It has been shown (Tweeten and Quance, 1969) that under specified assumptions[1/]:

$$E_{ii} = \sum_k E_{ik} E_{ki}.$$

The elasticity of output supply is the input demand elasticity multiplied by production elasticity summed over all inputs. The contribution of input k to the elasticity of supply is $E_{ik}E_{ki}$.

Various methods can be used to estimate the production function. One method is ordinary least squares. A typical formulation is

$$(7.4.7)\ \ln O_i = \ln b_0 + b_1 \ln X_1 + b_2 \ln X_2 + \ldots + b_n \ln X_n$$

where coefficients b_1, $b_2, \ldots,$ b_n are elasticities of production. Taking antilogorithms, equation 7.4.7 is the Cobb-Douglas production function.

$$(7.4.8)\ O_i = b_0 X_1^{b_1} X_2^{b_2} \ldots X_n^{b_n}.$$

Because input quantities tend to move together through time, the high correlation among inputs often causes statistical problems in estimating equation 7.4.7 directly from time series. Indirect methods such as the "dual" are sometimes used[2/].

In theory, the marginal product of input k is equal to the ratio of input price to product price

$$\frac{dO_i}{dX_k} = \frac{P_k}{P_i}.$$

Multiplying both sides by the ratio of input quantity X_k to output, it is apparent that in competetive equilibrium

$$\frac{dO_i}{dX_k}\frac{X_k}{O_i} = \frac{P_k X_k}{P_i O_i}$$

1/ See Gardner (1979) for calculations with interdependent markets.

2/ Extensive literature reports the theory and application of the so called "dual" method of indirect estimates of input demand and output supply elasticities. See Lau and Yotopoulos (1972), Binswanger (1974), and Beatti and Taylor (1985).

and the elasticity of production can be measured by the factor share, the right-hand side, which can be computed directly from secondary data sources. Because the elasticity of production equals the factor share only in competetive equilibrium, methods have been devised to estimate the elasticity of production from factor shares in an economy in disequilibrium (Tyner and Tweeten, 1965).

Input demand equations may be specified in a form similar to that for the supply equation 7.4.1 but with input quantity the dependent variable. Technology may be a less prominent variable in the input demand equation. Suppose that the input elasticity of demand has been calculated and that the price elasticity of demand for input with respect to input price P_k is equal (sign reversed) to the elasticity of demand with respect to outut price P_i. The contribution to supply elasticity of O_i is shown as:

Input	Elasticity of production	Short Run		Long Run	
		Elas. of input dem.	Cont. to sup. elas.	Elas. of input dem.	Cont. to sup. elas.
Fertilizer	.1	.5	.05	2.0	.20
Land	.3	.0	.00	.3	.09
Labour	.3	.1	.03	.5	.15
Machinery	.1	.0	.00	3.0	.30
Irrigation	.2	.1	.02	1.5	.30
	1.0		.10		1.04

A 1 percent increase in the price of commodity i increases fertilizer use by .5 percent in the short run and by 2.0 percent in the long run. Because fertilizer is a small proportion of all inputs, its elasticity of production is not large. But because demand for fertilizer is responsive to price, fertilizer contributes more to the supply elasticity for O_i than does labour which has a larger elasticity of production.

Analysts may also find it useful to disaggregate supply response into area and yield components. It can be shown (Tweeten and Quance, 1969, p.349) that the total supply elasticity E_{ii} can be expressed as

(7.4.9) $E_{ii} = E_{yi} + E_{ai}(1 + E_{ya})$

where E_{yi} is the elasticity of yield with respect to product price P_i, E_{ai} is the elasticity of crop area with respect to P_i, and E_{ya} is the elasticity of yield with respect to area. If crop area is expanded on marginal lands, E_{ya} is negative; if area is expanded on superior lands (say recently irrigated), E_{ya} is positive. If yield and area are independent, then the total supply elasticity of O_i is a simple sum of the yield and area elasticities.

7.4.2 Demand and utilization analysis

Graphs, moving averages, simple linear regression, or other procedures can be used to analyse past trends or predict future trends in consumption or exports. Because the latter two variables are influenced by prices and other factors, a more comprehensive analysis may be desired.

