Supplementary material A

Tab. A1: Parameters used to calculate crop residues by the allocation fraction approach (equations 5 and 6) and the regression approach (equations 9 and 10)[%]

	fr_{yield}	fr_{stem}	fr_{root}	fr_{exud}	source	K	F	source
corn, silage	0.772	0	0.138	0.09	b1#1	1.04	0.005	f1
corn, grain	0.386	0.387	0.138	0.089	b1	1.35	0.06	f1
wwheat	0.322	0.482	0.118	0.078	b1#2	0.4	0.08	f1
swheat	0.322	0.482	0.118	0.078	b1#2	0.494	0.048	f2
wbarley	0.451	0.4	0.09	0.059	b1#2	0.68	0.068	f2
sbarley	0.451	0.4	0.09	0.059	b1#2	0.47	0.045	f1
wrye	0.335	0.482	0.11	0.073	b1[#3]	0.68	0.068	f2
srye	0.335	0.482	0.11	0.073	b1[#3]	0.68	0.068	f2
triticale	0.26	0.506	0.142	0.092	b1	0.72	0.072	f2
oats	0.319	0.283	0.241	0.157	b1	0.72	0.054	f2
potatoes	0.727	0.232	0.025	0.016	b2	0.128	0.104	f1
silage beets	0.619	0.353	0.017	0.011	b2	0.4	0	f2
sugar beets	0.619	0.353	0.017	0.011	b2	0.4	0	f2
wrape	0.313	0.383	0.174	0.13	g	0.284	0.043	f2
srape	0.313	0.383	0.174	0.13	g[#1]	0.284	0.043	f2[#2]
wturnip	0.313	0.383	0.174	0.13	g[#1]	0.284	0.043	f2[#2]
sturnip	0.313	0.383	0.174	0.13	g[#1]	0.284	0.043	f2[#2]
mustard	0.209	0.537	0.154	0.118	g	0.198	0.134	f2[#2]
oilradish	0.209	0.537	0.154	0.118	g[#4]	0.403	0	f2
cabbage	0.6	0	0.24	0.16	f2[#1]	0	0.042	f2
alfalfa	0.42	0	0.35	0.23	b1	2.00	0.036	f2

lupine	0.571	0	0.26	0.169	b1	0.75	0	f2
peas	0.217	0.507	0.167	0.109	$g^{\#2}$	0.111	0.083	f2
field bean	0.217	0.507	0.167	0.109	$g^{\#3}$	0.999	0	f2
Grassland/pasture	0.233	0	0.465	0.302	b1	2.8	0	f2
Grassland w. legumes	0.298	0	0.426	0.276	b1	2.51	0	0
Grassland annual	0.441	0	0.339	0.22	b1	2.8	0	
swedish turnip	0.313	0.383	0.174	0.13	$b2^{\#1}$	0.4	0	$f2^{\#3}$

b1 Bolinder et al. (2007)

b2 Bolinder et al. (2015)

b1#1 estimated from grain_ corn

b1#2 winter and summer cereals with similar cofficients

$b1^{\#3}$ small grain cereals

$b2^{\#1}$ parameters of sugar beet

g Gran et al. (2009)

$g^{\#1}$ parameters of winter rape

$g^{\#2}$ under rainfall

$g^{\#3}$ like peas

$g^{\#3}$ like mustard

$f2^{\#1}$ derived from Franko (2010) assuming fr_{exu} =0.66 fr_{root} (Wiesmeier et al. ,2014)

$f2^{\#2}$ parameters of winter rape

$f2^{\#3}$ parameters of sugar beets

Supplementary material B

Tab. B1: Fractions of crop residues rooted to DPM, RPM, HUM pool of RothC with the 90% confidence interval in brackets[%]

	Allocation fractions			Regression		
	DPM	**RPM**	**HUM**	**DPM**	**RPM**	**HUM**
Default	59	41	0	59	41	0
Straw	98.6(96;100)	1.4(0;4)	0(0;0)	97.5(93;100)	2.5(0;7)	0(0;0)
Green manure	98.2(94;100)	1.8(0;6)	0(0;0)	98.1(95;100)	1.9(0;5)	0(0;0)
Roots	65.0(57;72)	35.0(57;75)	0(0;0)	57(48;66)	43(34;52)	0(0;0)

Tab. B2: Fractions of organic amendments rooted to DPM, RPM, HUM pool of RothC with the 90% confidence interval in brackets [%]

