
Discharge (l s <sup>-1</sup> )	2.1	4.2	8.3	13.9	20.8	27.8

In Italy the range of hydrants corresponds to the following discharges:

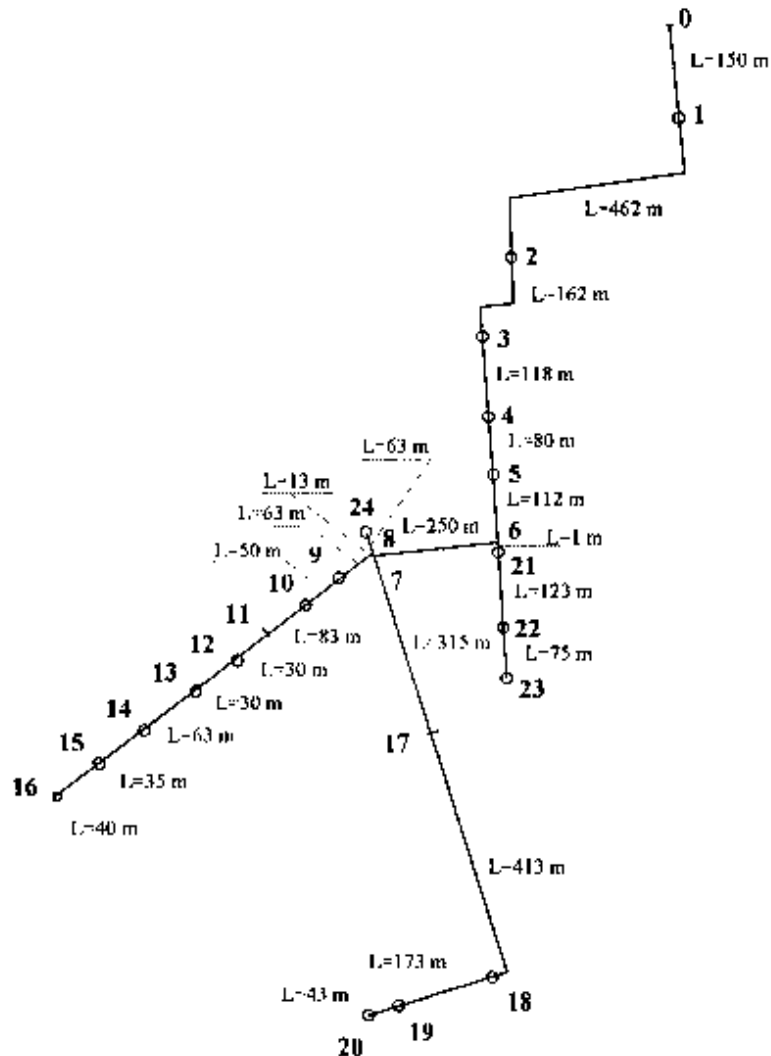
Class of hydrant	0	1	2	3	4	5
Discharge (l s <sup>-1</sup> )	2.5	5	10	15	20	25

In Box 1, an example of the Clément formula is worked out and the results are shown in Table 2 (generated by COPAM).

**TABLE 2**

**Discharges flowing into each section of the network under study (output of the COPAM package: computation with the first Clément model)**

Section Number	Initial Node	Final Node	Number of Hydrants	Area (ha)	1st Clément discharge (l/s)
1	0	1	19	57.00	60.00
2	1	2	18	54.00	60.00
3	2	3	17	51.00	50.00
4	3	4	16	48.00	50.00
5	4	5	15	45.00	50.00
6	5	6	14	42.00	50.00
7	6	7	11	33.00	40.00
8	7	8	8	24.00	40.00
9	8	9	7	21.00	40.00
10	9	10	6	18.00	40.00
11	10	11	5	15.00	40.00
12	11	12	5	15.00	40.00
13	12	13	4	12.00	40.00
14	13	14	3	9.00	30.00
15	14	15	2	6.00	20.00
16	15	16	1	3.00	10.00
17	7	17	3	9.00	30.00
18	17	18	3	9.00	30.00
19	18	19	2	6.00	20.00
20	19	20	1	3.00	10.00
21	6	21	3	9.00	30.00
22	21	22	2	6.00	20.00
23	22	23	1	3.00	10.00
24	8	24	1	3.00	10.00

**BOX 1: APPLICATION OF THE FIRST CLÉMENT MODEL FOR THE COMPUTATION OF FLOWS IN AN ON-DEMAND NETWORK**


Data:

$q_s$  = specific continuous discharge (24/24 hours) =  $0.327 \text{ l s}^{-1} \text{ ha}^{-1}$

$A_p$  = area of the plot to be irrigated = 3 ha

$A$  = total irrigable area =  $3 \cdot 19 = 57 \text{ ha}$

$d$  = nominal discharge of the hydrants =  $10 \text{ l s}^{-1}$

$U(P_q)$  = operation quality = 1.645

$r$  = use coefficient = 0.667

$R$  = total number of hydrants = 19

$N_{ba}$  = minimum number of hydrants in simultaneous operation = 4

\*\*\*\*\*



**BOX 1 Cont'd**

$$p = \text{elementary probability} = \frac{q_s A}{rRd} = \frac{0.327 \cdot 57}{0.667 \cdot 19 \cdot 10} = 0.147$$

1) Number of hydrants simultaneously operating downstream the section 0-1:

$$N_{0-1} = R_p + U(P_q) \sqrt{R_p (1-p)} = 19 \cdot 0.147 + 1.645 \sqrt{19 \cdot 0.147 \cdot (1-0.147)} = 5.33$$

We assume  $N_{0-1} = 6^{(1)}$

The design discharge downstream the section 0-1 is:

$$Q_{0-1} = N_{0-1} \cdot d = 6 \cdot 10 = 60 \text{ l s}^{-1}$$

2) Number of hydrants in simultaneous operation downstream the section 1-2:

$$N_{1-2} = R_{1-2} p + U(P_q) \sqrt{R_{1-2} p (1-p)} = 18 \cdot 0.147 + 1.645 \sqrt{18 \cdot 0.147 \cdot (1-0.147)} = 6.11$$

We assume  $N_{1-2} = 6$

The design discharge downstream the section 1-2 is:

$$Q_{1-2} = N_{1-2} \cdot d = 6 \cdot 10 = 60 \text{ l s}^{-1}$$

3) Number of hydrants in simultaneous operation downstream the section 2-3:

$$N_{2-3} = R_{2-3} p + U(P_q) \sqrt{R_{2-3} p (1-p)} = 17 \cdot 0.147 + 1.645 \sqrt{17 \cdot 0.147 \cdot (1-0.147)} = 4.90$$

We assume  $N_{2-3} = 5$

The design discharge downstream the section 2-3 is:

$$Q_{2-3} = N_{2-3} \cdot d = 5 \cdot 10 = 50 \text{ l s}^{-1}$$

and so on for the calculation of the discharges in the other sections.

In order to facilitate the calculation of large networks, a computer software program called COPAM (Combined Optimization and Performance Analysis Model) has been developed by Lamaddalena (1997). This software (enclosed) has several options that will be explained in this paper.

All the computer programs for computation of irrigation systems require detailed information on the pipe network transporting water from the source to the demand points (hydrants). In the following section some general information on the use of COPAM, as well as the installation of the program and the preparation of the input data files, are presented.

### **Installation of COPAM**

Basic Windows knowledge is required for installing the COPAM Package.

Create an appropriate directory in the hard disk (it can be called "Copam"),

Insert the install disk in the appropriate drive,

Copy all files from the install disk to the directory previously created (e.g. Copam). Verify that the "\*.dll" files have been copied,

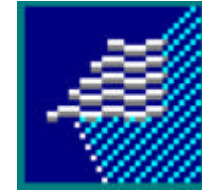
From Windows Explorer, open the directory Copam and create a shortcut on the desktop for the file (icon) “copam.exe”,

Click the icon label once to change the name. It is suggested to call it COPAM (Figure 11).

### Starting COPAM

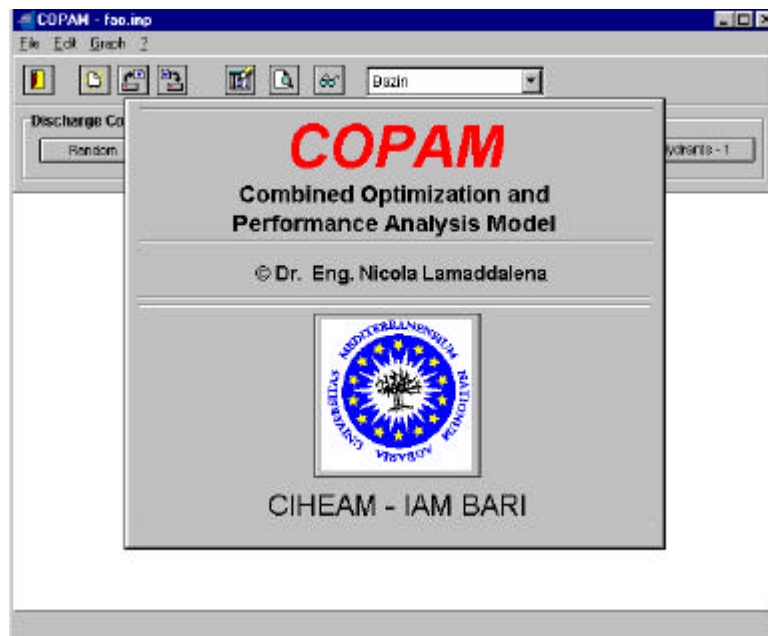
Double click on the COPAM icon. Figure 12 will occur for few seconds:

**FIGURE 11**  
**COPAM icon**



COPAM

**FIGURE 12**  
**First screen of the COPAM package**



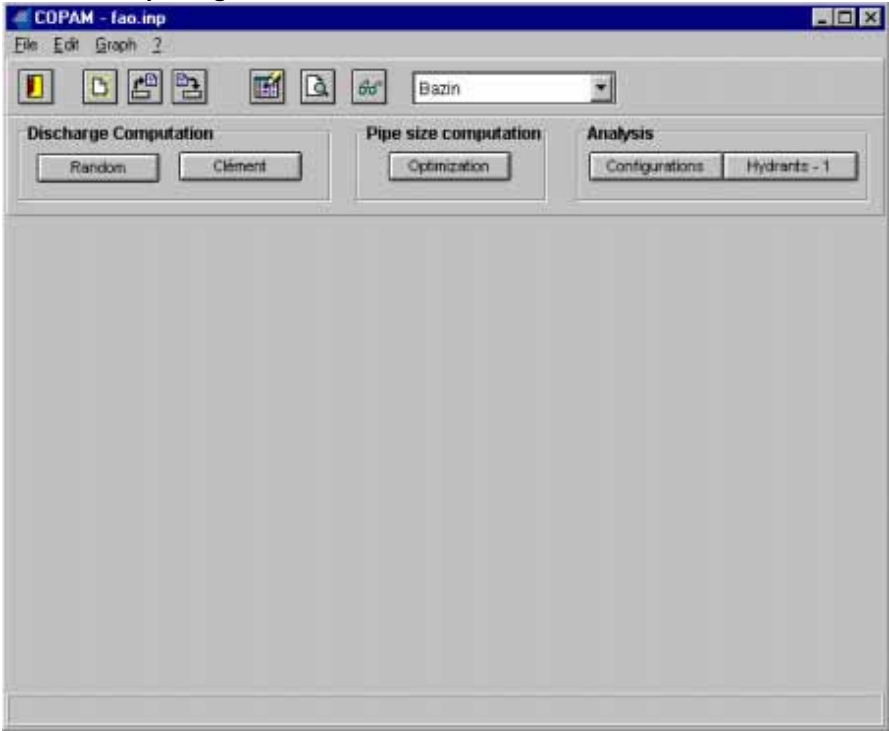
Then Figure 13 will appear.

Three different sets of programs are available in the COPAM package:

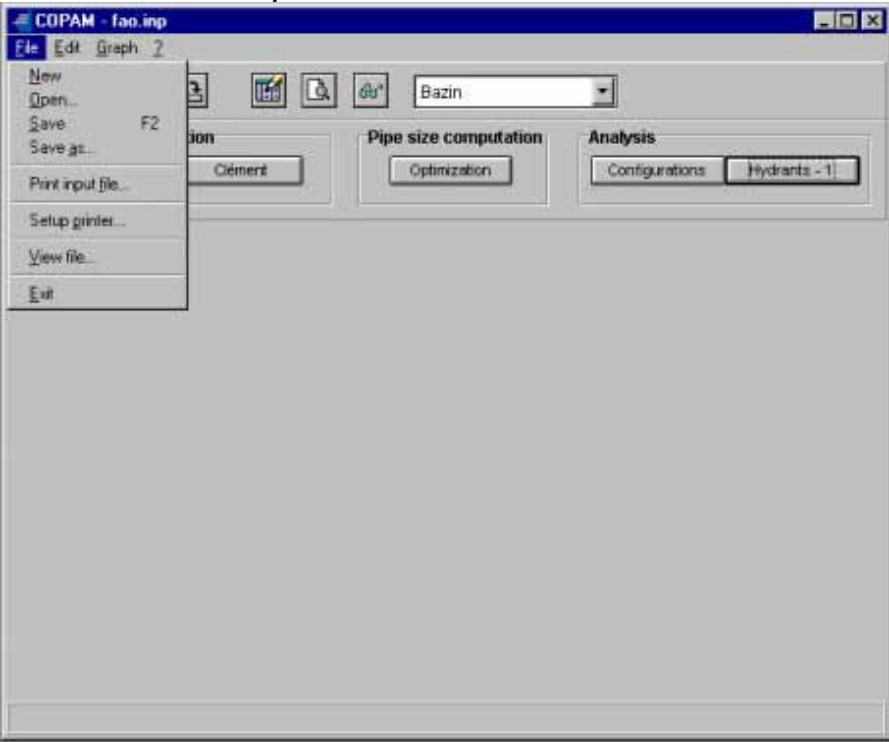
- Discharges computation
- Pipe size computation
- Analysis

Two programs are available under the set “Discharges computation”: Clément and Random. One program is available under the set “Pipe size computation”: optimization. Two programs are available under the set “Analyses”: Configurations and Hydrants. For all the above software, the basic input file is the same and it is explained below.

**FIGURE 13**  
Layout of the COPAM package



**FIGURE 14**  
File menu bar and its sub-menu options



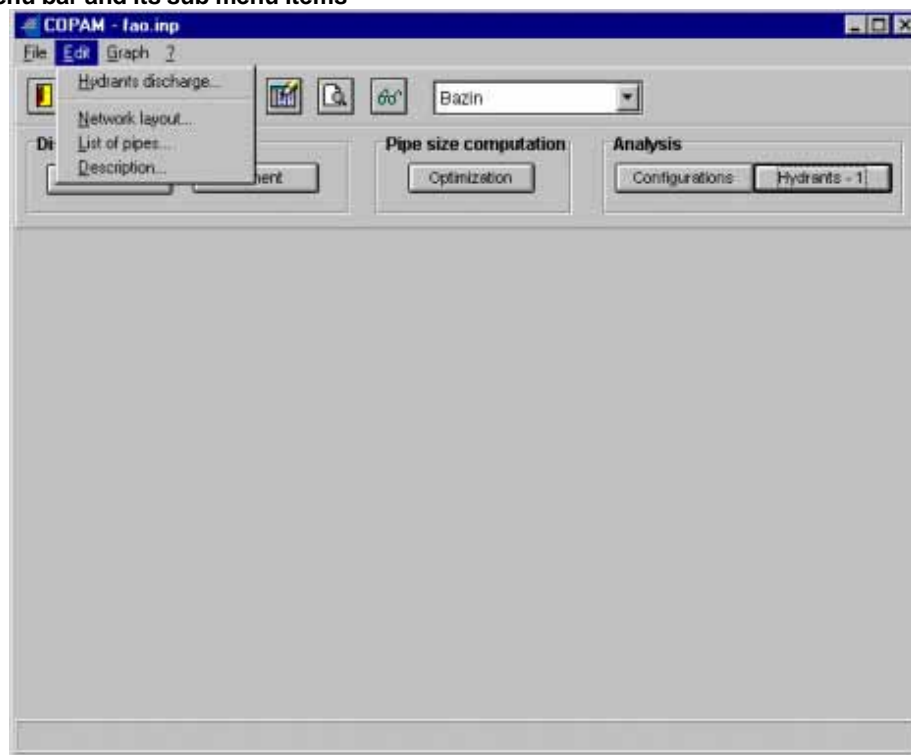
### *Preparation of the input file*

The line of words beginning “File” is a menu bar. When you click on any of the words with your mouse, a sub-menu will drop down. For example, if you click on the File menu, you will see the sub-menu in Figure 14.

Different options are available in the File menu: New, Open, Save, Save as, Print input file, Setup printer, View file, Exit. All these options are familiar for windows users.

When you want to create a new file, the option “New” is clicked. Then the menu bar “Edit” is selected and the sub-menu in Figure 15 will drop down.