The demand function for a good i may be expressed as

$$(7.4.10) \quad C_i = f(\frac{P_i}{PG}, \frac{P_j}{PG}, \frac{Y}{PG}, G, T)$$

where C_i is quantity demanded, P_i is the price of good i, P_j is the price of related commodities, PG is the general price level, Y is income, G represents government interventions such as provision of food stamps, and T is a trend variable measuring such factors as changes in tastes and preferences. If demand is measured on a per caput basis, C_i and Y_i are expressed per person. If total demand is measured, then C_i and Y are for the total population and a variable N for population must be added to the right-hand side of equation 7.4.10.

The demand equation may be estimated from time series, sometimes with price rather than quantity dependent. The function may be estimated by an ordinary least squares multiple regression equation (see Tweeten, 1967) or an interdependent system of demand equations for related commodities using two-stage least squares or other simultaneous equation system. In other instances, analysts estimate price and income responses from food consumption data obtained from a cross-sectional survey of consumers.

The quantity C_i may be expressed at the retail level or farmgate level. The quantity may be traditional physical units or a nutrient such as protein or total calories. Measures of protein or total calorie demand are particularly useful to examine the impact of public policies on nutrition by income or occupational groups in the population.

As in the case of supply analysis, it is often convenient to express the response of quantity to price or income as elasticities. The elasticity of demand is the percentage increase in consumption associated with a 1 percent increase in price or income. Own-price elasticity is

$$E_{ii} = \frac{\Delta C_i}{C_i} / \frac{\Delta P_i}{P} = \frac{dC_i}{dP_i} \frac{P_i}{C_i} = \frac{d \ln C_i}{d \ln P_i}.$$

Cross-price elasticity is

$$E_{ij} = \frac{\Delta C_i}{C_i} / \frac{\Delta P_j}{P_j} = \frac{dC_i}{dP_j} \frac{P_j}{C_i} = \frac{d \ln C_i}{d \ln P_j}$$

and income elasticity is

$$E_{iY} = \frac{\Delta C_i}{C_i} / \frac{\Delta Y}{Y} = \frac{dC_i}{dY} \frac{Y}{C_i} = \frac{d \ln C_i}{d \ln Y_i}$$

where Δ refers to the change in a variable and d refers to a very small change. If consumers react to real prices and incomes, it can be shown that the sum of the price and income elasticities theoretically is zero. For normal goods, the own-price elasticity is negative and the income elasticity is positive. The cross-price elasticity E_{ij} may be divided into responses for several related commodities, some of which are substitutes for i and have positive elasticities and some of which are complements and have negative elasticities. Analysts have devised procedures to specify an entire matrix of own-price and cross-price elasticities of demand for food in a country (Brandow, 1961: George and King, 1971). Ordinarily, the distinction between short run and long run is not as important for demand as for supply because consumers tend to adjust purchases fairly rapidly to a change in price or income.

Income elasticities vary with income levels. For example, the income elasticity for the poorest 10 percent of the population in India and Indonesia has been found to be .8 and for the richest 10 percent to be .3 (Knudsen and Scandizzo, 1979). Thus a 10 percent increase in real income is predicted to increase food consumption by 8 percent among the very poor and by 3 percent among the very rich in these countries. If food has few substitutes so that the cross-price elasticity of demand is zero, then the income elasticity is also a measure of the own-price elasticity for food but with the sign reversed. If this were the case in India and Bangladesh, a 10 percent increase in real food price would be expected to reduce food intake by 8 percent for the very poor and by 3 percent for the very rich in these countries. The analyst equipped with estimates of income and price elasticities for total food, for individual food items such as grains and root crops, for calories, and for protein could examine and predict the likely consequences for various segments of the population of government programmes to influence nutrition with a subsidy to producers, a price ceiling to consumers, or with targeted food assistance. The latter might be through food stamps or through direct distribution of a food consumed largely by the poor and made available in fair-price shops.

7.4.2.1 Marketing margins

Governments face a food price dilemma in that to benefit producers, many of whom are poor, they want high supply prices but to benefit consumers, many of whom also are poor, they want low food prices (Timmer et al, 1983). The marketing margin, which is the difference between the farm and retail price of food, ideally is kept low to reduce the food price dilemma. Those who market food from producer to consumer incur costs for procurement, processing, transportation, storage, wastage, and selling which must be covered by receipts or the place, form, and time utility created by marketing activities will not be forthcoming. Consumers and/or producers may be worse off from either too high or too low marketing margins.