	Allocation fractions			Regression		
	DPM	RPM	HUM	DPM	RPM	HUM
Default	49	49	2	49	49	2
FYM	72.7(62;83)	27.3(17;38)	0(0;0)	66.2(57;77)	33.8(23;43)	0(0;0)
FYM compost	57.3(45;68)	42.7(31;55)	0(0;0)	52.8(43;53)	47.2(37;57)	0(0;0)
Cattle slurry	66.8(22;97)	33.2(3;78)	0(0;0)	74.0(40;97)	26(3;60)	0(0;0)
Pig slurry	37.0(0;95)	63(5;24)	0(0;76)	46.2(0,96)	53.8(4;39)	0(0;61)
Sewage sludge	0(0;0)	60.9(43;80)	39.1(20;57)	0(0;0)	48.8(29;67)	51.2(33;71)
Waste compost	7.8(0;91)	92.2(8;15)	0(0;85)	40.3(0;95)	59.7(5,38)	0(0;62)
Peat	0(0;0)	45.1(34;57)	54.9(43;66)	0(0;0)	41.4(30;53)	58.6(47;70)
Sawdust	47.3(31;62)	52.7(38;69)	0(0;0)	43(28;59)	57(41;72)	0(0;0)

Supplementary material C

C_{DPM} DPM pool

C_{RPM} RPM pool

$C_1 = C_{BIO}$ BIO pool

$C_2 = C_{HUM}$ HUM pool

C_{IOM} IOM pool

cue carbon use efficiency as fraction of pool outflux that is not respired

f_i fraction of decomposed material directed to pool *i*

k_j decomposition rate of pool *i* (with *BIO* = C_1 and *HUM* = C_2)

ξ rate modifying factor driven by temperature, precipitation and coverage by vegetation

I input rate [t C ha^{-1} a^{-1}]

γ_{DPM} fraction of *I* entering C_{DPM}

f_{DPM} fraction of *DPM* on *C(0)* (carbon stock at *t=0*)

The differential equation describing the change of RothC pools with time are according to Sierra and Müller (2015):

$$\frac{dC_{DPM}}{dt} = \gamma_{DPM} I - \xi k_{DPM} C_{DPM} \quad \text{(C.1)}$$

$$\frac{dC_{RPM}}{dt} = \gamma_{RPM} I - \xi k_{RPM} C_{RPM} \quad \text{(C.2)}$$

$$\frac{dC_{BIO}}{dt} = \alpha_1 \xi k_{DPM} C_{DPM} + \alpha_1 \xi k_{RPM} C_{RPM} + C_1(\alpha_1 \xi k_1 - \xi k_1) + \alpha_1 \xi k_2 C_2 \quad \text{(C.3)}$$

$$\frac{dC_{HUM}}{dt} = \alpha_2 \xi k_{DPM} C_{DPM} + \alpha_2 \xi k_{RPM} C_{RPM} + \gamma_{C2} I + \alpha_2 \xi k_1 C_1 + C_2(\alpha_2 \xi k_2 - \xi k_2) \quad \text{(C.4)}$$

with:

$$\alpha_i = cue \times f_i \quad \text{(C.5)}$$

The analytical solution of this system is derived based on an analytical solution of the ICBM/2B model introduced by Kätterer and Andren (2001). With:

$$a_{11} = k_1\, \xi(\alpha_1 - 1) \quad \text{(C.6)}$$

$$a_{12} = \alpha_1 k_2\, \xi \quad \text{(C.7)}$$

$$a_{21} = \alpha_2 k_1 \xi \quad \text{(C.8)}$$

$$a_{22} = k_2\, \xi(\alpha_2 - 1) \quad \text{(C.9)}$$

$$b_{11} = \gamma_{RPM} I \alpha_1 \quad \text{(C.10)}$$

$$b_{12} = \alpha_1(C_{RPM}(0) k_{RPM} \xi - \gamma_{RPM} I) \quad \text{(C.11)}$$

$$b_{13} = \gamma_{DPM} I \alpha_1 \quad \text{(C.12)}$$

$$b_{14} = \alpha_1(C_{DPM}(0) k_{DPM} \xi - \gamma_{DPM} I) \quad \text{(C.13)}$$

$$b_{21} = \gamma_{RPM} I \alpha_2 + \gamma_{C2} I \quad \text{(C.14)}$$

$$b_{22} = \alpha_2(C_{RPM}(0) k_{RPM} \xi - \gamma_{RPM} I) \quad \text{(C.15)}$$

$$b_{23} = \gamma_{DPM} I \alpha_2 \quad \text{(C.16)}$$

$$b_{24} = \alpha_2(C_{DPM}(0) k_{DPM} \xi - \gamma_{DPM} I) \quad \text{(C.17)}$$

Eigenvalues of the 2x2 matrix are then (Kätterer and Andren 2001):

$$\lambda_{1,2} = \frac{a_{11}+a_{22}}{2} \pm \sqrt{\left(\frac{a_{11}+a_{22}}{2}\right)^2 + a_{12}a_{21} - a_{11}a_{22}} \qquad \text{(C.18)}$$