**FIGURE 15**  
Edit menu bar and its sub menu items



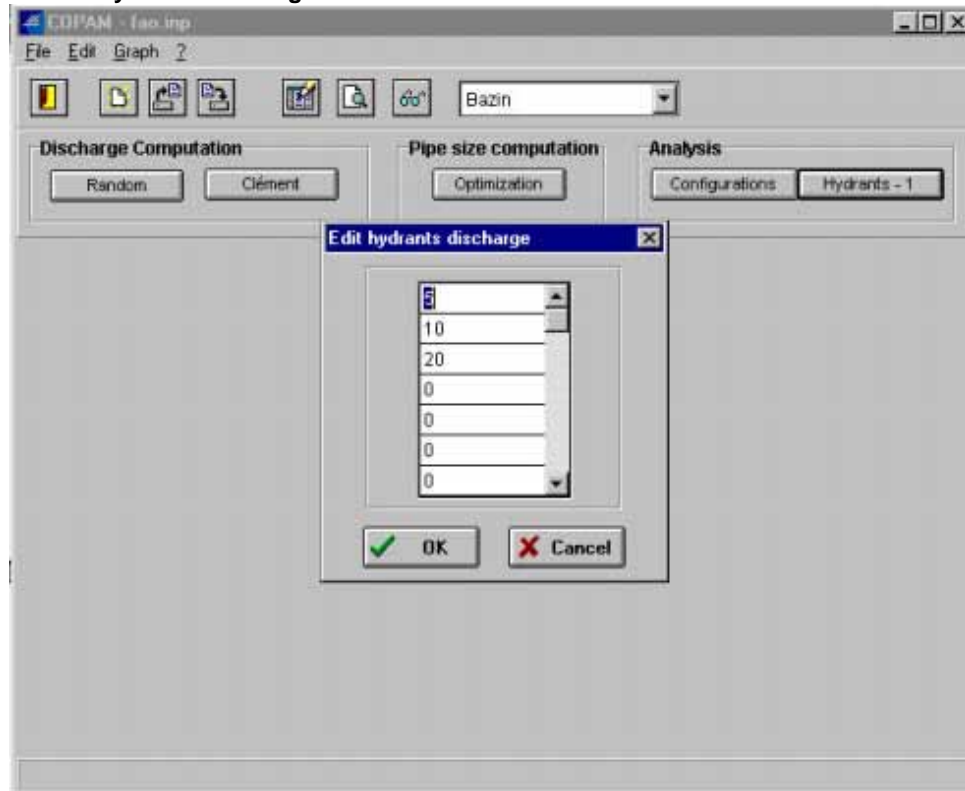
By clicking on each option, the input data may be inserted using the Edit menu. It is strongly recommended to prepare the input data before entering in the program.

First select the sub-menu “Edit/Hydrants Discharge” by clicking on it. Figure 16 will appear and the list of the nominal discharges of the hydrants is inserted in the appropriate edit box. The list of hydrants discharge is introduced in an increasing order and the user should enter any standard values of discharge selected for the project, as used in the respective country.

The command “OK” will close the option and store the information.

By clicking on the option “Network layout”, input data are inserted in the sub-menu “Edit/network layout”.

**FIGURE 16**  
**Sub menu Edit/Hydrants discharge**

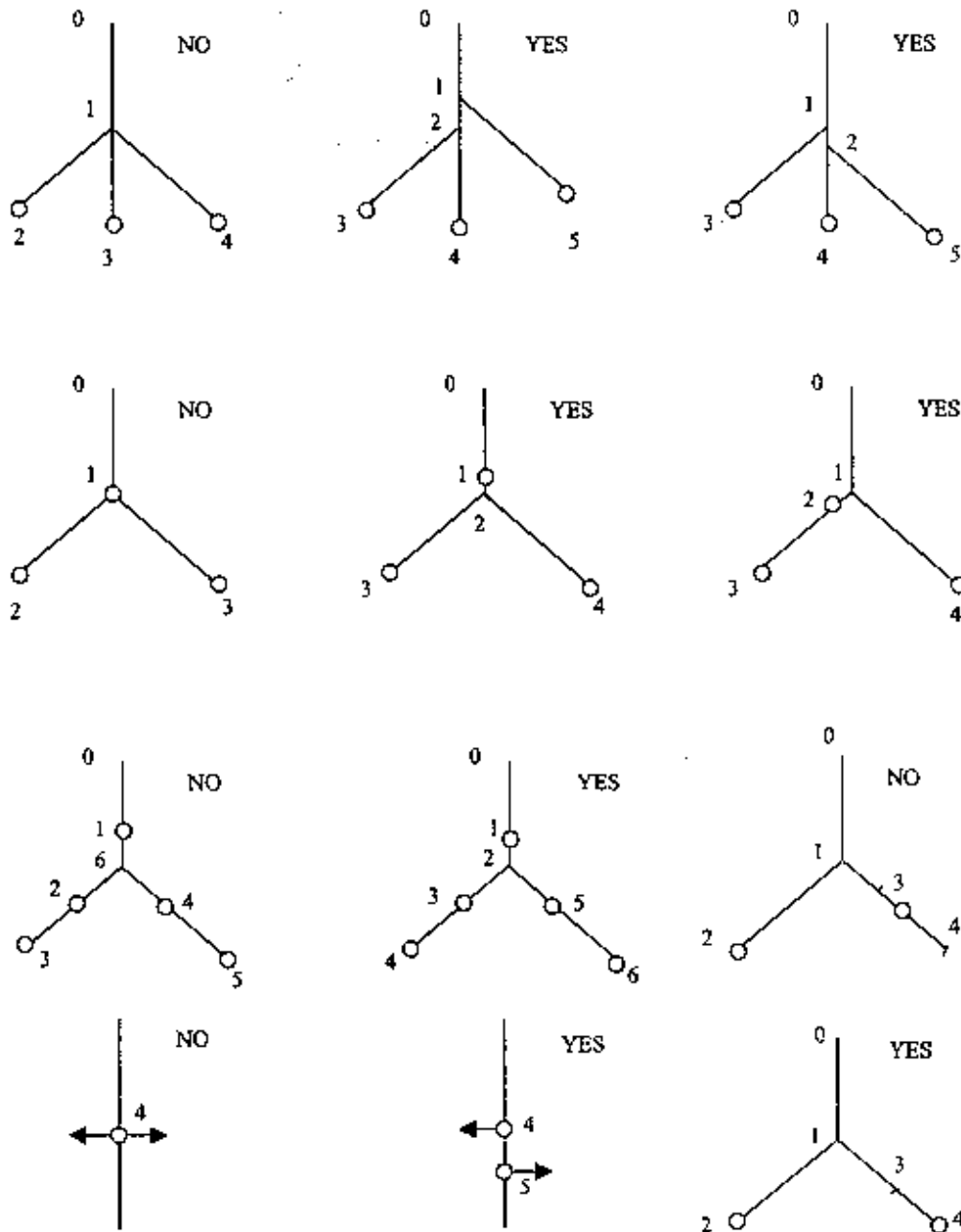


It is assumed the network is of the branching type. Each node (hydrants and/or linking of sections) is positioned by a number. The node numbering is extremely important for the correct execution of the program. It has to be allocated as follows:

- The upstream node (source) must have number 0
- The other nodes are numbered consecutively, from upstream to downstream. Any node may be jumped.
- The number of the section is equal to the number of its downstream node.
- All terminal nodes of the branches must have a hydrant.
- No more than two sections may be derived by an upstream node. If so, an imaginary section with minimum length (i.e.:  $l_{\min} = 1$  m) must be created and an additional node must be considered. This node must have a sequential number.
- No hydrants may be located in a node with three sections joined. If so, an additional node with a sequential number must be added.
- If hydrants with two or more outlets exist in the network, one number for each outlet needs to be allocated by creating an imaginary section with minimum length.

When the numbering has been completed, the following information has to be entered in the “Edit/Network layout”:

**FIGURE 17**  
Examples of node numbering

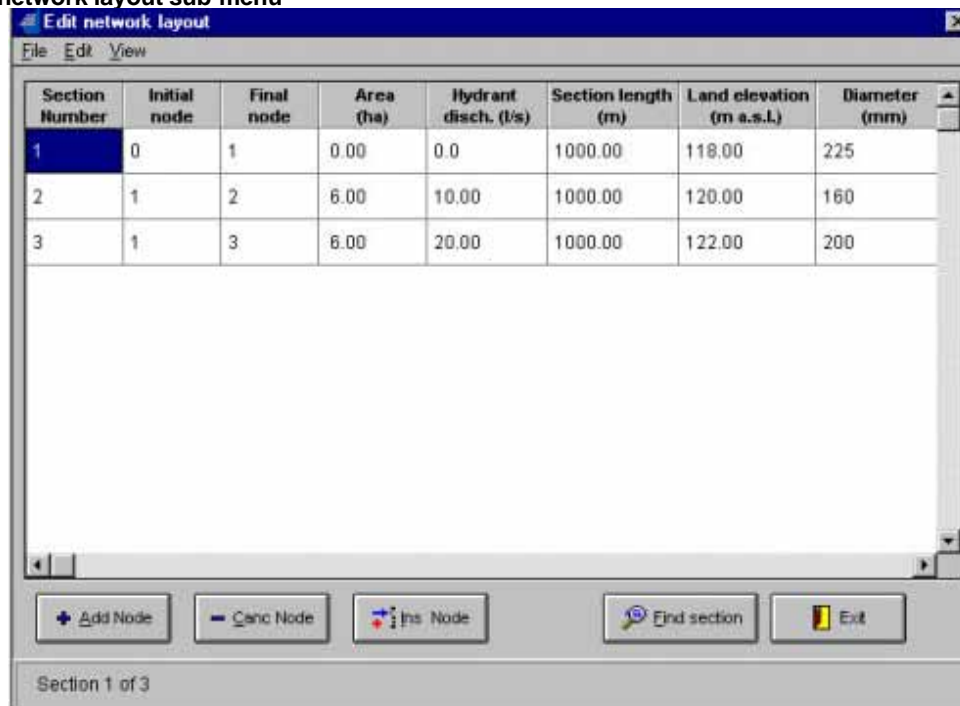


- area irrigated by each hydrant (in hectares); if no hydrant occurs in the node, Area=0 has to be typed,
- hydrant discharge (in  $l\ s^{-1}$ ). It may be selected by clicking in the combo box,
- section length (in m),
- land elevation of the downstream node (in m a.s.l.),

- nominal diameter of the section pipe (in mm). This information is needed when the program is used for the analysis of the network. In the design stage, Diameter=0 must be considered.

In Figure 18, an example of the sub-menu “Edit/network layout” is reported. Additional options are available on the bottom of the screen. They may be activated by clicking on the button: Add Node, Canc Node, Ins Node and Find Section, respectively for adding a new node, delete a node, insert a node and for finding a section. The exit button closes the sub menu.

**FIGURE 18**  
Edit/network layout sub-menu



Section Number	Initial node	Final node	Area (ha)	Hydrant disch. (l/s)	Section length (m)	Land elevation (m a.s.l.)	Diameter (mm)
1	0	1	0.00	0.0	1000.00	118.00	225
2	1	2	6.00	10.00	1000.00	120.00	160
3	1	3	6.00	20.00	1000.00	122.00	200

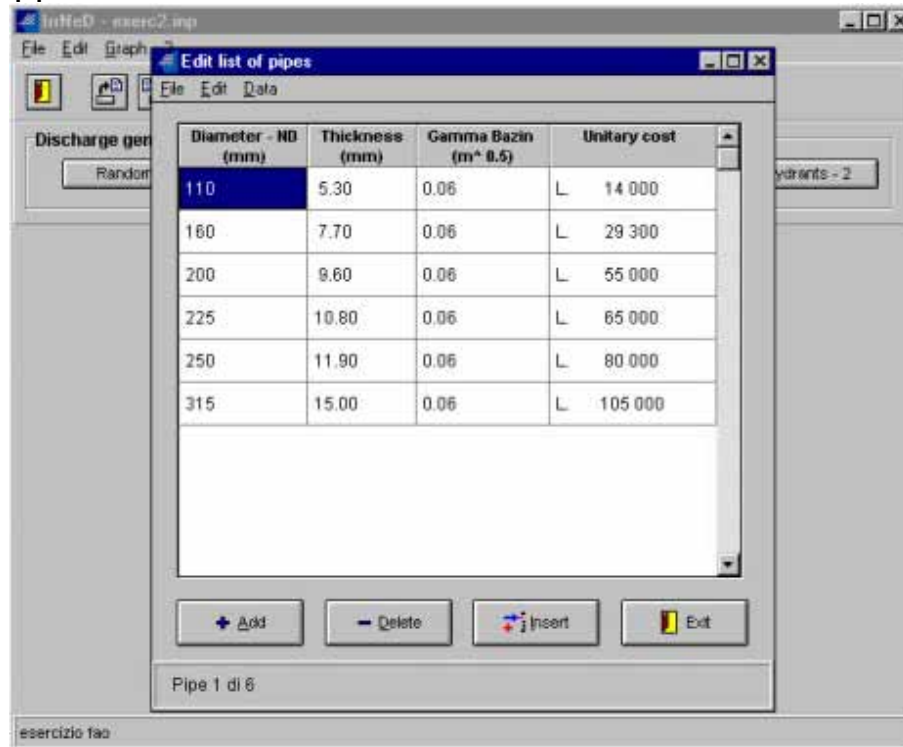
Section 1 of 3

When the network layout is completed, the “Edit/list of pipes” sub menu is selected and Figure 19 will appear.

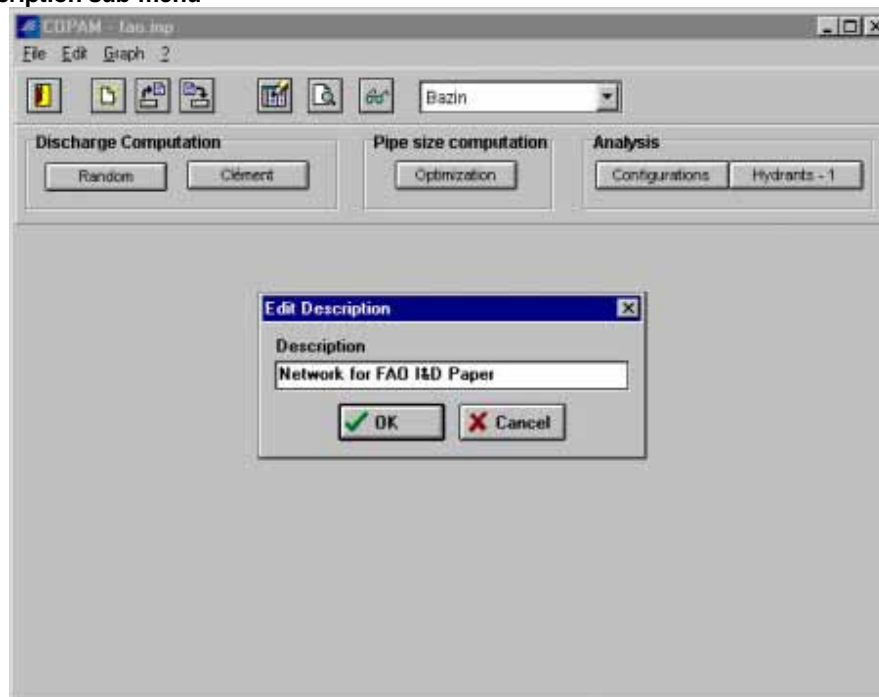
The list of commercial diameters (in mm) is inserted in the “Edit/list of pipes” sub-menu. The list has to be completed by the thickness (in mm) of the pipes, the roughness ( $\gamma$ , Bazin coefficient) and the unitary cost of the pipe. An internal procedure of the COPAM package will link, automatically, the currency to the regional setting properties of your computer. The pipes unitary costs are typed in increasing order. The nominal diameters are typed in the grid. When the nominal diameter corresponds to the internal diameter, the pipe thickness is considered equal to zero. The types of pipes are identified by the Bazin roughness coefficient.

In the sub-menu “Edit/Description” (Figure 20), the description of the file may be typed. This information is important when a large number of data files are managed to aid with recognition.