Policy analysts can help to inform policy makers by using information from surveys to estimate total and components of marketing margins. These estimates help identify inefficiencies in the marketing system. Marketing margins in the private sector can be compared to marketing costs estimated for an alternative system of marketing such as a state-run agency or a parastatal corporation. For comparison, it is important to estimate not how efficiently a state agency would operate in theory but in practice. Governments which have taken over some or all marketing activities frequently have experienced actual marketing costs (paid for partly out of taxes) that are higher than the "inefficient" private sector they replace.

Marketing margins appropriately vary spatially because of transportation cost and seasonally because of storage costs. These elements need to be included when determining the marketing margin for a specific location and season.

7.4.2.2 Export demand

Exports may be an important component of demand and need to be accounted for in demand analysis of the impact of price on total quantity demanded or the impact on quantity demanded of a change in price. Domestic demand tends to be inelastic so that a given percentage change in price results in a smaller percentage change in quantity (in opposite direction), hence revenue is greater with a lower quantity and higher price. Export demand for any one country tends to be elastic in that a given percentage increase in price induces a larger percentage decrease in quantity. Hence export revenue is less with a higher price.

The total demand quantity q_d for a commodity is domestic demand quantity q_c plus export demand quantity q_x or $q_d = q_c + q_x$. The total demand elasticity is

$$(7.4.11)\quad \frac{dq_d}{dp}\frac{p}{q_d} = \frac{dq_c}{dp}\frac{p}{q_c}\left(\frac{q_c}{q_d}\right) + \frac{dq_x}{dp}\frac{p}{q_x}\left(\frac{q_x}{q_d}\right).$$

It is apparent that the total demand elasticity is a weighted sum of domestic and export elasticities with the respective weights equal to the domestic share and export share of total demand quantity.

Figure 7.2 illustrates demand curves for domestic consumption C and for exports X. Total demand CaD is the horizontal summation of the demand curves for C and X. If price is p, domestic demand quantity is q_c, exports q_x, and total demand is $q_d = q_c + q_x$. If the government intervenes to raise price to p_1 above the world price, the export market is lost. The domestic demand quantity and total demand quantity are q_{d1}. If decision makers wish to retain the export market when price is increased to p_1, then subsidies could be used to hold the export price to p.

7.4.3 Joining Demand and Supply Analyis

Neither supply alone nor demand alone is adequate to analyse many actual or prospective policies. Joint use of demand and supply to analyse policies is illustrated with Figure 7.3. The domestic demand is D and domestic supply is S. The import supply M is horizontal (perfectly elastic for a small country), implying that the country can import all it desires of the commodity at Price P_e. The supply curve,

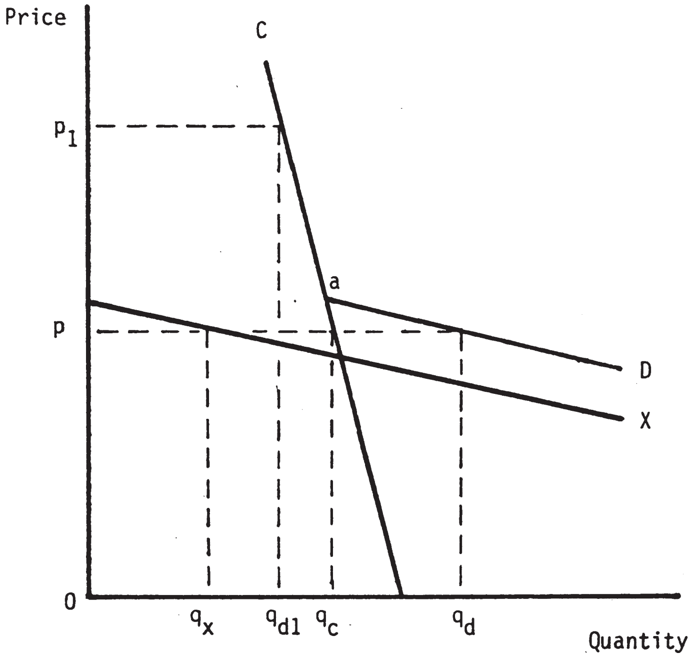

Figure 7.2 - Illustration of demand curves for domestic consumption C, exports X, and total demand CaD