The complementary function for C_2:

$$\theta_1 exp(\lambda_1 t) + \theta_2 exp(\lambda_2 t) \qquad \text{(C.19)}$$

The particular integral is:

$$c_0 + c_1 \exp(-k_{RPM}\xi t) + c_2 exp(-k_{DPM}\xi t) \qquad \text{(C.20)}$$

With coefficients:

$$c_0 = \frac{a_{12}(b_{21}+b_{23}) - a_{22}(b_{11}+b_{13})}{a_{11}a_{22} - a_{12}a_{21}} \qquad \text{(C.21)}$$

$$c_1 = \frac{a_{12}b_{22} - b_{12}(a_{22}+k_{RPM}\xi)}{(k_{RPM}\xi)^2 + (a_{11}+a_{22})k_{RPM}\xi + a_{11}a_{22} - a_{21}a_{12}} \qquad \text{(C.22)}$$

$$c_2 = \frac{a_{12}b_{24} - b_{14}(a_{22}+k_{DPM}\xi)}{(k_{DPM}\xi)^2 + (a_{11}+a_{22})k_{DPM}\xi + a_{11}a_{22} - a_{21}a_{12}} \qquad \text{(C.23)}$$

$$\theta_1 = B_0 - \theta_2 - c_0 - c_1 - c_2 \qquad \text{(C.24)}$$

$$\theta_2 = \frac{1}{\lambda_2 - \lambda_1}[a_{12}C_2(0) + (a_{11} - \lambda_1)C_1(0) + \lambda_1 c_0 + (\lambda_1 + k_{RPM}\xi)c_1 + (\lambda_1 + k_{DPM})c_2 + b_{11} + b_{12} +$$
$$b_{13} + b_{14}] \qquad \text{(C.25)}$$

For these definitions C_{DPM} , C_{RPM}, C_1, C_2 become:

$$C_{DPM}(t) = I\gamma_{DPM}\left(\frac{1}{k_{DPM}\xi} - \frac{exp(-k_{DPM}\xi t)}{k_{DPM}\xi}\right) + C_{DPM}(0)\exp(-k_{DPM}\xi t) \quad \text{(C.26)}$$

$$C_{RPM}(t) = I\gamma_{RPM}\left(\frac{1}{k_{RPM}\xi} - \frac{exp(-k_{RPM}\xi t)}{k_{RPM}\xi}\right) + C_{RPM}(0)\exp(-k_{RPM}\xi t) \quad \text{(C.27)}$$

$$C_1(t) = \theta_1 exp(\lambda_1 t) + \theta_2 \exp(\lambda_2 t) + c_0 + c_1 exp(-k_{RPM}\xi t) + c_2 exp(-k_{DPM}\xi t) \qquad \text{(C.28)}$$

$$C_2(t) = \frac{1}{a_{12}}\left[(\lambda_1 - a_{11})\theta_1 exp(\lambda_1 t) + (\lambda_2 - a_{11})\theta_2 exp(\lambda_2 t) - a_{11}c_0 - b_{11} - b_{13} - \left(c_1\left(k_{rpm}\xi + a_{11}\right) + b_{12}\right)exp\left(-k_{rpm}\xi t\right) - \left(c_2\left(k_{dpm}\xi + a_{11}\right) + b_{14}\right)exp\left(-k_{dpm}\xi t\right)\right] \qquad \text{(C.29)}$$

Further transformation of equation C.21 to C.25 shows that each pool at time t (except IOM) is linear correlated to the initial carbon stock C(0) and carbon input I:

Factor c_0:

From C.20 and:

$$b_{11} + b_{13} = I(\alpha_1\gamma_{RPM} + \alpha_1\gamma_{DPM}) \qquad \text{(C.30)}$$

$$b_{21} + b_{23} = I(\alpha_2\gamma_{RPM} + \gamma_{C2} + \alpha_2\gamma_{DPM}) \qquad \text{(C.31)}$$

Follows for c_0 :

$$c_0 = I\,\frac{a_{12}(\alpha_2\gamma_{RPM}+\gamma_{C2}+\alpha_2\gamma_{DPM})-a_{22}(\alpha_1\gamma_{RPM}+\alpha_1\gamma_{DPM})}{a_{11}a_{22}-a_{12}a_{21}} \qquad \text{(C.32)}$$

$$c_0 = I\,\frac{\alpha_2\gamma_{RPM}a_{12}+\gamma_{C2}a_{12}+\alpha_2\gamma_{DPM}a_{12}-a_{22}\alpha_1\gamma_{RPM}-a_{22}\alpha_1\gamma_{DPM}}{a_{11}a_{22}-a_{12}a_{21}} \qquad \text{(C.33)}$$