**FIGURE 19**  
Edit list of pipes sub-menu

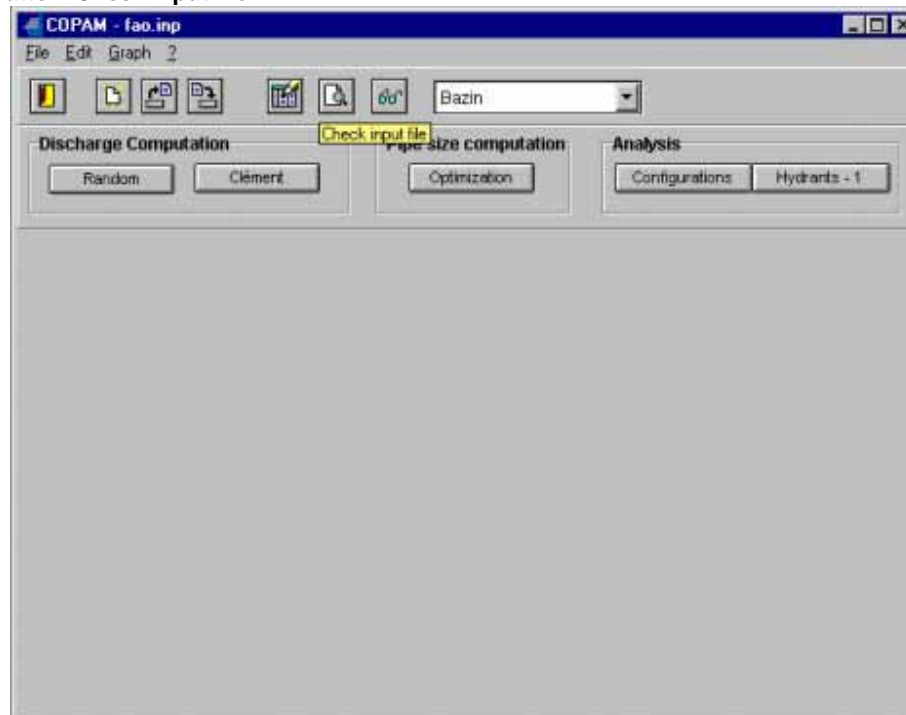


**FIGURE 20**  
Edit description sub-menu

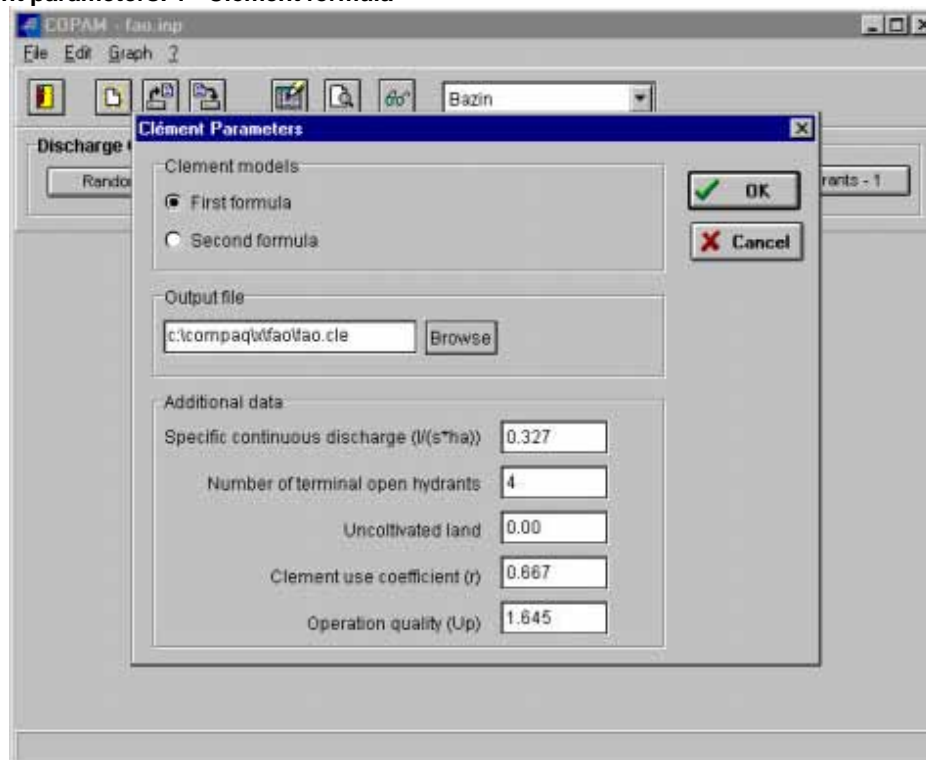




**FIGURE 21**  
Toolbar button "Check input file"



**FIGURE 22**  
Clément parameters: 1<sup>st</sup> Clément formula



An additional option is available in the COPAM package: the toolbar button “Check input file” (see Figure 21). It checks for the most common errors in the input file.

### ***Computation of discharges***

When the set “Discharge computation” is selected, two different programs are available: Clément and Random. The program “Clément” allow the computation of the discharges flowing into the network through the first and the second Clément models. When the first one is selected, additional parameters have to be typed in the “Clément parameters/sub-menu” (Figure 22). They are the:

- specific continuous discharge (in  $\text{l s}^{-1} \text{ ha}^{-1}$ )
- minimum number of terminal open hydrants
- percentage of uncultivated land (in %)
- Clément use coefficient ( $r$ )
- Clément operation quality,  $U(P_q)$ .

An example of the output file of the program “Clément” is the one reported in Table 2. The name of the output file is typed in the appropriate edit box and the extension “.cle” is automatically assigned to the file.

## **The Second Clément's model**

### ***Basic theory***

Considering the limitations in the first formulation, Clément developed a second model for calculating discharges in irrigation systems operating on-demand (Clément, 1966). This second Clément model is based on the Markovian stochastic theory of birth and death processes. It is summarized hereafter. The complete formulation is reported in Clément (1966) and Lamaddalena (1997). The derivation of the second Clément model is based on some fundamental concepts on the theory of the stationary Markovian processes.

Consider a set of customers arriving at a service station (hydrants in the case of an irrigation system). The pattern of arrivals is described by a distribution function of times of arrival. The customers require different times to be served and, thus, the times of service are described by another statistical distribution function. For an irrigation system we can consider:

- customers in service, which is the average number of arrivals during the average time period of operation of a hydrant;
- customers served, which is the average number of hydrants operating at a given instant  $t$ ;
- customers in the queue, which is the average number of arrivals when the system is saturated (during the average waiting time).

Consider a generic system characterized by a random function,  $X(t)$ , assuming values 1, 2, .....,  $N$ , which represents any possible state of the system.

In the case of an irrigation system, the state of the system is defined by the number of hydrants in operation, while birth and death correspond to opening or closing one hydrant, respectively.

Consider an irrigation network having  $R$  hydrants. Let us assume that the operating time for all hydrants follows the same distribution function with average duration of the hydrants operation equal to the average irrigation time,  $IT$ . Furthermore, let us assume that the network has been designed with the hypothesis of having  $N$  hydrants simultaneously open ( $N < R$ ).

Define  $P_{SAT}$  as the probability of saturation of a network equipped with  $(R-1)$  hydrants and  $u'$  the standard normal variable. It may be demonstrated (Clément, 1966; Lamaddalena, 1997) that:

$$u' = \frac{N - R p}{\sqrt{R p (1 - p)}} \quad (16)$$

and

$$P_{SAT} = \frac{1}{\sqrt{R p (1 - p)}} \frac{\Psi(u')}{\Pi(u')} \quad (17)$$

where  $\Psi(u')$  and  $\Pi(u')$  are, respectively, the Gaussian probability distribution function and the Gaussian cumulative distribution function. Making:

$$F(u') = \frac{\Psi(u')}{\Pi(u')} \quad (18)$$

the Equation 17 becomes

$$P_{SAT} = \frac{1}{\sqrt{R p (1 - p)}} F(u') \quad (19)$$

At this stage, it is easy to fix the standard normal variable,  $u'$ , according to the target probability,  $p$ , and to determine the corresponding values of  $\Psi(u')$  and  $\Pi(u')$  and also the value of  $F(u')$ <sup>1</sup>. In fact,  $p$  is given by the Equation 2,  $P_{SAT}$  can be selected (usually,  $P_{SAT} = 0.01$  is suggested) and  $F(u')$  can be calculated using:

$$F(u') = P_{SAT} \sqrt{R p (1 - p)} \quad (20)$$

from a diagram representing  $F(u')$  as a function of  $u'$ , or directly from the equation representing such a function (Figure 23). It is then possible to determine the corresponding value of  $u'$  and, using the Equation 16, we can finally calculate the number of hydrants simultaneously operating in the network:

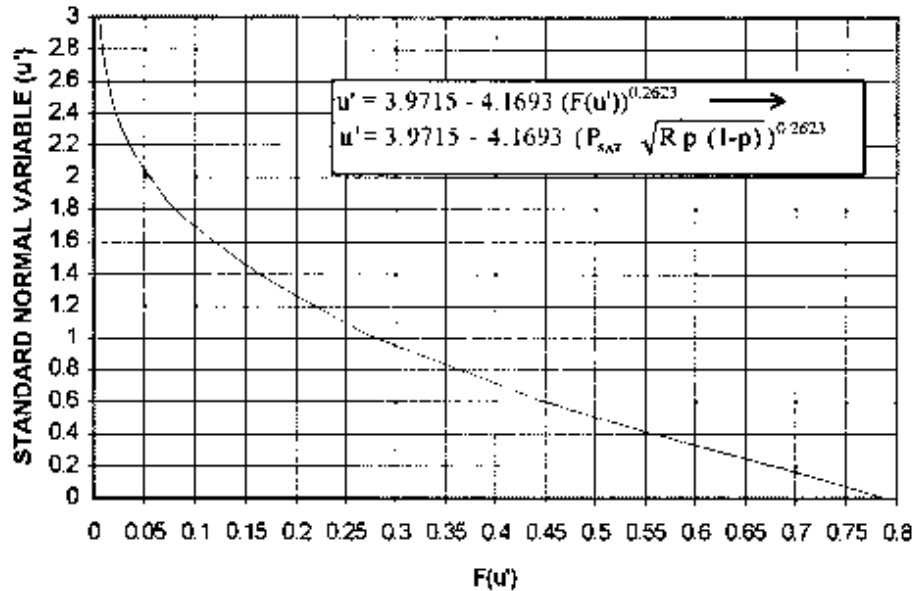
$$N = R p + u' \sqrt{R p (1 - p)} \quad (21)$$

where Equation 21 represents the second formula of Clément.

---

<sup>1</sup> It is possible to use the statistical tables giving, for each value of  $u'$ , the corresponding values of  $\Psi(u')$  and  $\Pi(u')$  and the corresponding functions. For the present work the functions  $\Psi(u')$  and  $\Pi(u')$  have been calculated and introduced in a computer program for solving the discharge calculation by using the 2<sup>nd</sup> Clément's formula.

FIGURE 23

Diagram representing  $u'$  as a function of  $F(u')$ 

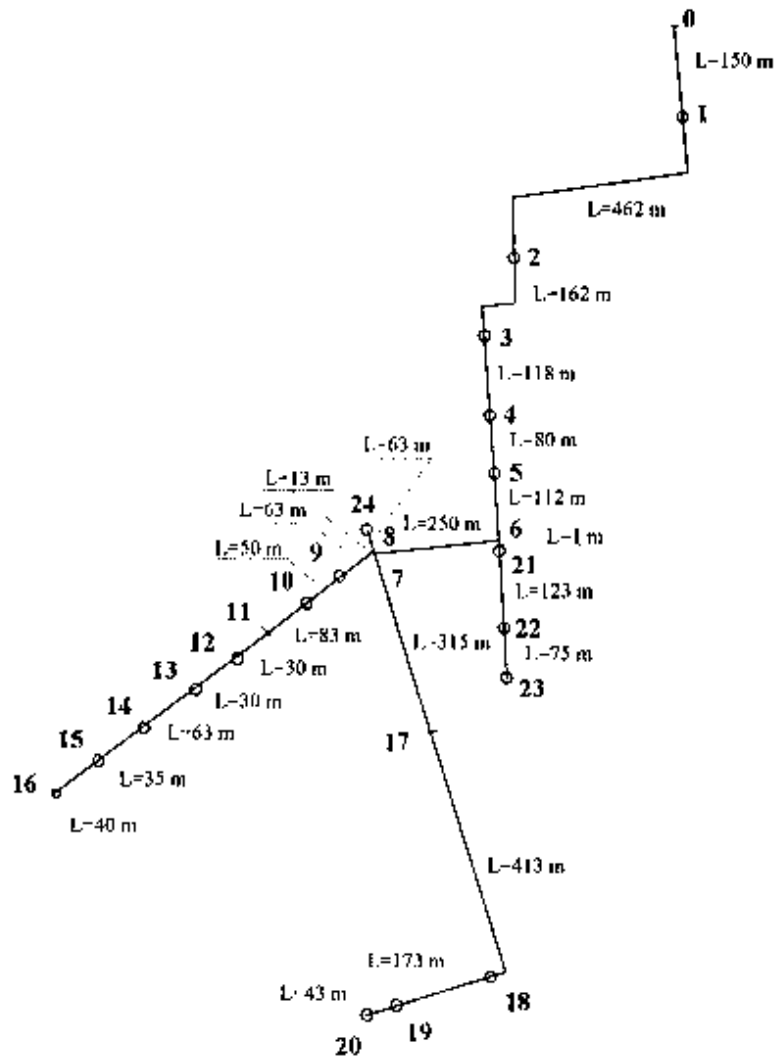
The structure of this second formula of Clément is similar to the first Clément formula but, in this case,  $u'$  is not a constant depending on the selected cumulative probability (corresponding to the quality of operation of the network), but it is a function of  $P_{SAT}$ ,  $p$  and  $R$ .

The second Clément model is based on the theory of birth and death processes. This hypothesis limits its applicability. In fact, this theory is well applied for designing telephone lines, where if the busy line is engaged (saturation) the customer has to call later. But for irrigation systems it is not so easy to establish saturation conditions. Furthermore, also when the system is saturated farmers may decide to irrigate with a lower pressure and/or discharge at the hydrant. Finally, the complexity in mathematical approach and the negligible differences in results pushed all designers to apply anytime the first model instead of the second one. In Table 3, the discharges flowing into each section of the network in the example are reported.

### *Applicability of the 2<sup>nd</sup> Clément model*

The computer package COPAM may be used for computing the discharges into each section of an irrigation network by using the 2<sup>nd</sup> Clément model. The 'Clément parameters/sub-menu' of the Clément program (see Figure 24) is filled with the parameters described above.

The application of the 1<sup>st</sup> and the 2<sup>nd</sup> Clément models for computing the discharges into a large Italian irrigation network (the one illustrated in the Annex 1) is shown hereafter. The following design data were used for these calculations. The network is equipped with 660 hydrants of  $10 \text{ l s}^{-1}$  and the design values of the irrigated area  $A_i$ , the specific continuous discharge,  $q_s$ , and the coefficient of utilization of the network,  $r$ , are respectively:  $A_i = 2030 \text{ ha}$ ,  $q_s = 0.327 \text{ l s}^{-1} \text{ ha}^{-1}$  and  $r = 0.66$ . The 1<sup>st</sup> Clément's model was applied using the cumulative probability  $P_q=95\%$  (corresponding to  $U(P_q) = 1.645$ ), while the 2<sup>nd</sup> Clément's model was applied using the probability of saturation  $P_{SAT}=1\%$ .

**BOX 2: APPLICATION OF THE SECOND CLÉMENT MODEL FOR THE COMPUTATION OF FLOWS IN AN ON-DEMAND NETWORK**


Data:

$q_s$  = specific continuous discharge (24/24 hours) =  $0.327 \text{ l s}^{-1} \text{ ha}^{-1}$

$A_p$  = area of the plot to be irrigated = 3 ha

$A$  = total irrigable area =  $3 \cdot 19 = 57 \text{ ha}$

$d$  = nominal discharge of the hydrants =  $10 \text{ l s}^{-1}$

$P_{SAT}$  = probability of saturation = 0.01

$r$  = use coefficient = 0.667

$R$  = total number of hydrants = 19

$N_{ba}$  = minimum number of hydrants in simultaneous operation = 4

\*\*\*\*\*

**BOX 2 Cont'd**

$$p = \text{elementary probability} = \frac{q_s A}{r R d} = \frac{0.327 \cdot 57}{0.667 \cdot 19 \cdot 10} = 0.147$$

$$u' = \text{standard normal variable} = 3.9715 - 4.1693 (P_{\text{SAT}} \sqrt{R p (1-p)})^{0.2623}$$

1) Number of hydrants simultaneously operating downstream the section 0-1:

$$N_{0-1} = R p + u' \sqrt{R p (1-p)} =$$

$$19 \cdot 0.147 + \left\{ 3.9715 - 4.1693 \left[ (0.01 \sqrt{19 \cdot 0.147 \cdot (1-0.147)}) \right]^{0.2623} \right\} \sqrt{19 \cdot 0.147 \cdot (1-0.147)}$$

$$= 6.77$$

We assume  $N_{0-1} = 7$ <sup>1</sup>

The design discharge downstream the section 0-1 is:

$$Q_{0-1} = N_{0-1} \cdot d = 7 \cdot 10 = 70 \text{ l s}^{-1}$$

2) Number of hydrants in simultaneous operation downstream the section 1-2:

$$N_{1-2} = R p + u' \sqrt{R p (1-p)} =$$

$$18 \cdot 0.147 + \left\{ 3.9715 - 4.1693 \left[ (0.01 \sqrt{18 \cdot 0.147 \cdot (1-0.147)}) \right]^{0.2623} \right\} \sqrt{18 \cdot 0.147 \cdot (1-0.147)} =$$

$$= 6.53$$

We assume  $N_{1-2} = 7$

The design discharge downstream the section 1-2 is:

$$Q_{1-2} = N_{1-2} \cdot d = 7 \cdot 10 = 70 \text{ l s}^{-1}$$

3) Number of hydrants in simultaneous operation downstream the section 2-3:

$$N_{2-3} = R p + u' \sqrt{R p (1-p)} =$$

$$17 \cdot 0.147 + \left\{ 3.9715 - 4.1693 \left[ (0.01 \sqrt{17 \cdot 0.147 \cdot (1-0.147)}) \right]^{0.2623} \right\} \sqrt{17 \cdot 0.147 \cdot (1-0.147)} =$$

$$6.29$$

We assume  $N_{2-3} = 6$

The design discharge downstream the section 2-3 is:

$$Q_{2-3} = N_{2-3} \cdot d = 6 \cdot 10 = 60 \text{ l s}^{-1},$$

and so on for the calculation of the discharges in the other sections.

<sup>1</sup> In this example the mathematical approximation was used for computing the discharges in each section of the network.