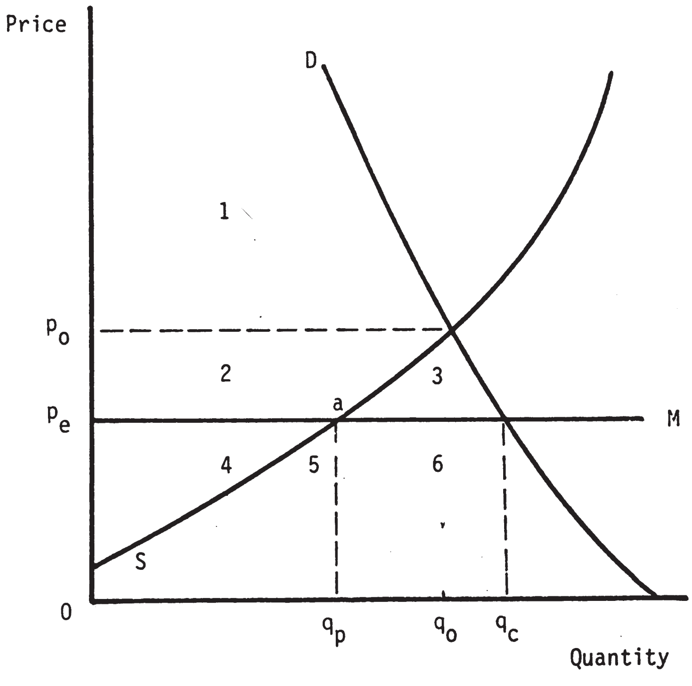

Figure 7.3 - Illustration of domestic demand curve D, domestic supply curve S, import supply curve M, and total supply SaM

showing the schedule of prices and quantities that will be supplied at the lowest price, is SaM and the total demand curve is D. Equilibrium in an open market is at the price where supply quantity just equals demand quantity and is at price p_e and quantity q_c in Figure 7.3. Of the total quantity, q_p is from domestic production and q_c-q_p is imported.

Suppose that policy makers decide to pursue a policy of self-sufficiency by eliminating imports and raising the domestic price to p_o. The new quantity supplied and demanded is q_o. It is less by q_c-q_o than the former quantity demanded because consumers cut back purchases along the demand curve D. The new domestic quantity supplied q_o exceeds the former quantity q_p by the amount q_o-q_p which is the domestic supply response along S to the increase in price from p_e to p_o.

Implications of numerous other policies such as taxes, subsidies, or quotas could be examined using demand and supply curves. Simulation (to be discussed in a later section) of an entire system of demand and supply equations provides even greater information, permitting the analyst to measure the impact of a policy change for many commodities whose markets interact.

7.4.4 Classical Welfare Analysis

Analysis thus far has provided little insight into the merits of a particular policy as measured by its impact on producers, consumers, taxpayers, and on national income. The analyst is rarely if ever in a position to designate one best policy because decision makers' objectives tend to be numerous and sometimes obscure. But various analytical tools have been devised to help identify effects on national income and the distribution of that income among producers, consumers, taxpayers, and society. The procedure is called classical welfare analysis[3/].

3/ For additional discussion of classifical welfare analysis, see Timmer et al. (1983, pp.190-94) and Tweeten (1979, ch.16)

The demand curve shows the maximum price consumers would pay for various quantities of a commodity. The benefit of an incremental unit of output is measured by its demand price. If the maximum price were charged for each quantity, the area beneath the demand curve D in Figure 7.3 to the left of q traces out the total revenue available from consuming q.

The supply curve shows the minimum price at which producers are willing to supply the various quantities of a commodity. They will tend to supply additional output if incremental variable costs are covered. The opportunity cost (benefit foregone by not consuming other goods and services) of an incremental unit of output to a competitive supplier is measured by the supply price. If the minimum price were paid for each possible quantity in Figure 7.3, it follows that the total variable cost (benefit foregone of other goods and services) of producing any given quantity q is the area beneath the supply curve in Figure 7.3 to the left of q.

