Unbiased Estimation of Production Total Using Specific Sampling Designs

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ABSTRACT: Two sampling designs are used for the unbiased estimation of an estimable parameter with simple random sampling without replacement design. A comparison of precision is also given.

The Midzuno[1952]-Sen[1952] (MS) and the Singh-Srivastava (SS) [1980] sampling designs providing unbiased ratio and regression estimators for the production mean $\overline{Y}$, or production total (with simple technical amends) $Y = N\overline{Y} = \sum_{i=1}^{N} Y_i$ where Y_i is the production of the unit i. Such sampling designs are based on statistical algorithms for the implementation, and both require an auxiliary variable as cultivated land area X_i of each yield unit i.

In this article, we have shown that the MS and the SS sampling designs can be used for the simultaneous unbiased estimation of all estimable population parameters with simple random sampling without replacement (srs) design. Further, the variances of these general estimators have been investigated.

Theorem 1. If $E(srs,\ t) = T$, where t is an unbiased estimator of the parameter T for srs design, then

$$E\left(MS, t' = t\,\frac{\overline{X}}{\overline{x}} \right) = E\left(SS, t'' = t\,\frac{S_x^{\,2}}{s_x^{\,2}} \right) = T ,$$

where t' and t'' are unbiased ratio-type strategies for the respective (MS and SS) sampling designs.□

Here x and X are the sample and the population values of the auxiliary variable, $\overline{x}$ and s_x^2 are the sample mean and the sample quasi-variance:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad s_x^{\,2} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 ,$$

and $\overline{X}$ and S_x^2 are the population mean and population quasi-variance:

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \quad \text{and} \quad S_x^{\,2} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2 ,$$

n and N being the cultivated land sample size and the cultivated land population size, respectively.

Theorem 2. The variances of the strategies (MS, t') and (SS, t'') are

$$V(MS, t') = E(MS, t'^2) - T^2 = E(srs, tt') - T^2$$

and

$$V(SS, t'') = E(SS, t''^2) - T^2 = E(srs, tt'') - T^2 . \square$$

Corollary 1. $V(MS, t') \leq V(SS, t'') \Leftrightarrow E(srs, tt') \leq E(srs, tt'') . \square$

References

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