$$c_0 = I\,\frac{\alpha_2\gamma_{RPM}a_{12}-a_{22}\alpha_1\gamma_{RPM}+\gamma_{C2}a_{12}+\alpha_2\gamma_{DPM}a_{12}-a_{22}\alpha_1\gamma_{DPM}}{a_{11}a_{22}-a_{12}a_{21}} \qquad \text{(C.34)}$$

$$c_0 = I\gamma_{RPM}\,\frac{\alpha_2 a_{12}-a_{22}\alpha_1}{a_{11}a_{22}-a_{12}a_{21}} + I\gamma_{DPM}\,\frac{\alpha_2 a_{12}-a_{22}\alpha_1}{a_{11}a_{22}-a_{12}a_{21}} + I\gamma_{C2}\,\frac{a_{12}}{a_{11}a_{22}-a_{12}a_{21}} \qquad \text{(C.35)}$$

Which is simplified by substitution:

$$c_0 = I\gamma_{RPM}c_{01} + I\gamma_{DPM}c_{02} + I\gamma_{C2}c_{03} \qquad \text{(C.36)}$$

Factor c1:

The combination of equations C.22 (c_1), C.15 (b_{22}) and C.11 (b_{12}) shows also that c_1 is linearly correlated to C and I:

$$c_1 = \frac{a_{12}b_{22} - b_{12}(a_{22} + k_{RPM}\xi)}{(k_{RPM}\xi)^2 + (a_{11} + a_{22})k_{RPM}\xi + a_{11}a_{22} - a_{21}a_{12}} \qquad \text{(C.22)}$$

$$b_{12} = C_{RPM}(0)k_{RPM}\xi\alpha_1 - I\gamma_{RPM}\alpha_1 \qquad \text{(C.37)}$$

$$b_{22} = C_{RPM}(0)k_{RPM}\xi\alpha_2 - \gamma_{RPM}I\alpha_2 \qquad \text{(C.38)}$$

$$c_1 = \frac{C_{RPM}(0)k_{RPM}\xi\alpha_2 a_{12} - I\gamma_{RPM}a_{12}\alpha_2 - (C_{RPM}(0)k_{RPM}\xi\alpha_1 - I\gamma_{RPM}\alpha_1)(a_{22} + k_{RPM}\xi)}{(k_{RPM}\xi)^2 + (a_{11} + a_{22})k_{RPM}\xi + a_{11}a_{22} - a_{21}a_{12}} \qquad \text{(C.39)}$$

$$c_1 = C_{RPM}(0)\frac{k_{RPM}\xi\alpha_2 a_{12} - k_{RPM}\xi\alpha_1(a_{22} + k_{RPM}\xi)}{(k_{RPM}\xi)^2 + (a_{11} + a_{22})k_{RPM}\xi + a_{11}a_{22} - a_{21}a_{12}} - I\gamma_{RPM}\frac{\alpha_2 a_{12} - \alpha_1(a_{22} + k_{RPM}\xi)}{(k_{RPM}\xi)^2 + (a_{11} + a_{22})k_{RPM}\xi + a_{11}a_{22} - a_{21}a_{12}}$$

(C.40)

Substituting coefficients by c_{11} and c_{12} results in:

$$c_1 = C_{RPM}(0)c_{11} - I\gamma_{RPM}c_{12} \qquad \text{(C.41)}$$

$$c_1 = C(0)f_{RPM}c_{11} - I\gamma_{RPM}c_{12} \qquad \text{(C.42)}$$

Factor c_2:

$$c_2 = \frac{a_{12}b_{24} - b_{14}(a_{22} + k_{DPM}\xi)}{(k_{DPM}\xi)^2 + (a_{11} + a_{22})k_{DPM}\xi + a_{11}a_{22} - a_{21}a_{12}} \qquad \text{(C.23)}$$

From (C.17) and (C.13) follows:

$$b_{24} = C_{DPM}(0)k_{DPM}\xi\alpha_2 - I\gamma_{DPM}\alpha_2 \qquad \text{(C.43)}$$

$$b_{14} = C_{DPM}(0)k_{DPM}\xi\alpha_1 - \gamma_{DPM}I\alpha_1 \qquad \text{(C.44)}$$

$$c_2 = \frac{C_{DPM}(0)k_{DPM}\xi\alpha_2 a_{12} - I\gamma_{DPM}\alpha_2 a_{12} - C_{DPM}(0)k_{DPM}\xi\alpha_1(a_{22}+k_{DPM}\xi) + \gamma_{DPM}I\alpha_1(a_{22}+k_{DPM}\xi)}{(k_{DPM}\xi)^2 + (a_{11}+a_{22})k_{DPM}\xi + a_{11}a_{22} - a_{21}a_{12}}$$