Net revenue or net benefit is the difference between total revenue and total cost. Given that potential total revenue from consuming q_c is area 1 to 6 in Figure 7.3 and total variable cost is area 5 + 6, it follows that the maximum net revenue from producing (or importing) and consuming the commodity is area 1 + 2 + 3 + 4. Any other quantity reduces net revenue and national income. For example, termination of imports to pursue a policy of self-sufficiency at quantity q_o would raise costs of production and reduce gross revenue so that net revenue would be only area 1 + 2 + 4 -- a loss of net revenue indicated by area 3. If a subsidy or other policy were successful in increasing quantity supplied and demanded above q_c, net revenue would be reduced because the incremental cost M of additional supplies exceeds the incremental benefits (along curve D to the right of q_c) of additional consumption. Thus consumption q_c, domestic production q_p, and imports of $q_c - q_p$ provide greatest economic efficiency as measured by net revenue, the contribution of a commodity to national income.

For the supply and demand curves in Figure 7.3 to be valid efficiency measures, they must measure incremental costs and revenues for society[4/]. Private incremental costs (supply curve) and benefits

4/ Classical welfare analysis has been criticised because the marginal utility of income is not constant over the range of the supply and demand curves being considered, the curves are inaccurate outside the range of historical data, and because producers surplus based (demand curve) will suffice if private firms and individuals respond to incentives and if there are not externalities in the form of social cost (benefits) diverging from private costs (benefits). This assumption will not hold in the case of environmental impacts - costs to society of soil erosion or pesticide contamination of food for example tend to exceed costs paid by the individuals and firms. Net social costs are appropriately measured by areas bounded by social supply and demand curves between the social equilibrium quantity and the actual quantity.

Demand and supply curves in Figure 7.3 are also useful to show the distribution of net revenue. If price is p_e, gross revenue to producers is area 4 + 5. Variable costs equal to area 5 are incurred by producers so their net revenue (return to fixed inputs) is area 4 which is defined as producers surplus.

Consumers would be willing to incur outlays equal to area 1 to 6 for quantity q_c but only have to pay the value in area 4 + 5 + 6, hence they have net benefits called consumers surplus equal to area 1 + 2 + 3 when price is p_e and quantity is q_c. It is apparent that total net revenue is producers surplus plus consumers surplus.

A change in supply-demand quantity from q_c influences not only net revenue but how it is divided. For example, a policy realizing self-sufficiency at price p_o and quantity q_o raises net revenue to producers by area 2 and reduces net benefits to consumers by area 2 + 3. Thus consumers lose and producers gain from self-sufficiency. Area 3, the net revenue loss to the country, is the consumers' loss in excess of producers' gain.

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on a behavioural supply relationship is an inexact measure of returns to fixed factors (net producer income). These criticisms do not invalidate classical welfare analysis if options considered are within or close to the range of historical data experience, do not entail a major change in income, and if separate estimates (often available) of variable costs and receipts are available to verify and, if necessary, adjust estimates of producers surplus.

It is often convenient to include a third group of gainers or losers from public policy - taxpayers. With taxpayers included, the net gain (loss) to society is the sum of net gains (losses) to producers, consumers, and taxpayers. It is frequently convenient to consider only deviations from the most efficient allocation so that extreme values of demand and supply curves outside the range of historical experience (and which are likely to be inaccurate) do not influence measures of inefficiency.

Figure 7.4 illustrates the impact of subsidies to reduce the food price dilemma by lowering food prices to consumers and raising prices to producers. The domestic demand and supply curves are respectively D and S and the horizontal (small country) export demand curve is X. Without intervention in markets, total demand DaX intersects supply S at equilibrium price p_e and quantity q_p. Of this quantity, q_c goes to consumers and $q_p - q_c$ to exports.

A subsidy of $p_e - p_c$ per unit to domestic consumers raises consumption from q_c to q_c'. The cost of the subsidy to taxpayers is $q_c'(p_e - p_c)$ or area 1 + 2 in Figure 7.4. Compared to equilibrium without intervention at p_e and q_c, the net gain to consumers (consumers surplus) is area 1. Thus the net social cost in foregone national income is area 2 from the subsidy to consumers.