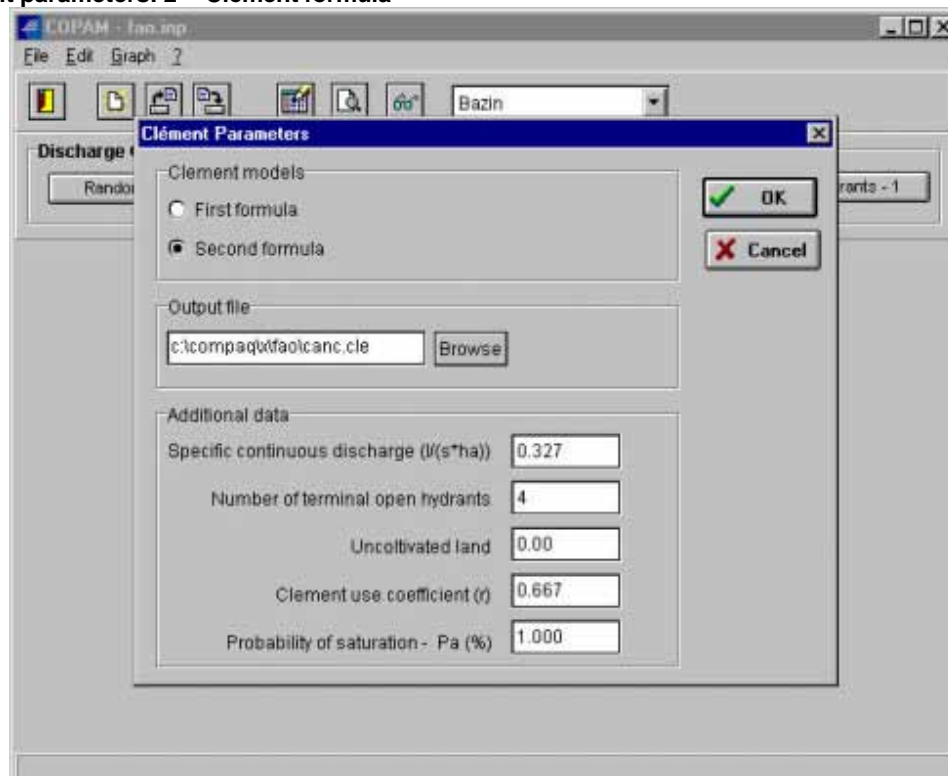
TABLE 3

Discharges flowing into each section of the network under study (output of the COPAM package: computation with the second Clément model)

Section Number	Initial Node	Final Node	Number of Hydrants	Area (ha)	2nd Clément discharge (l/s)
1	0	1	19	57.00	70.00
2	1	2	18	54.00	70.00
3	2	3	17	51.00	60.00
4	3	4	16	48.00	60.00
5	4	5	15	45.00	60.00
6	5	6	14	42.00	60.00
7	6	7	11	33.00	50.00
8	7	8	8	24.00	40.00
9	8	9	7	21.00	40.00
10	9	10	6	18.00	40.00
11	10	11	5	15.00	40.00
12	11	12	5	15.00	40.00
13	12	13	4	12.00	40.00
14	13	14	3	9.00	30.00
15	14	15	2	6.00	20.00
16	15	16	1	3.00	10.00
17	7	17	3	9.00	30.00
18	17	18	3	9.00	30.00
19	18	19	2	6.00	20.00
20	19	20	1	3.00	10.00
21	6	21	3	9.00	30.00
22	21	22	2	6.00	20.00
23	22	23	1	3.00	10.00
24	8	24	1	3.00	10.00

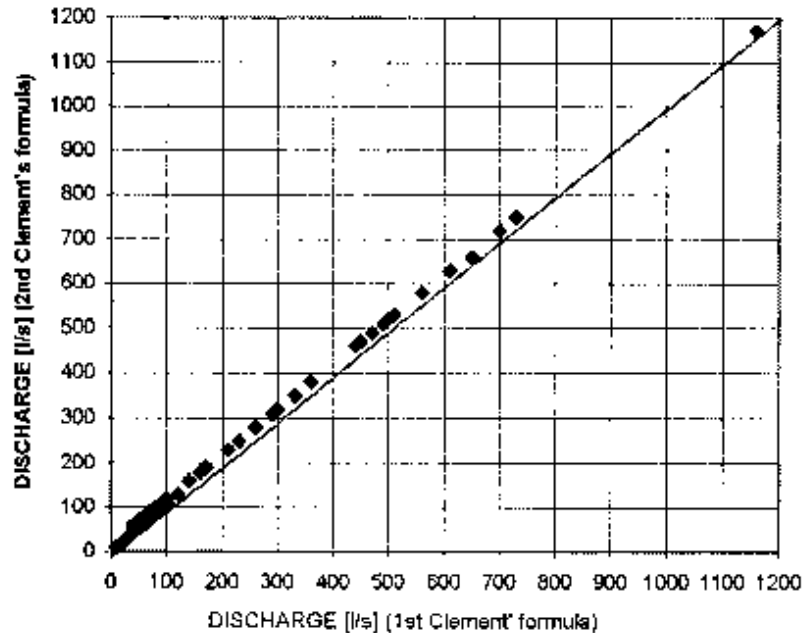
FIGURE 24

Clément parameters: 2<sup>nd</sup> Clément formula

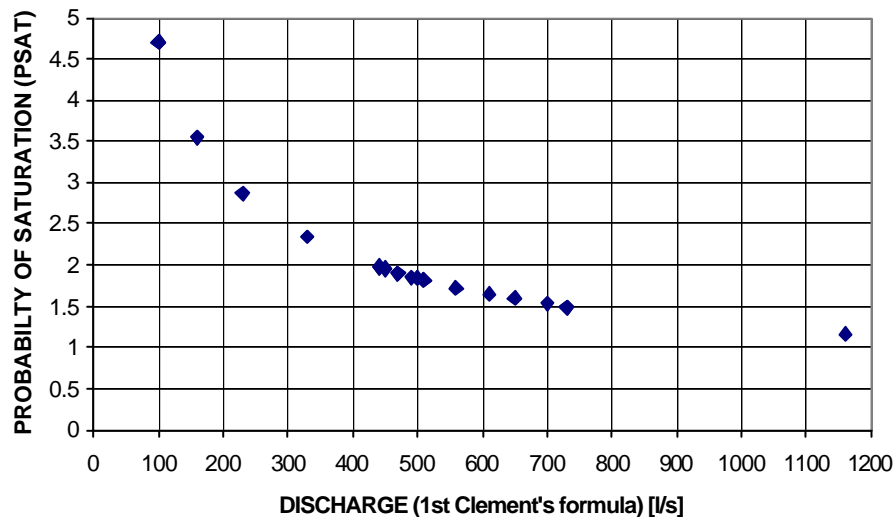


**FIGURE 25**

Comparison between discharges computed with the 1<sup>st</sup> and the 2<sup>nd</sup> Clément equations applied to the Italian irrigation network illustrated in Annex 1

**FIGURE 26**

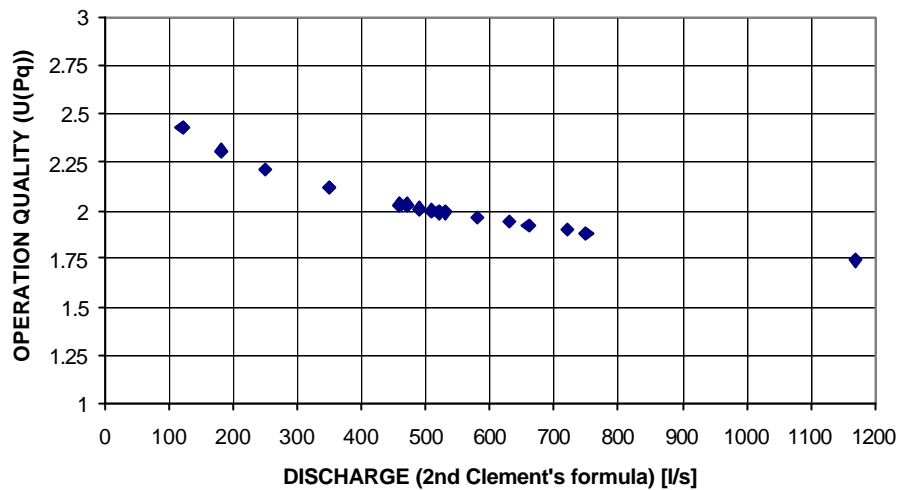
Probability of saturation versus the 1<sup>st</sup> Clément model discharges



The results of the application are summarized in Figure 25 which shows that, for the case under study, using the second model slightly higher discharges are obtained. Further analyses have shown that when the probability of saturation  $P_{SAT}$  is computed using the discharges given by the 1<sup>st</sup> Clément model (using the above design parameters) it becomes evident (Figure 26) that  $P_{SAT}$  would be near to 5% for low discharges but would approach  $P_{SAT} = 1\%$  for higher discharges. When  $U(P_q)$  is computed from the discharges corresponding to the 2<sup>d</sup> Clément model, similar results are obtained with a limit value of  $U(P_q)$  close to  $U(P_q) = 1.645$  (Figure 27).



**FIGURE 27**  
Operation quality,  $U(P_q)$  versus the 2<sup>nd</sup> Clément model discharges



This example calculation showed that results from both Clément equations are compatible and differences are negligible in practice. Because of the limitation of the theory of birth and death processes applied to the irrigation networks and the mathematical complexity almost all designers use the first model instead of the second.

#### SEVERAL FLOW REGIMES MODEL (SFR MODEL)

The configuration of discharges (flow regimes) in an irrigation network greatly affects design and operation conditions. In the next sections a model for simulating the water demand in a pressurized irrigation system operating on-demand is described.

#### Model for generating random discharges

The following approach is proposed in this section for simulating the possible operation conditions of an on-demand irrigation system. It operates using the random generation of  $K$  hydrants simultaneously opened among the total number  $R$  (with  $K < R$ ). This generation is performed to consider the temporal variability of the discharges flowing into the irrigation network. Each generation, according to the definition reported in the technical literature (CTGREF, 1979; Bethery *et al.*, 1981) produces a hydrants configuration. Each hydrants configuration corresponds to a discharge configuration. Therefore, the discharge in the sections of the network is calculated as the sum of the discharges withdrawn from the downstream opened hydrants.

Before illustrating the procedure used for the generation of discharge configurations, it is important to underline that the use of a computer for generating random numbers is considered conceptually impossible. In fact, any program will produce an output that is predictable, hence not truly random because the numbers are not unrelated. Nevertheless, practical computer random number generators are in common use (Knuth, 1981). To distinguish purely random physical process from computer-generated sequences, the term pseudo-random is used.

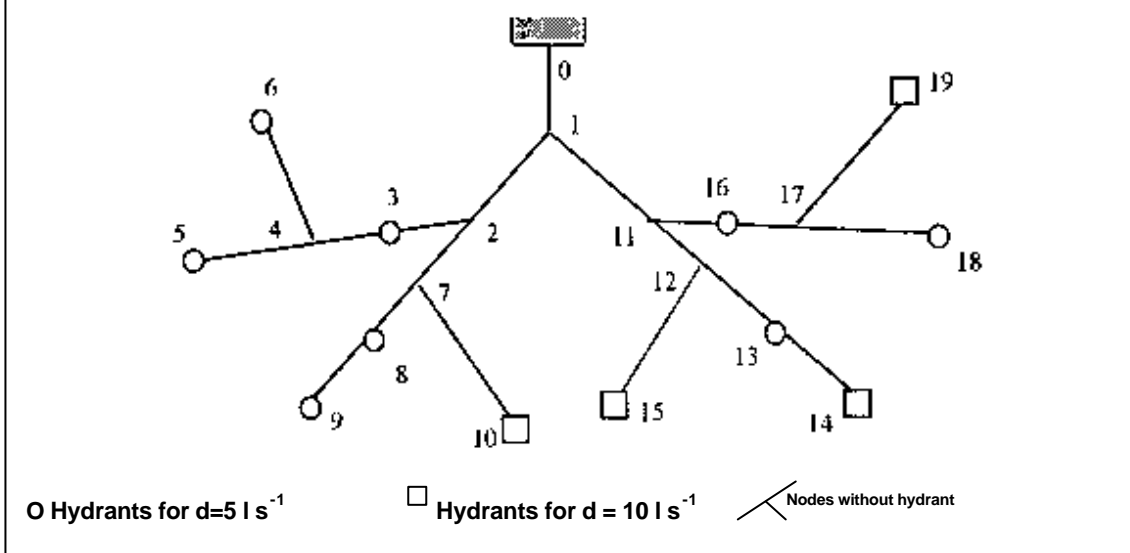
There exists a body of random number generators which mutually do satisfy the definition over a very broad class of application programs. But what is random enough for one application may not be random enough for another. The chi-square test may be used to verify the goodness of the random generation for a particular application (Press *et al.*, 1989; Knuth, 1981).

For generating random discharge configurations, uniform deviates are applied. The uniform deviates are just random numbers which lie within a specified range, with any one random number in the range just as likely as any other (Press *et al.*, 1989).

The procedure utilized is as follows (see Figure 28):

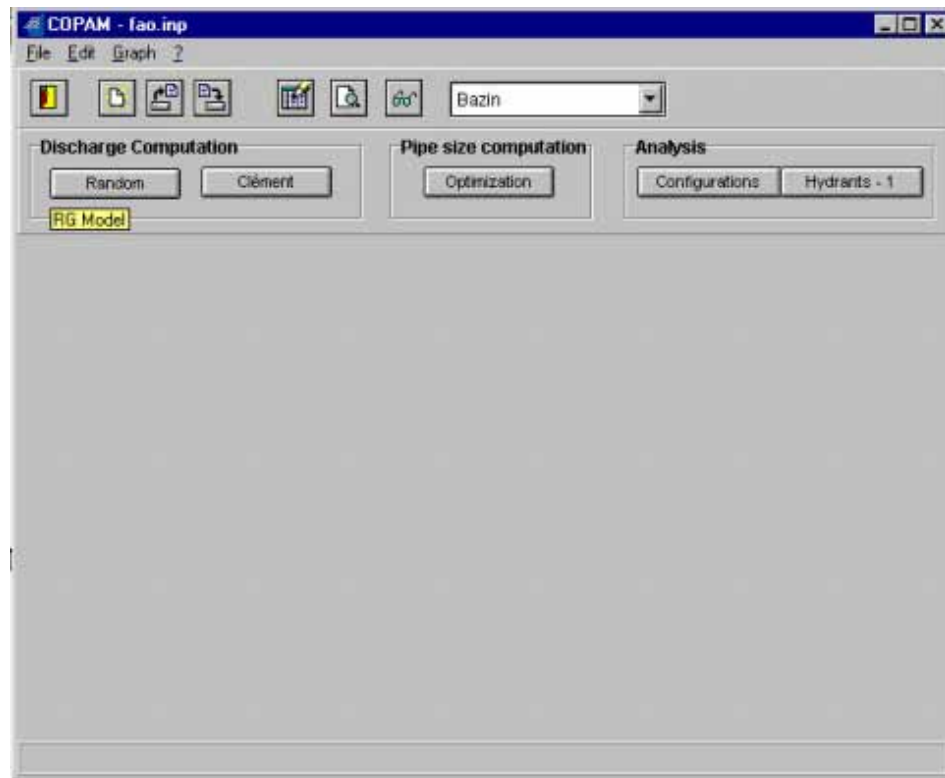
- 1- Every node of the system receives an identification number,  $j$ .
- 2- Since some nodes do not correspond to hydrants, a code identification of the hydrants is assigned ( $ch=0$  when there is no hydrant at the node,  $ch=1$  when there is a hydrant with nominal discharge of  $5 \text{ l s}^{-1}$ ,  $ch=2$  when there is a hydrant with nominal discharge of  $10 \text{ l s}^{-1}$ ,  $ch=3$  when there is a hydrant with nominal discharge of  $20 \text{ l s}^{-1}$ ).
- 3- Definition of a list of discharges to be tested.
- 4- Definition of the number of hydrants configuration to be generated, corresponding to the discharges defined in the previous list.
- 5- Aggregation of the hydrants discharges in each configuration, in order to calculate the discharges flowing into the sections of the network (discharge configuration).

**FIGURE 28**  
Typical layout of a network



A computer program written in Turbo Pascal, version 6.0 generates random sequences using the internal procedure called Random (Schildt, 1986). The random number generator is initialized by the procedure Randomize. The reliability of the Turbo Pascal random number generator has been verified by Schildt (1986). Additional tests have been also performed by El Yacoubi (1994) and Lamaddalena (1997). The program for the random generation of discharge configurations (RGM: Random Generation Model) is integrated in the COPAM package. It is selected by clicking on the appropriate button (Figure 29). Then Figure 30 will appear on the screen. The input data for the irrigation network were illustrated in the previous section.

**FIGURE 29**  
Random generation program



The RG Model may be used for two different purposes:

- analysis of existing irrigation systems;
- design of new irrigation systems.

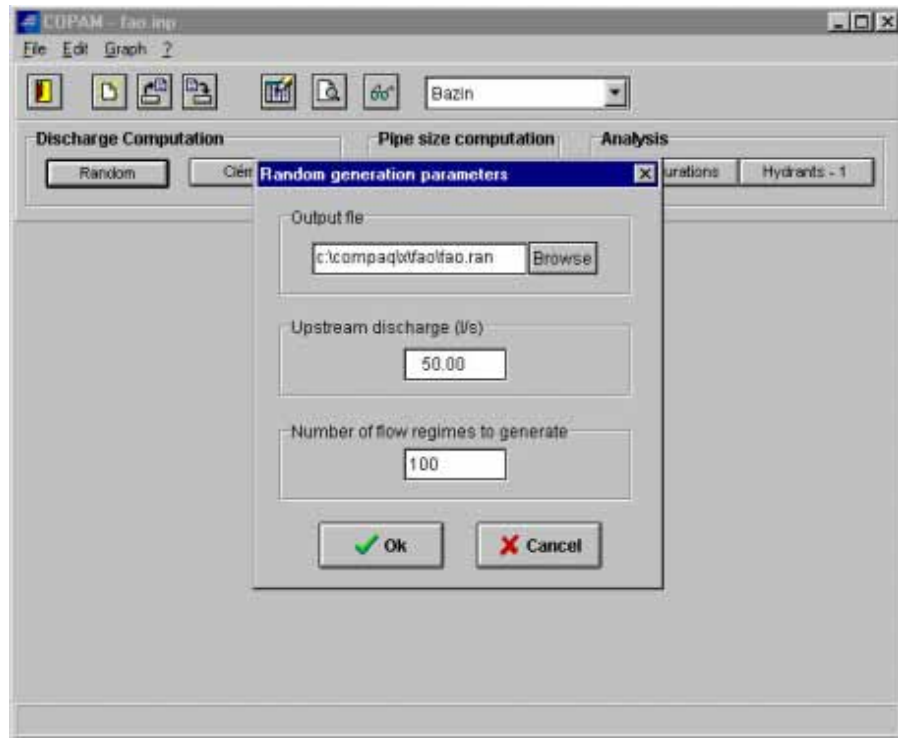
In the first case, this model is based on the knowledge of the demand hydrograph at the upstream end of the network. In fact, it allows the selection of the upstream discharge corresponding to various hydrant configurations. The value of the upstream discharge is inserted in the appropriate edit box (see Figure 30). Corresponding to the selected discharge, a number of hydrants simultaneously operating (hydrant configuration) is automatically withdrawn. This procedure is repeated for several configurations and is used for analysing the system, as illustrated in the chapter 5. The number of configurations (or flow regimes) to generate is typed in the appropriate edit box (see Figure 30). It must be multiple of 10.