(C.45)

$$c_2 = C_{DPM}(0)\frac{k_{DPM}\xi\alpha_2 a_{12} - k_{DPM}\xi\alpha_1(a_{22}+k_{DPM}\xi)}{(k_{DPM}\xi)^2+(a_{11}+a_{22})k_{DPM}\xi+a_{11}a_{22}-a_{21}a_{12}} - I\frac{\gamma_{DPM}\alpha_2 a_{12} - \gamma_{DPM}\alpha_1(a_{22}+k_{DPM}\xi)}{(k_{DPM}\xi)^2+(a_{11}+a_{22})k_{DPM}\xi+a_{11}a_{22}-a_{21}a_{12}}$$

(C.46)

Substituting coefficients by c_{21} and c_{22} results in:

$$c_2 = C_{DPM}(0)c_{21} - \gamma_{DPM}Ic_{22} \quad \text{(C.47)}$$

$$c_2 = C(0)f_{DPM}c_{21} - \gamma_{DPM}Ic_{22} \quad \text{(C.48)}$$

Factor θ_2:

$$\theta_2 = \frac{1}{\lambda_2-\lambda_1}[a_{12}C_2(0) + (a_{11}-\lambda_1)C_1(0) + \lambda_1 c_0 + (\lambda_1 + k_{RPM}\xi)c_1 + (\lambda_1 + k_{DPM}\xi)c_2 + b_{11} + b_{12} + b_{13} + b_{14}] \quad \text{(C.49)}$$

$$\theta_2 = \frac{1}{\lambda_2-\lambda_1}[C_2(0)a_{12} + C_1(0)(a_{11}-\lambda_1) + I\gamma_{RPM}c_{01}\lambda_1 + I\gamma_{DPM}c_{02}\lambda_1 + I\gamma_{C2}c_{03}\lambda_1 + C(0)f_{RPM}c_{11}(\lambda_1+k_{RPM}\xi) - I\gamma_{RPM}c_{12}(\lambda_1+k_{RPM}\xi) + C(0)f_{DPM}c_{21}(\lambda_1+k_{DPM}\xi) - I\gamma_{DPM}c_{22}(\lambda_1+k_{DPM}\xi) + I\gamma_{RPM}\alpha_1 + C_{RPM}(0)k_{RPM}\xi\alpha_1 - I\gamma_{RPM}\alpha_1 + I\gamma_{DPM}\alpha_1 + C_{DPM}(0)k_{DPM}\xi\alpha_1 - I\gamma_{DPM}\alpha_1] \quad \text{(C.50)}$$

$$\theta_2 = \frac{1}{\lambda_2-\lambda_1}[C(0)f_2 a_{12} + C(0)f_1(a_{11}-\lambda_1) + I\gamma_{RPM}c_{01}\lambda_1 + I\gamma_{DPM}c_{02}\lambda_1 + I\gamma_{C2}c_{03}\lambda_1 + C(0)f_{RPM}c_{11}(\lambda_1+k_{RPM}\xi) - I\gamma_{RPM}c_{12}(\lambda_1+k_{RPM}\xi) + C(0)f_{DPM}c_{21}(\lambda_1+k_{DPM}\xi) - I\gamma_{DPM}c_{22}(\lambda_1+k_{DPM}\xi) + C(0)f_{RPM}k_{RPM}\xi\alpha_1 + C(0)f_{DPM}k_{DPM}\xi\alpha_1] \quad \text{(C.51)}$$

$$\theta_2 = C(0)\frac{f_2 a_{12}+f_1(a_{11}-\lambda_1)+f_{RPM}c_{11}(\lambda_1+k_{RPM}\xi)+f_{DPM}c_{21}(\lambda_1+k_{DPM}\xi)+f_{RPM}k_{RPM}\xi\alpha_1+f_{DPM}k_{DPM}\xi\alpha_1}{\lambda_2-\lambda_1} +$$

$$I\gamma_{DPM}\frac{c_{02}\lambda_1-c_{22}(\lambda_1+k_{DPM}\xi)}{\lambda_2-\lambda_1} + I\gamma_{RPM}\frac{c_{01}\lambda_1-c_{12}(\lambda_1+k_{RPM}\xi)}{\lambda_2-\lambda_1} + I\gamma_{C2}\frac{c_{03}\lambda_1}{\lambda_2-\lambda_1} \quad \text{(C.52)}$$

Substituting coefficients by θ_{21}, θ_{DPM}, θ_{RPM}, θ_{C2}gives:

$$\theta_2 = C(0)\theta_{21} + I\gamma_{DPM}\theta_{DPM} + I\gamma_{RPM}\theta_{RPM} + I\gamma_{C2}\theta_{C2} \quad \text{(C.53)}$$

Factor $\boldsymbol{\theta_1}$:

Combing equation C.24, C.36, C.42 and C.53 results in:

$$\theta_1 = C_1(0) - \theta_2 - c_0 - c_1 - c_2 \qquad \text{(C.24)}$$

$$\theta_1 = C(0)f_1 - C(0)\theta_{21} - I\gamma_{DPM}\theta_{2\_DPM} - I\gamma_{RPM}\theta_{2\_RPM} - I\gamma_{C2}\theta_{2\_I}$$

$$-I\gamma_{DPM}c_{02} - I\gamma_{RPM}c_{01} - I\gamma_{C2}c_{03} - C(0)f_{RPM}c_{11} + I\gamma_{RPM}c_{12} - C(0)f_{DPM}c_{21} + \gamma_{DPM}Ic_{22} \quad \text{(C.54)}$$

$$\theta_1 = C(0)(f_1 - \theta_{21} - f_{RPM}c_{11} - f_{DPM}c_{21}) - I\gamma_{DPM}\left(\theta_{2\_DPM} + c_{02} - c_{22}\right) - I\gamma_{RPM}(\theta_{RPM} + c_{01} - c_{12}) - I\gamma_{C2}\theta_{2\_I} \qquad \text{(C.55)}$$

$$\theta_1 = C(0)(f_1 - \theta_{21} - f_{RPM}c_{11} - f_{DPM}c_{21}) - I\gamma_{DPM}\left(\theta_{2\_DPM} + c_{02} - c_{22}\right) - I\gamma_{RPM}\left(\theta_{2\_RPM} + c_{01} - c_{12}\right) - I\gamma_{C2}\theta_{2\_I} \qquad \text{(C.56)}$$

$$\theta_1 = C(0)(f_1 - \theta_{21} - f_{RPM}c_{11} - f_{DPM}c_{21}) - I\gamma_{DPM}\left(\theta_{2\_DPM} + c_{02} - c_{22}\right) - I\gamma_{RPM}\left(\theta_{2\_RPM} + c_{01} - c_{12}\right) - I\gamma_{C2}\left(\theta_{2\_I} + c_{03}\right) \qquad \text{(C.57)}$$

Substitution of coefficients of equation C.57 by θ_{11}, θ_{1_DPM}, $\theta_{1_RPM}$and θ_{1_I}

$$\theta_1 = C(0)\theta_{11} - I\gamma_{DPM}\theta_{1\_DPM} - I\gamma_{RPM}\theta_{1\_RPM} + I\gamma_{C2}\theta_{1\_I} \qquad \text{(C.58)}$$

From equations C.1 to C.58 follows that c0, c1, c2 and ζ are lineary correlated to initial C or I or both:

$$c_0 = I\gamma_{RPM}c_{01} + I\gamma_{DPM}c_{02} + I\gamma_{C2}c_{03} \qquad \text{(C.36)}$$

$$c_1 = C(0)f_{RPM}c_{11} - I\gamma_{RPM}c_{12} \qquad \text{(C.42)}$$

$$c_2 = C(0)f_{DPM}c_{21} - \gamma_{DPM}Ic_{22} \qquad \text{(C.48)}$$

$$\theta_2 = C(0)\theta_{21} + I\gamma_{DPM}\theta_{2\_DPM} + I\gamma_{RPM}\theta_{2\_RPM} + I\gamma_{C2}\theta_{2\_I} \qquad \text{(C.53)}$$

$$\theta_1 = C(0)f_1\theta_{11} - I\gamma_{DPM}\theta_{1\_DPM} - I\gamma_{RPM}\theta_{1\_RPM} - I\gamma_{C2}\theta_{1\_I} \qquad \text{(C.58)}$$

From equation C.28 and equations C.36, C.42, C.48, C.53 and C.58 it follows for BIO at time t:

$$C_1(t) = \left(C(0)\theta_{11} - I\gamma_{DPM}\theta_{1\_DPM} - I\gamma_{RPM}\theta_{1\_RPM} - I\gamma_{C2}\theta_{2\_I}\right)exp(\lambda_1 t) + \left(C(0)\theta_{21} + I\gamma_{DPM}\theta_{2\_DPM} + I\gamma_{RPM}\theta_{2\_RPM} + I\gamma_{C2}\theta_{2\_I}\right)\exp(\lambda_2 t) + I\gamma_{RPM}c_{01} + I\gamma_{DPM}c_{02} + I\gamma_{C2}c_{03} + (C(0)f_{RPM}c_{11} - I\gamma_{RPM}c_{12})exp(-k_{RPM}\xi t) + (C(0)f_{DPM}c_{21} - \gamma_{DPM}Ic_{22})exp(-k_{DPM}\xi t) \qquad \text{(C.59)}$$