A subsidy of $p_p - p_e$ per unit raises the price to producers to p_p and increases their output to q_p' from q_p. The cost to taxpayers of the subsidy is $q_p'(p_p - p_e)$ or area 3 + 4 + 5 in Figure 7.4. Compared to equilibrium at p_e and q_p, the gain in net income (producers surplus) is area 3 + 4, hence the subsidy to producers reduces national income by area 5. The cost to taxpayers of combined subsidies to producers and consumers is area 1 + 2 + 3 + 4 + 5 in figure 7.4. The results measured as gains and losses compared to no intervention are summarized as follows:

Gain to consumers	1
Gain to producers	3 + 4
<u>Loss to taxpayers</u>	<u>1 + 2 + 3 + 4 + 5</u>
Net loss to society	2 + 5

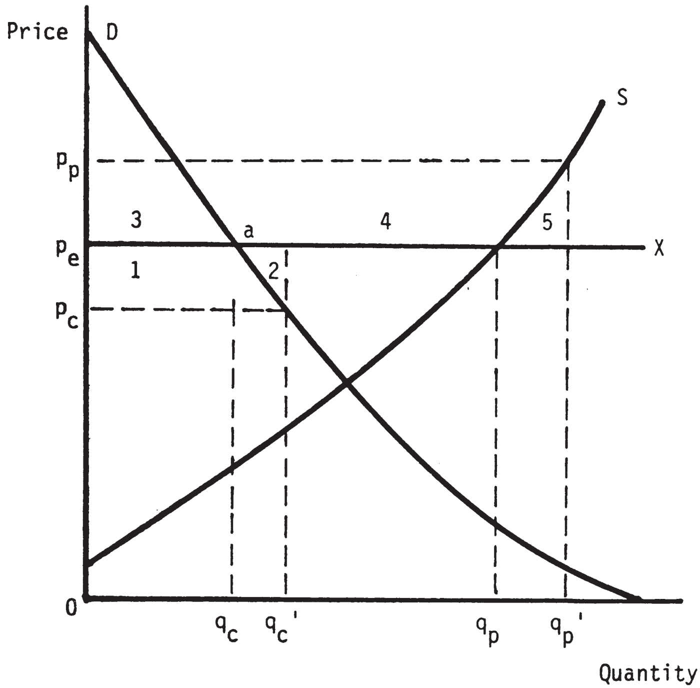

Figure 7.4 - Illustration of domestic supply curve S, domestic demand curve D, export demand curve X, and total demand DaX

Classical welfare analysis is suitable for evaluating impacts not only of taxes and subsidies but also of other distortions such as quotas and exchange rate interventions. An overvalued exchange rate is equivalent to an export tax and an import subsidy; an undervalued exchange rate is equivalent to an export subsidy and an import tax.

The efficiency of the transfer to consumers can be measured by the ratio of area 1 to the transfer cost 1 + 2, to producers by the ratio of area 3 + 4 to transfer cost 3 + 4 + 5, and to consumers plus producers by the ratio of area 1 + 3 + 4 to area 1 + 2 + 3 + 4 + 5. Administrative costs also need to be included in net social costs. An efficient transfer may reduce national income by 4 percent of the income transferred; transfer costs through taxes or subsidies on agricultural output in developing countries tend to be much larger.

If per caput income is sharply different between producers, consumers, and taxpayers (of course any one individual can be all three), efficiency measures may need to be adjusted for the marginal utility of money (see later section on project analysis). If producers have low incomes and taxpayers have high incomes, a transfer of income from taxpayers to producers may be appropriate to raise well being. A cash transfer is often more efficient than price interventions or input subsidies, but cash transfers are often difficult to administer in developing countries. Also food assistance targeted to the poor may be a more efficient transfer than general food subsidies to consumers.