All the generated flow regimes are stored in an output file with its name typed in the appropriate edit box. The extension “.ran” is automatically assigned to this output file.

When a new irrigation system is designed, the upstream discharge is not known a priori. Therefore, to allow the generation of different hydrants configurations, such discharge may be computed, for example, with the Clément models<sup>1</sup>. This is achieved through the COPAM

<sup>1</sup> Other models for generating the upstream discharges are under study. Nevertheless, despite the good improvements that have been made in the generation of the upstream withdrawn volumes, the problem of transformation from volumes to discharges has still not been solved because of a number of

**FIGURE 30**  
Random generation parameters



package as explained in the previous section. The discharge value is allocated in the edit box “Upstream discharge” of the “Random generation parameters”. After including the number of configurations to be generated in the appropriate edit box (it must be multiple of 10), operation of the network is simulated by generating the hydrants configurations. These hydrants configurations, stored in a pre-selected output file, are taken into account for computing the optimal pipe size of the network by using the approach illustrated in the chapter 4. Applications of the above methodologies for design and analysis of Italian irrigation networks are reported in Lamaddalena (1997).

---

uncertainties involved in that process. Some authors (Abdellaoui, 1986; El Yacoubi, 1994; Pereira et al., 1995; Lamaddalena, 1997) presented interesting approaches for the solution of such a problem but only a few applications are available and additional tests are needed for validation and for generalizing the applicability of such models. For this reason, the first Clement model is preferred for the computation of the discharge at the upstream end of the network and, in correspondence of such discharge, a large number of hydrant configurations may be generated. In this way, the actual operation conditions are simulated and are taken into account for the design (and/or the analysis) of the irrigation system.

## Chapter 4

# Pipe-size calculation

The problem of calculating the optimal pipe size diameters of an irrigation network has attracted the attention of many researchers and designers. Many optimization models based on linear programming (LP), non-linear programming (NLP) and dynamic programming (DP) techniques are available in literature. These models give important improvements to practical problems through less costly solutions and less computation time respect to the classical approaches.

One limitation of the optimization models is that they consider only One Flow Regime (OFR) during the process of the pipe size computation. With this kind of approach, there is no assurance that the system selected is the least costly and compatible with the required performance. Indeed, in on-demand irrigation systems, the distribution of flows in each section may strongly vary in time and space. Thus, several flow regimes (SFR) should be taken into account during the computation process.

In this chapter an optimization model using the Labye's Iterative discontinuous algorithm is presented and applied for the case of One Flow Regime and extended to the case of Several Flow Regimes.

### OPTIMIZATION OF PIPE DIAMETERS WITH OFRM

#### Review on optimization procedures

Research on optimization procedures to design water distribution networks have been reported since the 1960s. Karmeli *et al.* (1968), Schaake and Lai (1969), and Lieng (1971) developed optimization models for solving branched networks where a given demand pattern was used to define the flow uniquely in the pipes. Afterwards, important efforts have been made by Alperovits and Shamir (1977), Quindri *et al.* (1981), Morgan and Goulter (1985) for solving looped systems, where infinite number of distributions of flow can meet a specific demand pattern. Alperovits and Shamir (1977) used fundamental linear programming formulation but their approach is severely limited to small size systems. The approach by Quindri *et al.* (1981) may solve larger systems but only pipes are included in the design and no additional components are considered (e.g. pumping station, reservoirs, valves). In the approach by Morgan and Goulter (1985), the cost of components is not included in the objective function and multiple runs are required to determine the best solution.

Lansey and Mays (1989) proposed a methodology for determining the optimal design of water distribution systems considering the pipe size, the pumping station and reservoirs. In addition, the optimal setting for control and pressure-reducing valves can be determined. This methodology considers the design problem as a non-linear one, where all the typical components of a network are designed while analyzing multiple alternative discharges. This procedure still is computationally intensive and requires a large number of iterations to obtain the solution, especially when applied to large networks.

In view of the concerns sometimes expressed about the use of linear programming when large networks are designed, other approaches have been developed (Labye, 1966 and 1981) using the dynamic programming formulation. Later, another approach was developed (Ait Kadi, 1986) to optimize the network layout and the pipe sizes simultaneously for a network composed by a mainline and secondary parallel branches. This approach (Ait Kadi, 1986) involves two stages. In the first one, an initial solution is constructed by obtaining the optimum layout of the mainline, with each pipe of the network having the smallest allowable diameter. In the second stage, the initial solution is improved by reducing, iteratively, the upstream head and, consequently, varying the layout of the mainline and pipe sizes simultaneously. This iterative process is continued until the optimum head giving the minimum cost of the network is reached.

In recent years, several comprehensive reviews on the state-of-the-art in this field were undertaken (Walsky, 1985b; Goulter, 1987; Walters, 1988). Furthermore, a number of optimization models were assessed (Walsky *et al.*, 1987). An interesting result of the analysis by Walsky *et al.* (1987) was that models produce similar design, both in terms of costs and hardware components selected. The cost associated with the solutions determined by different models only varied by 12%. In addition, the most expensive systems were those with increased level of reliability, imparted by additional storage in the system. Linear programming and dynamic programming for calculating the optimal pipe diameters in irrigation networks have also been compared (Di Santo and Petrillo, 1980a,b; Ait Kadi, 1986).

Di Santo and Petrillo (1980a) demonstrated that a network solved by using both LP and the DP with Bellman's formulation has the same optimal solution. Results obtained by Ait Kadi (1986) for networks solved by using both LP and the Labye's Iterative Discontinuous Method (LIDM) were similar.

The above analyses (Di Santo and Petrillo, 1980a,b; Ait Kadi, 1986; Walsky *et al.*, 1987) indicate that the optimization models utilized were relatively robust and that the optimization is not sensitive to the technique itself. However, it is useful to improve the ability to design and analyze water distribution systems. In fact, system design must consider various critical demand hydrographs to ensure system reliability (Templeman, 1982; Hashimoto *et al.*, 1982). This is valid for both branched and looped networks where the same daily pattern demand may correspond to several configurations of flows in the pipes. In addition, it is useful to examine the extent to which the model techniques and approaches are incorporated into engineering practice.

Better designs have been obtained through optimization models but there has been no guarantee of global optimality, i.e. on what should be the objective of an optimal design. Assuming that an optimal design is the one which meets the applied demands at the least cost (Goulter, 1992), it should incorporate multiple demand conditions, failures in the system components and reliability.

For considering the reliability of the network, a definition of "failure" is necessary. In general, a network failure is an event in which a network is not able to provide sufficient flow or sufficient pressure to meet the demand. Under this definition, failure can occur either if a component (e.g., a pipe) is undersized, or if the actual demand exceeds the design demand. These two cases are independent for practical purposes. A considerable effort has been directed to the reliability question over the last few years.

Relatively little success has been achieved in obtaining comprehensive measures of network reliability that are computationally feasible and physically realistic (Goulter, 1987; Lansey and Basnet, 1990).

The measures that give good representation of reliability are computationally impractical, like the model by Su *et al.* (1987) where 200 min. of computer time is needed for solving a three

loop example. On the other hand, those approaches that are computationally suitable provide very poor description of the network performance. Also, empirical solutions may be considered for improving reliability performance, like the one by Bouchard and Goulter (1991) which proposed to add valves to the links in order to isolate branches during failure for repair.

Despite the efforts in developing optimization models, they are not widely used in practice. The main reasons for this are: they are often too complex to be used and designers are not comfortable with the optimization approaches.

Indeed, Walsky *et al.* (1987) have shown that, by using optimization models, important answers to practical problems are possible which can be verified using simulation techniques. Furthermore, optimization models require less computation time than the classical approaches. Usually, optimization models are difficult to use. The main reason is that they are developed in academic environments where the algorithm is much more important than the input-output interface. In addition, many older engineers have not had opportunity to study formal optimization techniques.

Considering the effort in developing optimization models, it seems reasonable to assert that research should be oriented to integrate simulation and optimization models rather than to develop new optimization algorithms. Techniques for designing optimal pipe diameters for irrigation networks under several discharge configurations need to be improved and tested. Furthermore, reliability of systems need to be well defined in order to be included in the optimization process. Finally, the need for improving the user interface in optimization models software is important (Walsky *et al.*, 1987).

An interesting commercial program for the calculation of the optimal pipe diameters for irrigation networks has been developed by CEMAGREF (1990). This program (XERXES-RENFORS, vers. 5.0) has a user-friendly interface. It computes the discharges with the first Clément formula, or by adding hydrants' discharges, while Labye's discontinuous method is used to compute the optimal diameters. Also, optimal pumping station and optimal reinforcement of the networks may be calculated. This program has interfaces in French and head losses are computed using the Calmon and Lechapt formula.

In the present publication, an effort has been made to develop and distribute the computer program (COPAM). It has an English user-friendly interface and all the calculations are easy. It integrates models for the optimal pipe size computation with models for the analysis of irrigation systems and allow to present the outputs under form of files and graphics. The program facilitates full understanding by the user with the capability to verify the results.

When the design of the pumping stations is required the economic aspects are an important component but the performance analysis of the network is also required. This latter aspect is covered in Chapter 5.

As far as the regulation of the pumping stations is concerned, the reader is referred to Irrigation and Drainage Paper No. 44 (page 128) where the subject is treated in detail. However, the regulation by variable speed pumps is a promising approach concerning energy saving and has been detailed in Chapter 5 (page 74).

### **Labye's Iterative Discontinuous Method (LIDM) for OFR**

The approach proposed by Labye (1981), called Labye's Iterative Discontinuous Method (LIDM), for optimizing pipe sizes in an irrigation network is described in this section. This method is developed in two stages.

In the first stage, an initial solution is constructed giving, for each section  $k$  of the network, the minimum commercial diameter ( $D_{\min}$ ) according to the maximum allowable flow velocity ( $v_{\max}$ ) in a pipe, when the pipe conveys the calculated discharge ( $Q_k$ ). The diameter for the section  $k$  is calculated by the relationship:

$$(D_{\min})_k = \sqrt{\frac{4Q_k}{\pi v_{\max}}} \quad (22)$$

After knowing the initial diameters, it is possible to calculate the piezometric elevation  $(Z_0)_{\text{in}}$  at the upstream end of the network, which satisfies the minimum head ( $H_{j,\min}$ ) required at the most unfavorable hydrant ( $j$ ):

$$(Z_0)_{\text{in}} = H_{j,\min} + ZT_j + \sum_{0 \rightarrow M_j} Y_k \quad (23)$$

where  $\sum_{0 \rightarrow M_j} Y_k$  are the head losses along the pathway ( $M_j$ ) connecting the upstream end of the network to the most unfavourable hydrant.

The initial piezometric elevation  $(Z_0)_{\text{in}}$  relative to the initial diameters solution is therefore calculated through the relationship (23).

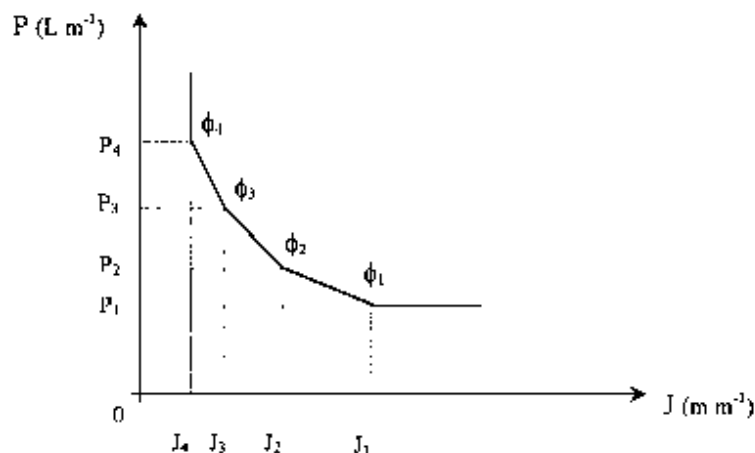
In the second stage, the optimal solution is obtained by iteratively decreasing the upstream piezometric elevation  $(Z_0)_{\text{in}}$  until reaching the effectively available upstream piezometric elevation,  $Z_0$ , by selecting, for each iteration, the sections for which an increase in diameter produces the minimum increase of the network cost. The selection process at each iteration is carried out as described below.

At any iteration  $i$ , the commercial pipe diameters (at most two diameters per section (Labye, 1966))  $D_{s+1}$  and  $D_s$  (with  $D_{s+1} > D_s$ ) are known. The coefficient:

$$b_s = \frac{P_{s+1} - P_s}{J_s - J_{s+1}} \quad (24)$$

is defined (Fig. 31), where  $P_s$  [ITL] and  $J_s$  [ $\text{m m}^{-1}$ ] are, respectively, the cost and the friction loss per unit length of pipe diameter  $D_s$  [m], and  $P_{s+1}$  [ITL] and  $J_{s+1}$  [ $\text{m m}^{-1}$ ] are, respectively, the cost and the friction loss per unit length of pipe diameter  $D_{s+1}$  [m].

**FIGURE 31**  
Characteristic curve of a section





The minimum cost variation,  $dP$ , of the elementary scheme  $(SN)^*$  (Figure 32) of any sub-network,  $(SN)$ , and a section  $k$  in series with  $(SN)$ , for any given variation,  $dH'$ , of the head  $H'$  [m], at the upstream end of  $(SN)^*$ , is obtained by solving the following "local" linear programming (Ait Kadi *et al.*, 1990):

$$\min. dP = -b_{s,SN} dH - b_{s,k} dY_k \quad (25)$$

subject to:

$$dH + dY_k = dH' \quad (26)$$

where  $dH$  [m] and  $dY_k$  [m] are, respectively, the variation of the head at the upstream end of  $(SN)$  and the variation of the friction loss in section  $k$ .

The optimal solution of the equations (25) and (26) is:

$$dH = dH' \quad \text{and} \quad dY_k = 0 \quad \text{if} \quad b_{s,SN} < b_{s,k} \quad (27)$$

$$dH = 0 \quad \text{and} \quad dY_k = dH' \quad \text{if} \quad b_{s,SN} > b_{s,k} \quad (28) \text{ ----}$$

Therefore, the minimum cost variation,  $dP$ , of  $(SN)^*$  can be written as:

$$dP = -b^* dH' \quad \text{with} \quad (29)$$

$$b^* = \min(b_{s,SN}, b_{s,k}) \quad (30)$$

Hence, proceeding from any terminal section of the pipe network, the equation (30) can be used to determine the section that will vary at each iteration. Note that in this process,  $b_{s,SN}$  of the assembly of two sections in derivation is equal to (Fig. 32):

$$b_{s,SN} = b_{s,1} + b_{s,2} \quad (31)$$

whereas, for two sections in series it would be equal to:

$$b_{s,SN} = \min(b_{s,1}, b_{s,2}) \quad (32)$$

In the case of a terminal section with a head in excess at its downstream end ( $H_j > H_{j,min}$ ), the value of  $b_{s,SN}$  to be used in the process is equal to zero as long as the excess head prevails.

The magnitude of  $dH_i$ , for each iteration  $i$ , is determined as:

$$dH_i = \min(EH_i, \Delta Y_i, \Delta Z_i) \quad (33)$$

where:

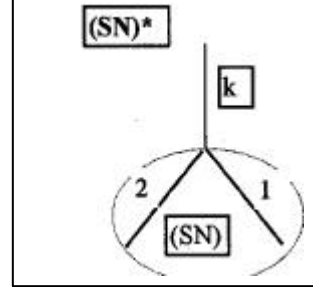
$EH_i$  is the minimum value of the excess head prevailing in all the nodes where the head will change;

$\Delta Y_i$  is the minimum value of  $(Y_{k,i} - Y^*)$  for those sections which will change in diameters, with  $Y_{k,i}$  being the value of the head loss in the section  $k$  at iteration  $i$ , and  $Y^*$  is, for this section, the value of the head loss corresponding to the largest diameter over its entire length if the section has two diameters, or the next greater diameter if the section has only one diameter. Note that for those terminal sections with head in excess ( $H_j > H_{j,min}$ ),  $\Delta Y_i$  is equal to the value of this excess ( $H_j - H_{j,min}$ ).

$\Delta Z_i$  is the difference between the upstream piezometric elevation,  $(Z_0)_i$ , at iteration  $i$ , and the piezometric elevation,  $Z_0$ , effectively available at the upstream end of the network.

The iterative process is continued until  $Z_0$  is reached, obtaining the optimal solution. An example is described in Box 3.