$$C_1(t) = C(0)\theta_{11}exp(\lambda_1 t) - I\gamma_{DPM}\theta_{1\_DPM}exp(\lambda_1 t) - I\gamma_{RPM}\theta_{1\_RPM}exp(\lambda_1 t) - I\gamma_{C2}\theta_{1\_I}exp(\lambda_1 t) + C(0)\theta_{21}\exp(\lambda_2 t) + I\gamma_{DPM}\theta_{2\_DPM}\exp(\lambda_2 t) + I\gamma_{RPM}\theta_{2\_RPM}\exp(\lambda_2 t) + I\gamma_{C2}\theta_{2\_I}\exp(\lambda_2 t) + I\gamma_{RPM}c_{01} + I\gamma_{DPM}c_{02} + I\gamma_{C2}c_{03} + C(0)f_{RPM}c_{11}exp(-k_{RPM}\xi t) - I\gamma_{RPM}c_{12}exp(-k_{RPM}\xi t) + C(0)f_{DPM}c_{21}exp(-k_{DPM}\xi t) - \gamma_{DPM}Ic_{22}exp(-k_{DPM}\xi t) \qquad \text{(C.60)}$$

$$C_1(t) = C(0)\theta_{11}exp(\lambda_1 t) - I\gamma_{DPM}\theta_{1\_DPM}exp(\lambda_1 t) - I\gamma_{RPM}\theta_{1\_RPM}exp(\lambda_1 t) - I\gamma_{C1}\theta_{2\_I}exp(\lambda_1 t) + C(0)\theta_{21}\exp(\lambda_2 t) + I\gamma_{DPM}\theta_{2\_DPM}\exp(\lambda_2 t) + I\gamma_{RPM}\theta_{2\_RPM}\exp(\lambda_2 t) + I\gamma_{C2}\theta_{2\_I}\exp(\lambda_2 t) + I\gamma_{RPM}c_{01} + I\gamma_{DPM}c_{02} + I\gamma_{C2}c_{03} + C(0)f_{RPM}c_{11}exp(-k_{RPM}\xi t) - I\gamma_{RPM}c_{12}exp(-k_{RPM}\xi t) + C(0)f_{DPM}c_{21}exp(-k_{DPM}\xi t) - \gamma_{DPM}Ic_{22}exp(-k_{DPM}\xi t) \qquad \text{(C.61)}$$

$$C_1(t) = C(0)\left(\theta_{11}exp(\lambda_1 t) + \theta_{21}\exp(\lambda_2 t) + f_{RPM}c_{11}exp(-k_{RPM}\xi t) + f_{DPM}c_{21}exp(-k_{DPM}\xi t)\right) + I\gamma_{DPM}\left(-\theta_{1_{DPM}}exp(\lambda_1 t) + \theta_{2_{DPM}}\exp(\lambda_2 t) + c_{02} - c_{22}exp(-k_{DPM}\xi t)\right) +$$

$$I\gamma_{RPM}\left(-\theta_{1_{RPM}} exp(\lambda_1 t) + \theta_{2_{RPM}} \exp(\lambda_2 t) + c_{01} - c_{12} exp(-k_{RPM}\xi t)\right) + I\gamma_{C2}\left(-\theta_{1_I} exp(\lambda_1 t) + \theta_{2_I} \exp(\lambda_2 t) + c_{03}\right) \quad \text{(C.62)}$$

Resulting equation for BIO

By substituting coefficients we see that equation C.62 is basically a linear equation:

$$C_{BIO}(t) = C(0)s_{BIO} + I\gamma_{DPM}u_{BIO.DPM} + I\gamma_{RPM}u_{BIO.RPM} + I\gamma_{HUM}u_{BIO.HUM} \quad \text{(C.63)}$$

with coefficients:

$$s_{BIO} = \left(\theta_{11} exp(\lambda_1 t) + \theta_{21}\exp(\lambda_2 t) + f_{RPM}c_{11} exp(-k_{RPM}\xi t) + f_{DPM}c_{21} exp(-k_{DPM}\xi t)\right)$$

(C.64)

$$u_{BIO.DPM} = \left(-\theta_{1_{DPM}} exp(\lambda_1 t) + \theta_{2_{DPM}}\exp(\lambda_2 t) + c_{02} - c_{22} exp(-k_{DPM}\xi t)\right) \quad \text{(C.65)}$$

$$u_{BIO.RPM} = \left(-\theta_{1_{RPM}} exp(\lambda_1 t) + \theta_{2_{RPM}}\exp(\lambda_2 t) + c_{01} - c_{12} exp(-k_{RPM}\xi t)\right) \quad \text{(C.66)}$$

$$u_{BIO.HUM} = I\gamma_{C2}\left(-\theta_{1_I} exp(\lambda_1 t) + \theta_{2_I}\exp(\lambda_2 t) + c_{03}\right) \quad \text{(C.67)}$$