7.4.5 Stabilizing Prices and Consumption

In concluding this section which has dealt with use of demand and supply in policy analysis, it is well to consider briefly an analysis of stabilization of prices and consumption through a buffer stock reserve. Supply and utilization data from the commodity balance equation 7.3.1 can be used to determine statistically appropriate buffer stocks to stabilize consumption given the variability in domestic production, exports, and imports. If the commodity is utilized only for exports and domestic consumption, the change in stock from year to year is $C_t + X_t - O_t$ from equation 7.3.1. The estimated variance in exports is

$$(7.4.12)\quad S_x^2 = \sum_{t=1}^{n} \frac{(X_t - \bar{X})^2}{n - 1}$$

where $\bar{X}$ is the mean of exports from sample year 1 to year n. The estimated variance in domestic production is

$$(7.4.13)\ S_o^{\ 2} = \sum_{t=1}^{n} \frac{(O_t - \bar{O})^2}{n-1}$$

where $\bar{O}$ is the mean of production from year 1 to year n. Given a normal distribution, the estimated variance in stocks $S_s^{\ 2}$ given that C_t is constant is

$$(7.4.14)\ S_s^{\ 2} = S_x^{\ 2} + S_o^{\ 2} - 2rS_xS_o$$

where r is the simple correlation coefficient between exports and production.

Given empirical estimates of $S_s^{\ 2}$, stocks required to avoid consumption shortfalls with specified probability can be readily calculated. Buffer stocks of quantity S_s (one standard deviation) will be sufficient to avoid a shortfall in consumption in all but one of six years and buffer stocks of $2S_s$ (two standard deviations) will be sufficient to avoid a shortfall in consumption in all but one of 50 years on the average over the long run. (If trends in exports and production can be predicted, then the values of $S_x^{\ 2}$ and $S_o^{\ 2}$ can be computed from dispersion around their respective trends rather than around their respective means).

If $S_x^{\ 2}$ is 100,000 tonnes, $S_o^{\ 2}$ is 1,000,000 tonnes and the correlation between X and O is .1, then the estimated variance in stocks $S_s^{\ 2}$ from equation 7.4.14 is [100,000 + 1,000,000 - 2(.1)(316)(1,000)] or 1,036,800 tonnes and S_s is 1,018 tonnes. Thus buffer stocks held as carryover from one crop year to the next of $2S_s$ or 2,036 tonnes are adequate to cover a shortfall in production due to unfavourable weather or other shocks to production or exports in 49 out of 50 years. Pipeline stocks needed to fill normal distribution channels and seasonal stocks drawn down from harvest to the end of the marketing season supplement buffer stocks.

It may be noted that the foregoing analysis is to determine statistically appropriate stocks rather than economically optimal stocks. Simulation analysis discussed later has been used to determine economically optimal buffer stock policies to stabilize consumption (see Eaton and Steele 1976).

7.5 Index Numbers, Composite Prices, and Productivity

In empirical estimates of demand and supply, composite measures of prices paid or prices received are sometimes necessary because the price of each commodity or of each resource cannot be included in statistical equations. Analysts frequently wish to measure trends in composite food prices or in the general price level. Also analysts sometimes wish to measure composite farm input and output to gauge productivity gains (ratio of aggregate output to input) and trends in total food production and consumption. In short, aggregation of individual prices or quantities is basic to policy analysis. At issue is how to form meaningful aggregates.

7.5.1 Index Number Formulation

Physical quantities or prices for individual commodities cannot merely be added together but must be weighted properly to form meaningful indices. To construct a quantity index, quantities of individual items are weighted by prices. To construct a price index, individual price components are weighted by quantities. Aggregate input is real resource costs measured in actual quantities of components multiplied by constant prices of inputs; aggregate output is real revenue measured by actual quantities multiplied by constant prices of products. Indices in any given year are expressed as a percentage of the index in some base year.

7.5.1.1 Laspeyres Index

The most widely used aggregation procedure is to weight prices (quantities) by base period quantities (prices) to form a Laspeyres Index. The procedure is illustrated with a simple example where the commodities (inputs or outputs) are Z and Y and p is price and q is quantity (Table 7.1).

The conventional notation is to designate price or quantity in the base year by the subscript zero (0) and in any given year other than the base by the subscript one (1). The Laspeyres quantity index L(q) for the data in Table 7.1 is

$$L(q) = \frac{p_{0Z}q_{1Z} + p_{0Y}q_{1Y}}{p_{0Z}q_{0Z} + p_{0Y}q_{0Y}} = \frac{\sum p_0 q_1}{\sum p_0 q_0}$$