**FIGURE 32**  
**Characteristic curve of a section**



**BOX 3**

Using the LIDM, compute the optimal pipe sizes for the network in figure. The following range of flow velocity is considered:  $v_{\max} = 2.5 \text{ ms}^{-1}$ ,  $v_{\min} = 0.2 \text{ ms}^{-1}$ . The available upstream piezometric elevation is:  $Z_0 = 165 \text{ m a.s.l.}$

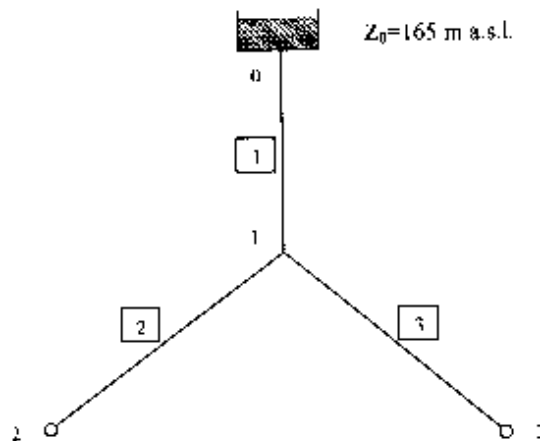


TABLE 3.1

Characteristic of the network

Section Number	Initial Node	Final Node	Discharge ( $\text{l s}^{-1}$ )	Length (m)	Land Elevation (m a.s.l.)	$H_{\min}$ (m)
1	0	1	35	1000	110	20
2	1	2	15	1000	120	30
3	1	3	20	1000	122	30

TABLE 3.2

List of commercial pipes

$\varnothing$ (mm)	Thickness (mm)	$\gamma_{\text{Bazin}}$ ( $\text{m}^{0.5}$ )	Cost (ITL $\text{m}^{-1}$ )
110	5.3	0.06	14 000
160	7.7	0.06	29 300
200	9.6	0.06	55 000
225	10.8	0.06	65 000
250	11.9	0.06	80 000
315	15.0	0.06	105 000

Darcy's equation was used for calculating the friction slope  $J$  [ $\text{m m}^{-1}$ ] in the pipes:

$$J = u Q^2$$

where

$$u = 0.000857 (1 + 2\gamma D^{-0.5})^2 D^{-5}$$

$\gamma$  is the roughness parameter of Bazin, expressed in  $\text{m}^{0.5}$ .  $Q$  ( $\text{m}^3 \text{s}^{-1}$ ) is the discharge flowing into the section and  $D$  (m) is the diameter of the section.

**BOX 3 Cont'd**

By applying the Darcy's equation, the following values of  $u$  are obtained:

$\bar{E}$ (mm)	$u$
110	168.343079
100	23.462156
200	7.293085
225	3.945896
250	2.269560
315	0.683712

The following flow velocities are obtained for each section:

Section 1		Section 2		Section 3	
$Q = 35 \text{ l s}^{-1}$		$Q = 15 \text{ l s}^{-1}$		$Q = 20 \text{ l s}^{-1}$	
$\bar{E}$ (mm)	$v$ (ms <sup>-1</sup> )	$\bar{E}$ (mm)	$v$ (ms <sup>-1</sup> )	$\bar{E}$ (mm)	$v$ (ms <sup>-1</sup> )
110	4.51	110	1.93	110	2.58
160	2.13	160	0.91	160	1.22
200	1.36	200	0.58	200	0.78
225	1.08	225	0.46	225	0.62
250	0.85	250	0.37	250	0.50
315	0.55	315	0.24	315	0.31

The list of the commercial pipe diameters available for each section, according the allowable range of flow velocities is:

Section Number	$\bar{E}$ (mm)	$\bar{E}$ (mm)	$\bar{E}$ (mm)	$\bar{E}$ (mm)	$\bar{E}$ (mm)	$\bar{E}$ (mm)
1	-	160	200	225	250	315
2	110	160	200	225	250	315
3	-	160	200	225	250	315

Section Number	Discharge (l s <sup>-1</sup> )	$\bar{E}_{\max}$ (mm)	$\bar{E}_{\min}$ (mm)
1	35	315	160
2	15	315	110
3	20	315	160

The friction losses are:

Section 1		Section 2		Section 3	
$\bar{E}$ (mm)	Jx1000	$\bar{E}$ (mm)	Jx1000	$\bar{E}$ (mm)	Jx1000
160	28.74	110	37.88	160	9.39
200	8.93	160	5.28	200	2.92
225	4.83	200	1.64	225	1.58
250	2.78	225	0.84	250	0.91
315	0.84	250	0.51	315	0.27
		315	0.15		

**BOX 3 Cont'd**

The values of the coefficient  $\beta$  are:

$$b_s = \frac{P_{s+1} - P_s}{J_s - J_{s+1}}$$

Section 1	
F (mm)	b/1000
160	1297
200	2439
225	7304
250	1287
315	

Section 2	
F (mm)	b/1000
110	469
160	7064
200	13278
225	39769
250	70064
315	

Section 3	
F (mm)	b/1000
160	3974
200	7469
225	22370
250	39411
315	

*- Initial solution*

For each section k of the network, the initial solution is constructed giving the minimum commercial diameter ( $D_{\min}$ ) according to the maximum allowable flow velocity ( $v_{\max}$ ) in the pipe.

After knowing the initial diameters, it is possible to calculate the piezometric elevation ( $Z_0$ )<sub>in</sub> at the upstream end of the network, which satisfies the minimum head ( $H_{j,\min}$ ) required at the most unfavourable hydrant (j):

$$Z_{0-2} = 150 + J_{110,2} * L_2 + J_{160,1} * L_1 = 150 + 37.88 + 28.74 = 216.63 \text{ m a.s.l.}$$

$$Z_{0-3} = 152 + J_{160,3} * L_3 + J_{160,1} * L_1 = 152 + 9.39 + 28.74 = 190.13 \text{ m a.s.l.}$$

The most unfavourable hydrant in this example is number 2 and, therefore, the initial solution is represented by the upstream piezometric elevation  $Z_{0,\text{in}} = 216.62 \text{ m a.s.l.}$  and the initial minimum diameters:

Section Number	$\Phi_{\min}$ (mm)
1	160
2	110
3	160

By starting from the node 0 along the path 0 → 3, it is possible to compute the pressure head on the node 2 and 3. On the node 2 there is no excess head, while on the node 3 there is excess equal to:

$$\Delta H_3 = 216.62 - 28.74 - 9.39 - 152 = 26.49 \text{ m}$$

This implies that  $\beta_3=0$ . According to the theory of LIDM, we have to compare  $\beta_1=1297$  and  $\beta_2=469$  in series. The minimum value correspond to  $\beta_2$ , so we have to change the diameter of the section 2 from  $\phi = 110 \text{ mm}$  to  $\phi = 160 \text{ mm}$ .

At this stage, we have to decrease  $Z_{0,\text{in}}$  so we select the minimum value among:

- Excess head on node where the head change: excess on node 3 = 26.49 m

**BOX 3 Cont'd**

$$- Z_{0,in} - Z_0 = 216.62 - 165 = 51.62 \text{ m}$$

$\Delta Y \longrightarrow$  by changing the diameter of the section 2 from  $\phi = 110 \text{ mm}$  to  $\phi = 160 \text{ mm}$ :

$$\Delta Y = 37.88 - 5.28 = 32.60 \text{ m}$$

The minimum value is 26.49 m

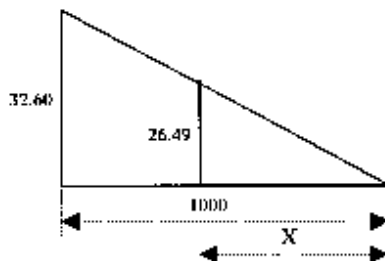
- The solution at the first iteration is:

$$Z_{0,1} = 216.62 - 26.49 = 190.13 \text{ m a.s.l.}$$

This gives the diameters shown below:

Section Number	$E_{min}$ (mm)
1	160
2	160 *
3	160

\* By changing the whole diameter on the section 2 we would recover 32.60 m and, in fact, we decrease the upstream piezometric elevation only 26.49 m. This implies that we have a mixage on section 2 between the diameters  $\phi = 110 \text{ mm}$  and  $\phi = 160 \text{ mm}$ . The lengths of such mixage are:



Note that  $1000 / 32.60 = X / 26.49$  and

$$X = \frac{1000 \cdot 26.49}{32.60} = 813 \text{ m}$$

This means that 813 m of the section 2 needs a diameter  $\phi = 160 \text{ mm}$  and  $(1000-813) = 187 \text{ m}$  needs a diameter  $\phi = 110 \text{ mm}$ .

At this stage, there is no excess head on the nodes 2 and 3. Therefore, the sections 2 and 3 are in parallel, and the values of the coefficients  $\beta$  for identifying the sections to be changed are

$$\text{- for the sections 2 and 3: } \beta = \beta_2 + \beta_3 = 469 + 3974 = 4453$$

$$\text{- for the sections 1: } \beta_1 = 1297$$

The minimum value is  $\beta_1$ , therefore we have to increase the diameter on the section 1 from  $\phi = 160 \text{ mm}$  to  $\phi = 200 \text{ mm}$ .

**BOX 3 Cont'd**

We select a the minimum value for  $Z_{0,1}$  among:

- Excess head on the nodes where the head change: no head variations on the nodes 2 and 3 occur;

$$- Z_{0,1} - Z_0 = 190.13 - 165 = 25.13 \text{ m}$$

$\Delta Y \longrightarrow$  by changing the diameter of the section 1 from  $\phi = 160 \text{ mm}$  to  $\phi = 200 \text{ mm}$ :

$$\Delta Y = 28.74 - 8.93 = 19.81 \text{ m}$$

The minimum value is 19.81 m, therefore we have to change diameter on the whole section 1 from  $\phi = 160 \text{ mm}$  to  $\phi = 200 \text{ mm}$ .

Therefore, the solution at the second iteration is:

$$Z_{0,2} = 190.13 - 19.81 = 170.32 \text{ m a.s.l.}$$

This gives the diameters shown below:

Section Number	$\bar{E}_{\min}$ (mm )
1	200
2	160-110
3	160

At this stage, there is no excess head on the nodes 2 and 3. Therefore, the sections 2 and 3 are considered in parallel, and the values of the coefficients  $\beta$  for identifying the sections to be changed are

$$- \text{for the sections 2 and 3: } \beta = \beta_2 + \beta_3 = 469 + 3974 = 4453$$

$$- \text{for the sections 1: } \beta_1 = 2439$$

The minimum value is  $\beta_1$ , so we increase the diameter on the section 1 from  $\phi = 200 \text{ mm}$  to  $\phi = 225 \text{ mm}$ .

We select a minimum value  $Z_{0,1}$  among:

- Excess head on the nodes where the head change: no head variations on the nodes 2 and 3 occur;

$$- Z_{0,1} - Z_0 = 170.32 - 165 = 5.32 \text{ m}$$

$\Delta Y \longrightarrow$  by changing the diameter of the section 1 from  $\phi = 200 \text{ mm}$  to  $\phi = 225 \text{ mm}$ :

$$\Delta Y = 8.93 - 4.83 = 4.10 \text{ m}$$

The minimum value is 4.10 m, so we change the diameter on the whole section 1 from  $\phi = 200 \text{ mm}$  to  $\phi = 225 \text{ mm}$ .

Therefore, the solution at the second iteration is:

$$Z_{0,2} = 170.32 - 4.10 = 166.22 \text{ m a.s.l.}$$

This gives the diameters shown below:

Section Number	$\bar{E}_{\min}$ (mm )
1	225
2	160-110
3	160

**BOX 3 Cont'd**

At this stage, there is still no any excess head on the nodes 2 and 3, so sections 2 and 3 are considered in parallel, and the values of the coefficients  $\beta$  for identifying the sections to be changed are

- for the sections 2 and 3:  $\beta = \beta_2 + \beta_3 = 469 + 3974 = 4453$

- for the sections 1:  $\beta_1 = 7304$

The minimum value is  $\beta = \beta_2 + \beta_3$ , so we increase the diameter on the sections 2 and 3.

We select the minimum  $Z_{0,2}$  value among:

- Excess head on the nodes where the head change: no head variation on the nodes 2 and 3 occur;

-  $Z_{0,2} - Z_0 = 66.22 - 165 = 1.22$  m

$\Delta Y \longrightarrow$  by changing the diameter of the section 3 from  $\phi = 160$  mm to  $\phi = 200$  mm:

$\Delta Y_3 = 9.38 - 2.92 = 6.46$  m

$\Delta Y \longrightarrow$  by changing the whole diameter of the section 2 from  $\phi = 110$  mm to  $\phi = 160$  mm:

$\Delta Y_2 = 32.60 - 26.49 = 6.11$  m

The minimum value is 1.22 m, so we change part of the diameter  $\phi = 110$  mm in the section 2 to  $\phi = 160$  mm and we change part of the diameter  $\phi = 160$  mm in the section 3 into  $\phi = 200$  mm. We will have mixage on the sections 2 and 3.

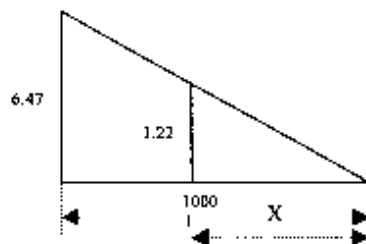
The solution at the third iteration is:

$Z_{0,3} = 166.22 - 1.22 = 165$  m a.s.l.

This gives the diameters shown below:

Section Number	$\bar{E}_{\min}$ (mm)
1	225
2	160-110
3	200-160

Mixage on the section 3:



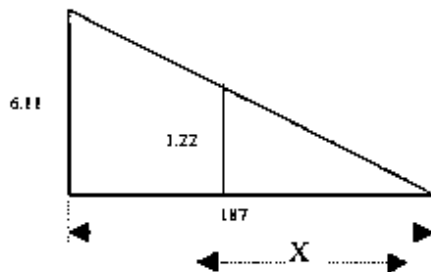
Note that  $1000 / 6.47 = X / 1.22$ , so

$$X = \frac{1000 \cdot 1.22}{6.47} = 189 \text{ m}$$

This means that 189 m of the section 3 needs a diameter  $\phi = 200$  mm and  $(1000 - 189) = 811$  m needs a diameter  $\phi = 160$  mm.

**BOX 3 Cont'd**

Mixage on the section 2:

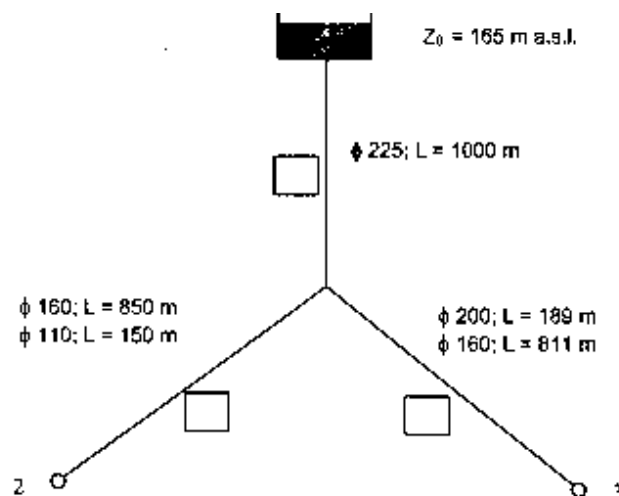


Note that  $187 / 6.11 = X / 1.22$ , so

$$X = \frac{187 \cdot 1.22}{6.11} = 37 \text{ m}$$

This means that additional 37 m of the section 2 needs a diameter  $\phi = 160$  mm, so that  $(813+37)=850$  m needs a diameter  $\phi = 160$  mm, and  $(1000-850)=150$  m needs a diameter  $\phi = 110$  mm.

The final solution is:



The cost of the network is: 126 162 300 ITL

### OPTIMIZATION OF PIPE DIAMETERS WITH SFRM

The above described optimization method considers fixed discharges flowing into the pipes of the network. This is not true for irrigation systems, especially when on-demand delivery schedules are adopted. In these systems, the discharges are strongly variable in time because of the extreme variability of the configurations of hydrants operating simultaneously, which depend on cropping patterns, irrigated areas, meteorological conditions, and farmer's behaviour. For this reason the LIDM has been extended (Ait Kadi *et al.*, 1990; Lamaddalena, 1997) to take



into account the hydraulic happenings in the network which consists of Several Flow Regimes (SFRM). Using a computer program, flow regimes were generated using the RGM approach presented in chapter 3.

### Labye's Iterative Discontinuous Method extended for SFR (ELIDM)

The algorithm of the Labye's iterative discontinuous method extended for SFR (Labye, 1981; Ait Kadi *et al.*, 1990) is divided in two stages. In the first stage, an initial solution is constructed giving, for each section  $k$  of the network, the minimum commercial diameter  $(D_{\min})_k$  according to the maximum allowable flow velocity  $(v_{\max})$  when the pipe conveys the maximum discharge  $(Q_{\max,k})$  for all the configurations. The diameter for the section  $k$  is calculated by the relationship:

$$(D_{\min})_k = \sqrt{\frac{4 Q_{\max,k}}{\pi v_{\max}}} \quad (34)$$

After knowing the initial diameters, it is possible to calculate, for each configuration  $r$ , the piezometric elevation  $(Z_0)_{in,r}$  [m a.s.l.] at the upstream end of the network, satisfying the minimum head,  $H_{j,\min}$  [m], required at the most unfavorable hydrant,  $j$ :

$$(Z_0)_{in,r} = H_{j,\min} + ZT_j + \sum_{0 \rightarrow M_j} Y_{k,r} \quad (35)$$

where  $\sum_{0 \rightarrow M_j} Y_{k,r}$  are the head losses along the pathway  $(M_j)$  connecting the upstream end of the network to the most unfavorable hydrant when the flow regime occurs.

Within all the configurations of discharges, only those requiring an upstream piezometric elevation,  $(Z_0)_{in,r}$ , greater than the effectively available one,  $Z_0$ , are considered.

With the first of  $C$  possible configurations  $(r_1)$ , the initial piezometric elevation  $(Z_0)_{in,r_1}$ , relative to the initial diameters solution,  $(D_{\min})_k$ , is calculated through the relationship (35). The optimal solution for  $r_1$  is obtained by iteratively decreasing the upstream piezometric elevation. For each iteration, select sections for which a change in diameter produces the minimum increase in the network cost. The selection process for each iteration is described below.

At any iteration  $(I)$ , the commercial pipe diameters are known. There are no more than two diameters per section (Labye, 1966) with  $D_{s+1} > D_s$ . The coefficient:

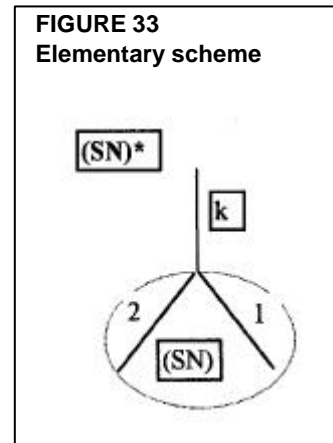
$$b_s = \frac{P_{s+1} - P_s}{J_s - J_{s+1}} \quad (24)$$

is defined, where  $P_s$  [ITL] and  $J_s$  [ $\text{m m}^{-1}$ ] are, respectively, the cost and the friction loss per unit length of pipe diameter  $D_s$  [m], and  $P_{s+1}$  [ITL] and  $J_{s+1}$  [ $\text{m m}^{-1}$ ] are, respectively, the cost and the friction loss per unit length of pipe diameter  $D_{s+1}$  [m].

The minimum cost variation,  $dP$ , of the elementary scheme  $(SN)^*$  (Figure 33) of any sub-network,  $(SN)$ , and a section  $k$  in series with  $(SN)$ , for any given variation,  $dH'$ , of the head  $H'$  [m], at the upstream end of  $(SN)^*$ , is obtained by solving the following "local" linear programming (Ait Kadi *et al.*, 1990):

$$\min. dP = - b_{s,SN} dH - b_{s,k} dY_k \quad (36)$$

**FIGURE 33**  
**Elementary scheme**



subject to:

$$dH + dY_k = dH' \quad (37)$$

where  $dH$  [m] and  $dY_k$  [m] are, respectively, the variation of the head at the upstream end of (SN) and the variation of the friction loss in section  $k$ . The optimal solution of the equations (36) and (37) is:

$$dH = dH' \quad \text{and} \quad dY_k = 0 \quad \text{if} \quad \mathbf{b}_{s,SN} < \mathbf{b}_{s,k} \quad (38)$$

$$dH = 0 \quad \text{and} \quad dY_k = dH' \quad \text{if} \quad \mathbf{b}_{s,SN} > \mathbf{b}_{s,k} \quad (39)$$

Therefore, the minimum cost variation,  $dP$ , of (SN)<sup>\*</sup> can be written as:

$$dP = - \mathbf{b}^* dH' \quad \text{with} \quad (40)$$

$$\mathbf{b}^* = \min (\mathbf{b}_{s,SN}, \mathbf{b}_{s,k}) \quad (41)$$

Hence, proceeding from any terminal section of the pipe network, the equation (40) can be used to determine the section that will vary at each iteration. Note that in this process (Figure 33),  $\mathbf{b}_{s,SN}$  of the assembly of two sections in derivation is equal to:

$$\mathbf{b}_{s,SN} = \mathbf{b}_{s,1} + \mathbf{b}_{s,2} \quad (42)$$

whereas, for two sections in series it is equal to:

$$\mathbf{b}_{s,SN} = \min (\mathbf{b}_{s,1}, \mathbf{b}_{s,2}) \quad (43)$$

It should be mentioned that in the case of a terminal section with a head in excess at its downstream end ( $H_j > H_{j,min}$ ), the value of  $\mathbf{b}_{s,SN}$  which is used in the process is equal to zero as long as the excess head prevails.

The magnitude of  $dH_{i,r}$ , for each iteration  $i$  and configuration  $r$ , is determined as:

$$dH_{i,r} = \min (EH_i, \Delta Y_i, \Delta Z_i) \quad (44)$$

where:

$EH_i$  is the minimum value of the excess head prevailing in all the nodes where the head will change;

$\Delta Y_i$  is the minimum value of  $(Y_{ki} - Y^*)$  for those sections that change in diameter, with  $Y_{k,i}$  being the value of the head loss in the section  $k$  at iteration  $i$ , and  $Y^*$  is, for this section, the value of the head loss corresponding to the largest diameter over its entire length if the section has two diameters, or the next greater diameter if the section has only one diameter. Note that for those terminal sections with head in excess ( $H_j > H_{j,min}$ ),  $\Delta Y_i$  is equal to the value of this excess ( $H_j - H_{j,min}$ ).

$\Delta Z_i$  is the difference between the upstream piezometric elevation,  $(Z_0)_{i,r1}$ , at iteration  $i$  when the flow regime  $r_1$  occurs, and the piezometric elevation,  $Z_0$ , effectively available at the upstream end of the network.

The iterative process is continued until  $Z_0$  is reached, obtaining the optimal solution for the examined configuration,  $r_i$ . This solution is considered as the initial solution for the next configuration ( $r_2$ ) and the iterative process is repeated. The process is continued until all the discharge configurations,  $C$ , have been considered. Diameters of the sections can only increase or remain constant for the previous analysed configurations to be satisfied. The final optimal solution should satisfy all the examined discharge configurations.

A computer program for calculating the optimal pipe size of an irrigation network under both several discharge configurations and in case of single flow regime has been developed (Lamaddalena, 1997) and integrated into the COPAM package. The ELIDM has been validated using the linear programming formulation extended for several flow regimes (Ben Abdellah, 1995; Lamaddalena, 1997). The commercial computer package (LINDO) was adapted for calculating the optimal solution for the irrigation network with linear programming.

Darcy's equation was used in ELIDM for calculating the friction slope  $J$  [ $\text{m m}^{-1}$ ] in the pipes:

$$J = 0.000857 (1 + 2\gamma D^{-0.5})^2 Q^2 D^{-5} = u Q^2 \quad (45)$$

where  $\gamma$  is the roughness parameter of Bazin, expressed in  $\text{m}^{0.5}$ . The other variables have been already defined in the text. In Table 4 the values of the roughness Bazin parameters for different types of pipes are reported. They are compared with the values of the equivalent homogeneous roughness  $\epsilon$  (mm). Information on the different types of pipes is reported in Annex 2.

**TABLE 4**  
**Bazin roughness parameter ( $\gamma$ ) for different types of pipes**

TYPE OF PIPE	Equivalent homogenous roughness $\epsilon$ ( mm )	$\gamma$ ( $\text{m}^{0.5}$ )
<b>1- Technically smooth tubes ( glass, brass, drawn copper, resin )</b>	$0 \div 0.02$	--
<b>2- Steel pipes</b>		
A ) Time degradable coverings		
- New pipes, varnished by centrifugation	0.05	--
- Bitumened by immersion	$0.1 \div 0.015$	$\leq 0.06$
- In current duty with light rust	$0.2 \div 0.4$	0.10
- With asphalt or tar applied by hands	$0.5 \div 0.6$	0.16
- With diffused tubercolisation	$1 \div 3$	0.23
B ) Non degradable coverings		
- Cement applied by centrifugation	$0.05 \div 0.15$	$\leq 0.06$
<b>3- Welded sheet-pipes</b>		
- In good conditions	$0.2 \div 0.3$	0.10
- In current duty with crusting	$0.4 \div 1.0$	0.16
<b>4- Nailed sheet-pipes</b>		
- 1 line of longitudinal nails	$0.3 \div 0.4$	0.10
- 2 lines of longitudinal nails	$0.6 \div 0.7$	0.16
- Idem with crusting	Till 3.0	0.30
- 4-6 lines of longitudinal nails	2.0	0.23
- 6 lines of longitudinal nails + 4 transversal	3.0	0.30
- Idem with crusting	Till 5.0	0.36
<b>5- Cast iron pipes</b>		
- With centrifuged-cemented covering	0.1	$\leq 0.06$
- New, covered internally with bitumen	0.15	0.06
- New, not covered	$0.2 \div 0.4$	0.10
- With light crusting	$0.4 \div 1.0$	0.16
- In current duty, partially rusted	$1.0 \div 2.0$	0.23
- strongly encrusted	$3.0 \div 5.0$	0.36
<b>6- Cement-pipes</b>		
- Asbestos cement	0.1	$\leq 0.06$
- New reinforced concrete, plaster perfectly smooth	$0.1 \div 0.15$	0.06
- Reinforced concrete with smooth plaster, in work for many years	2.0	0.23
- Tunnels with cement plaster, depending on the degree of finish	$2.0 \div 5.0$	$0.23 \div 0.36$

The applicability of the ELIDM in the case of a network with SFR is presented in the next section.

### Applicability

In this section, the applicability of the Labye iterative discontinuous method has been verified through the linear programming formulation. Both the methods have been applied to a network under several flow regimes.

The RGM presented in chapter 3 was previously used for generating different sets of random discharge configurations. These sets of configurations have been utilized for the calculation of the optimal diameters using the ELP and ELIDM programs.

The difference in cost of the network under study is shown in Figure 34. It is always less than 2% because in the ELP formulation all the discharge configurations are considered as a unique block and, thus, the exact solution is obtained. In the ELIDM program the configurations are considered one by one, with the final solution for the configuration  $r_i$  being the initial solution for the  $r_{i+1}$  configuration. Therefore, diameters of the sections can only increase or remain constant when the configurations change. For this reason, the order of the discharge configurations during the calculation has little effect on the optimal solution.

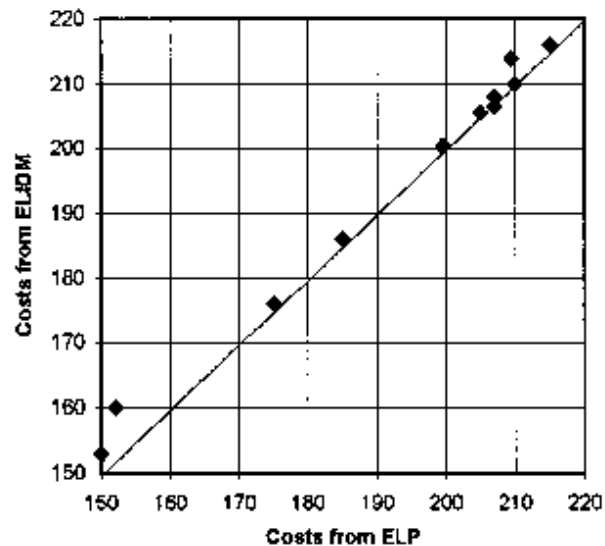
The differences between the ELP and the ELIDM are small. When several discharge configurations are considered, the latter method is preferred because it is more flexible and the computation time is very short (a few seconds for each discharge configuration). Furthermore, the computer software integrated in the COPAM package is user-friendly and allows the calculation of networks up to 1000 nodes under more than 1000 flow regimes.

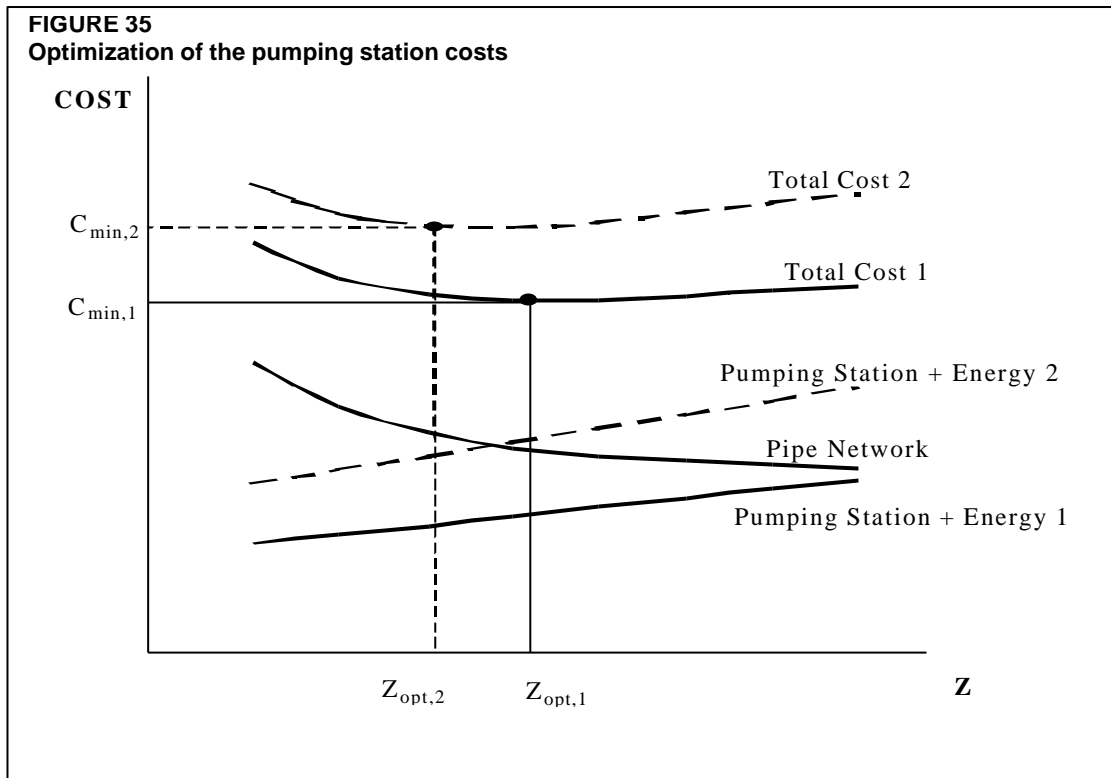
### THE INFLUENCE OF THE PUMPING STATION ON THE PIPE SIZE COMPUTATION

The basic relationship between the pumping station at the upstream end of an irrigation system and the pipe network is that the higher upstream pressure the lower the diameters of the network, and vice versa. The other important relationship is that the higher the pressure the higher the costs of the pumping station and the energy.

Through the optimization programs we can compute the variation of the network costs for different upstream piezometric elevations " $z$ ". On the other hand, the piezometric elevation can be translated into energy and pumping station costs. The total cost for the irrigation system (network, pumping station and energy) is given by the sum of both costs. The total cost is represented by a curve having a minimum value corresponding to an optimal solution (Figure 35). It is understood that all costs must be referred to the same life span and therefore must be actualized.

**FIGURE 34**  
Costs ( $10^6$  ITL) of the network in the example, calculated by using the ELP and the ELIDM for different sets of flow regimes





Notwithstanding the correctness of this mathematical approach, a practical limitation should be highlighted concerning the uncertainty in the energy cost evolution. In fact, there are examples of pumping stations that were designed when energy costs were relatively low (before the petrol crisis) and at that time an optimal upstream piezometric elevation was found that nowadays it is not economical and causes very high water fees (Figure 35).

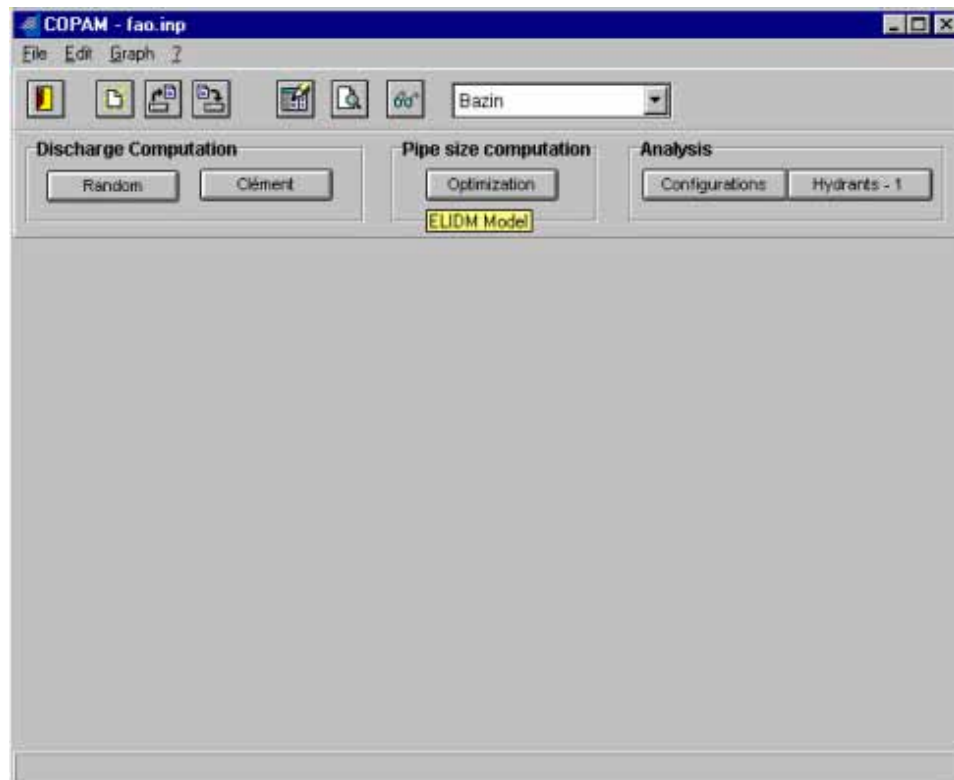
According to these considerations, the designer should use the optimization procedures with a critical view. For this reason the automatic optimization of the upstream piezometric elevation has not been included in COPAM. Nevertheless COPAM gives the possibility to compute the optimal cost of the network for several piezometric elevations but each one has to be computed one by one. The designer can use these results to evaluate different pumping station alternatives taking into account that related costs can only be estimated, in a reliable way, under present conditions.

### THE USE OF COPAM FOR PIPE SIZE COMPUTATION

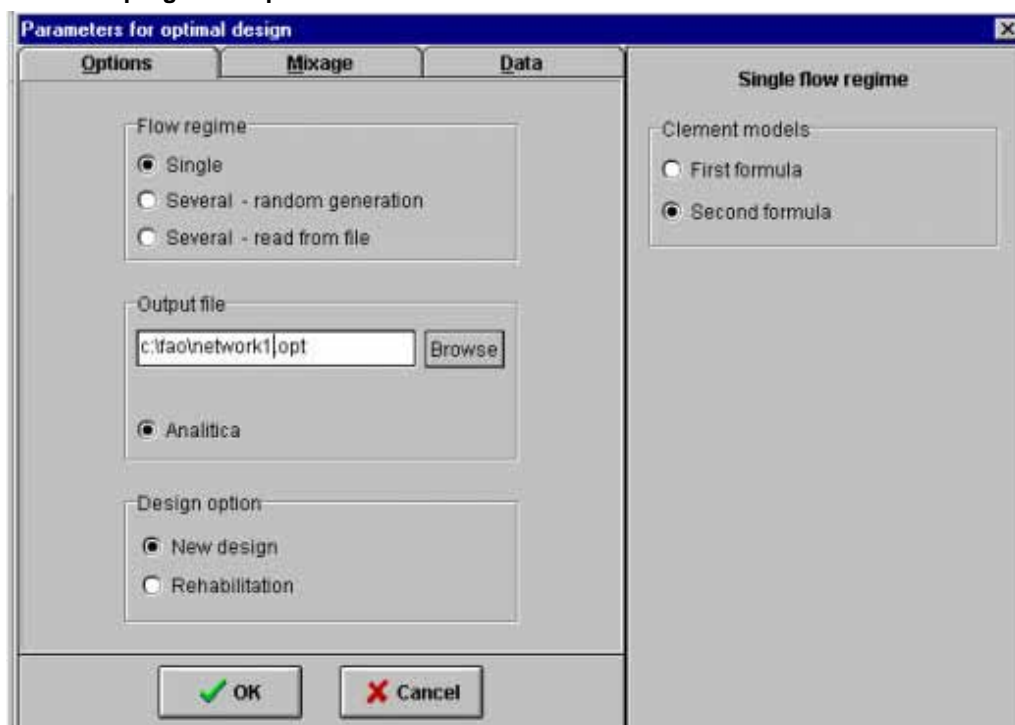
The use of the optimization program integrated in the COPAM package is easy. After completing the input file, as described in the chapter 3, by clicking on the “Optimization” button (Figure 36) the page control in Figure 37 will appear on the screen.

Three alternatives are available within the “Options” Tab control, concerning the flow regimes (Figure 37): single flow regimes, several flow regimes directly generated by the program, and several flow regimes previously generated and stored in a file to be read. The last option allows network optimization also in the case of rotation delivery schedule. In fact, the flow regimes are computed according to the planned irrigation schedule and stored in a file that will be read by the program. Concerning the option “single flow regimes”, the program allows the computation of discharges by using the 1<sup>st</sup> or 2<sup>nd</sup> Clément model.

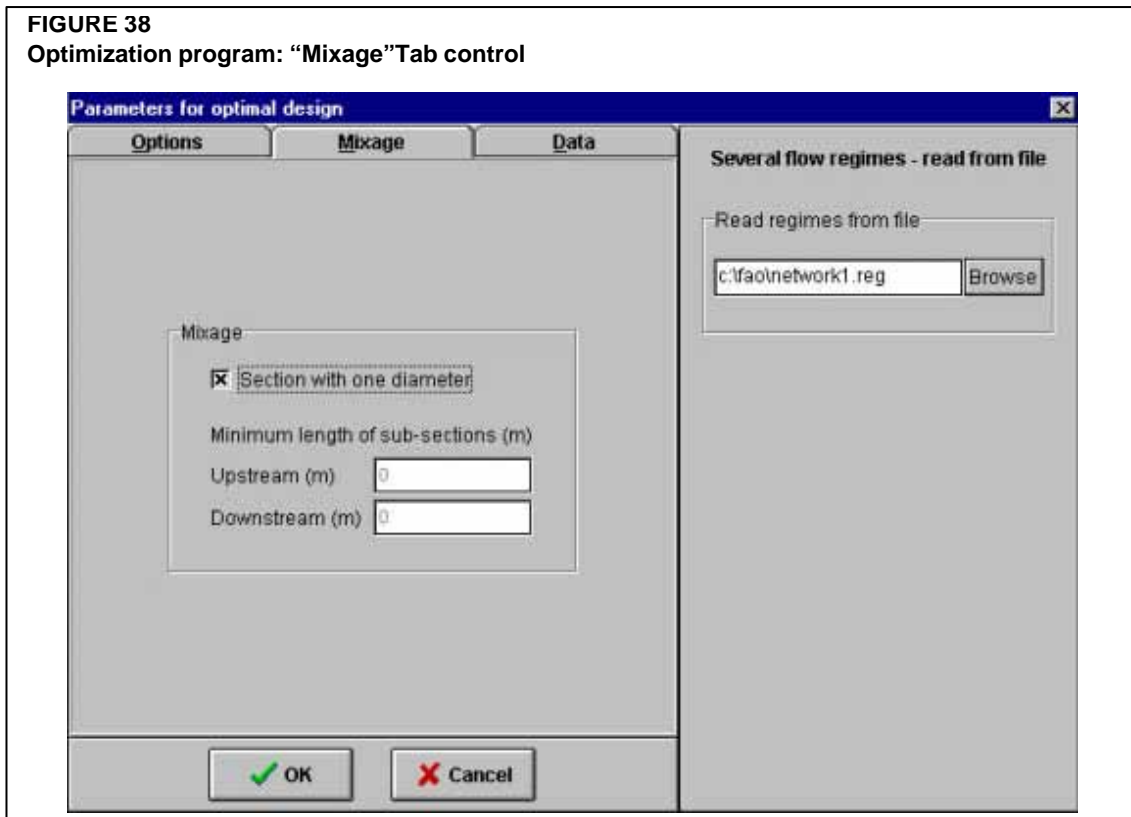
**FIGURE 36**  
Pipe size computation program



**FIGURE 37**  
Optimization program: "Options" Tab control



**FIGURE 38**  
Optimization program: “Mixage”Tab control



The name of the output file is typed in the appropriate edit box. The extension “.opt” is automatically assigned to this output file. All information concerning the computation is stored in that file (see the example of the output file in Annex 3). Finally, two additional design options are available in the program: new design and rehabilitation. In the first case, the program computes the optimal pipe size diameters starting from an initial solution obtained by using the smallest diameters respecting the maximum flow velocities constraints (as explained in the chapter 4). In the case of rehabilitation, the initial solution is given by the actual diameters of each section of the network.

The program, within the “Mixage” tab control (Figure 38), gives the possibility to select one diameter for each section or to consider the mixage with two diameters for each section where required. From a practical point of view, one diameter should be selected in order to avoid possible mistakes during the construction phase.

In the “Data” Tab control (Figure 39), all the parameters related to the 1<sup>st</sup> or 2<sup>nd</sup> Clément formula are introduced. These parameters were illustrated in chapter 3. The upstream piezometric elevation and the minimum pressure head required at the hydrants are typed in the appropriate edit boxes.

The software allows also the computation of the optimal pipe size diameters when the minimum pressure head at the hydrants is not a constant. In this case, the radio button “variable” is selected and, in the last column of the input file, the appropriate minimum pressure heads ( $H_{\min}$ ) are typed in correspondence of each hydrant (Figure 40).

**FIGURE 39**  
Optimization program: "Data" Tab control

**Parameters for optimal design**

**Options** **Mixage** **Data**

Minimum head at hydrants (m)

☒ Constant 20.0

☐ Variable

Upstream piezometric elevation (m a.s.l.) 128.00

Additional data

Specific continuous discharge (l/s\*ha) 0.327

Number of terminal open hydrants 4

Uncultivated land 0.00

Clement use coefficient (r) 0.667

Operation quality (Up) 1.645

OK Cancel

**Single flow regime**

Clement models

☒ First formula

☐ Second formula

**FIGURE 40**  
Input file: Edit network layout

**Edit network layout**

Final node	Area (ha)	Hydrant disch. (l/s)	Section length (m)	Land elevation (m a.s.l.)	Diameter (mm)	Hmin hydrants (m)
1	0.00	0.0	500.00	102.00	1200	25.00
2	0.00	0.0	775.00	102.00	800	25.00
3	0.00	0.0	375.00	102.00	800	25.00
4	0.00	0.0	425.00	102.00	800	30.00
5	0.00	0.0	775.00	88.00	700	30.00
6	0.00	0.0	1000.00	94.00	700	30.00
7	0.00	0.0	950.00	74.00	700	30.00
8	0.00	0.0	5.00	74.00	700	35.00
9	0.00	0.0	650.00	70.00	700	35.00

+ Add Node - Canc Node + Ins Node End section Exit

Section 11 of 903



**FIGURE 41**  
Optimization program: “Options” Tab control

When “Several flow regime” is selected within the “Options” Tab control (Figure 41), the final solution or the analytical solution for each flow regime may be printed on the output file by clicking on the appropriate radio button. The name of the output file is typed in the edit box. In this case the final solution is recommended, especially when many flow regimes are generated and the output file might be too long.

When the option “Several flow regimes” is selected, only the upstream piezometric elevation and the minimum pressure head are typed in the appropriate edit boxes in the “Data” Tab control (Figure 42).

In the case of flow regimes generated directly by the program (Option: Several-random generation in Figure 41), the name of the file where the flow regimes are stored needs to be specified in the appropriate box (Figure 41). Also, the maximum upstream discharge and the number of flow regimes to be generated (remember that this number must be multiple of 10) are specified in the appropriate boxes (Figure 41).

These last two inputs are not required when the flow regimes are read from a file (Option: Several-read from file, in Figure 43). Only the name of the file for reading the flow regimes is specified in the appropriate edit box.

### Sensitivity analysis of the ELIDM

The actual number of flow regimes in an irrigation network is large and the optimal solution varies according to the number of flow regimes considered (see Figure 34). Consequently, a sensitivity analysis is required to examine the variation of the optimal solution for several sets of flow regimes.

**FIGURE 42**  
Optimization program: “Data” Tab control

**Parameters for optimal design**

**Options** **Mixage** **Data**

Minimum head at hydrants (m)

☒ Constant 20.0

☐ Variable

Upstream piezometric elevation (m a.s.l.) 128.00

Additional data

Specific continuous discharge (l/(s\*ha)) 0.327

Number of terminal open hydrants 4

Uncultivated land 0.00

Clement use coefficient (r) 0.667

Probability of saturation - Pa (%) 1.000

**Several flow regimes - random**

Write regimes on file

c:\faotflowregime1.reg Browse

Maximum upstream discharge (l/s) 50

Number of flow regimes to generate 100

OK Cancel

**FIGURE 43**  
Optimization program: “Options” Tab control

**Parameters for optimal design**

**Options** **Mixage** **Data**

Flow regime

☐ Single

☐ Several - random generation

☒ Several - read from file

Output file

c:\compaq\w\faot\fao.opt Browse

☒ Final solution

☐ Analitica per ogni regime

Design option

☒ New design

☐ Rehabilitation

**Several flow regimes - read from file**

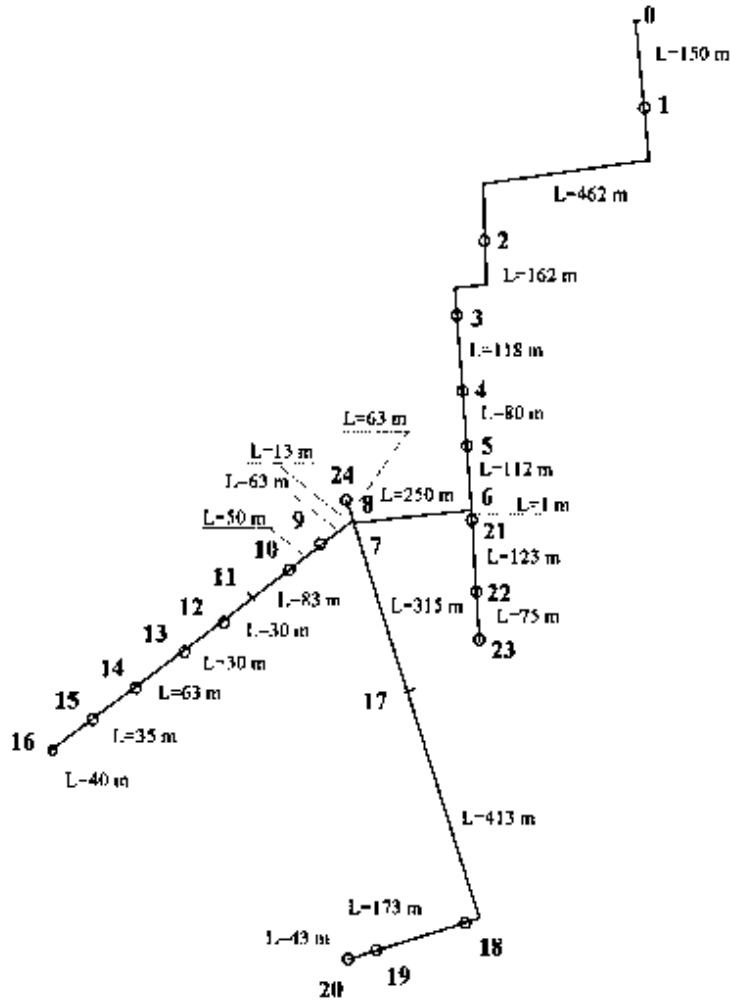
Read regimes from file

c:\compaq\w\faot\canc.ran Browse

OK Cancel

\*\*\*esempio di file input per il calcolo di una rete collettiva \*\*\*\*

**FIGURE 44**  
**Layout of the network**



The ELIDM was used for the irrigation network in Figure 44. Fifty different sets of flow regimes, corresponding to  $50 \text{ l s}^{-1}$ , were randomly generated by the RGM (described in the chapter 3), and used for investigating the sensitivity of the optimal solution versus the number of flow regimes. The input file of the network is reported in the Annex 3, as well as an example of the output file for one random flow regime.

Initially, ten random discharge configurations were generated for calculating the cost  $P_{1,10}$ . Subsequently, another ten random discharge configurations were generated for calculating the cost  $P_{1,10}$ , and so on for every  $P_{1,10}$  up to a total of 50 sets of flow regimes. This procedure was repeated for an increased number of configurations in each set, from 20 up to 100. Therefore, if  $C$  is the number of configurations in each set, varying from 10, 20, ..., 100, the average cost for each  $C$  is:

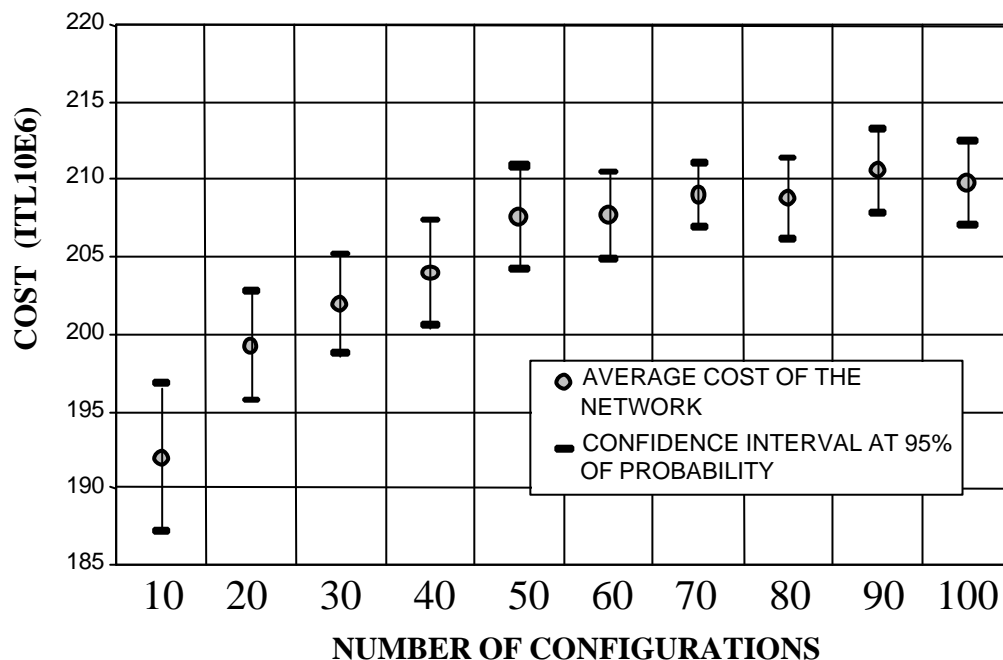
$$\bar{P}_C = \frac{\sum_{i=1}^{50} P_{i,C}}{50} \quad (46)$$

In Figure 45, the average cost of the network and the relative confidence interval at 95% of probability is reported for each set of discharge configurations.

Figure 45 shows that when a larger number of flow regimes is considered a more stable solution is obtained. When the number of flow regimes increases, the designed network is able to satisfy a larger population of possible configurations. Figure 45 also shows that when a larger number of flow regimes is considered, the optimal solution obtained for different repetitions of the same number of flow regimes falls within a narrower range of confidence interval. The results indicate that a minimum of 50 discharge configurations ( $C = 50$ ) is needed for calculating the network in the example.

**FIGURE 45**

**Cost of the network as a function of the number of configurations. The average costs and the confidence interval concern 50 sets of discharge configurations.**



The sensitivity analysis (Figure 45) shows how the cost of the network depends upon the number of flow regimes. Thus, the need to establish criteria for selecting the “most suitable” solution arises. We can define it as a “Satisfactory Solution” instead of an “Optimal Solution.” This satisfactory solution is related to performance criteria. In the next chapter, models for the analysis of on-demand irrigation systems and performance indicators are presented and applied.