From equation C.29 and equations C.36, C.42, C.48, C.53 and C.58 it follows for HUM at time t:

$$C_2(t) = \frac{1}{a_{12}}\left[(\lambda_1 - a_{11})\theta_1 exp(\lambda_1 t) + (\lambda_2 - a_{11})\theta_2 exp(\lambda_2 t) - a_{11}c_0 - b_{11} - b_{13} - \left(c_1\left(k_{rpm}\xi + a_{11}\right) + b_{12}\right) exp\left(-k_{rpm}\xi t\right) - \left(c_2\left(k_{dpm}\xi + a_{11}\right) + b_{14}\right) exp\left(-k_{dpm}\xi t\right)\right] \quad \text{(C.63)}$$

$$C_2(t) = \frac{(\lambda_1 - a_{11})\theta_1 exp(\lambda_1 t)}{a_{12}} + \frac{(\lambda_2 - a_{11})\theta_2 exp(\lambda_2 t)}{a_{12}} - \frac{a_{11}c_0}{a_{12}} - \frac{b_{11}}{a_{12}} - \frac{b_{13}}{a_{12}} - \frac{(c_1(k_{rpm}\xi + a_{11}) + b_{12}) exp(-k_{rpm}\xi t)}{a_{12}} - \frac{(c_2(k_{dpm}\xi + a_{11}) + b_{14}) exp(-k_{dpm}\xi t)}{a_{12}} \quad \text{(C.64)}$$

Summands of equation C.64 can be transformed into linear equations with C and I as independent variables:

First term

$$\frac{(\lambda_1-a_{11})\theta_1 exp(\lambda_1 t)}{a_{12}} = \frac{(\lambda_1-a_{11})exp(\lambda_1 t)}{a_{12}}\left(C(0)\theta_{11} - I\gamma_{DPM}\theta_{1\_DPM} - I\gamma_{RPM}\theta_{1\_RPM} - I\gamma_{C2}\theta_{2\_I}\right) \quad \text{(C.65)}$$

$$\frac{(\lambda_1-a_{11})\theta_1 exp(\lambda_1 t)}{a_{12}} = C(0)\frac{(\lambda_1-a_{11})exp(\lambda_1 t)\theta_{11}}{a_{12}} - I\gamma_{DPM}\frac{(\lambda_1-a_{11})exp(\lambda_1 t)\theta_{1_{DPM}}}{a_{12}} -$$

$$I\gamma_{RPM}\frac{(\lambda_1-a_{11})exp(\lambda_1 t)\theta_{1_{RPM}}}{a_{12}} - I\gamma_{C2}\frac{(\lambda_1-a_{11})exp(\lambda_1 t)\theta_{2\_I}}{a_{12}} \quad \text{(C.66)}$$

Second term

$$\frac{(\lambda_2-a_{11})\theta_2 exp(\lambda_2 t)}{a_{12}} = \frac{(\lambda_2-a_{11})exp(\lambda_2 t)}{a_{12}}\left(C(0)\theta_{21} + I\gamma_{DPM}\theta_{2\_DPM} + I\gamma_{RPM}\theta_{2\_RPM} + I\gamma_{C2}\theta_{2\_I}\right) \quad \text{(C.67)}$$

$$\frac{(\lambda_2-a_{11})\theta_2 exp(\lambda_2 t)}{a_{12}} = C(0)\frac{(\lambda_2-a_{11})exp(\lambda_2 t)f_1\theta_{21}}{a_{12}} + I\gamma_{DPM}\frac{(\lambda_2-a_{11})exp(\lambda_2 t)\theta_{2\_DPM}}{a_{12}} +$$

$$I\gamma_{RPM}\frac{(\lambda_2-a_{11})exp(\lambda_2 t)\theta_{2\_RPM}}{a_{12}} + I\gamma_{C2}\frac{(\lambda_2-a_{11})exp(\lambda_2 t)\theta_{2\_I}}{a_{12}} \quad \text{(C.68)}$$

Third Term

$$\frac{a_{11}c_0}{a_{12}} = \frac{1}{a_{12}}(I\gamma_{RPM}c_{01}a_{11} + I\gamma_{DPM}c_{02}a_{11} + I\gamma_{C2}c_{03}a_{11}) \quad \text{(C.69)}$$

Fourth Term

$$\frac{(c_1(k_{rpm}\xi + a_{11}) + b_{12})exp(-k_{rpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\left[c_1 k_{rpm}\xi exp\left(-k_{rpm}\xi t\right) + c_1 a_{11} exp\left(-k_{rpm}\xi t\right) +\right.$$

$$\left. b_{12} exp\left(-k_{rpm}\xi t\right)\right] \quad \text{(C.70)}$$

$$\frac{(c_1(k_{rpm}\xi + a_{11}) + b_{12})exp(-k_{rpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\left[(C(0)f_{RPM}c_{11} - I\gamma_{RPM}c_{12})k_{rpm}\xi exp\left(-k_{rpm}\xi t\right) +\right.$$

$$\left.(C(0)f_{RPM}c_{11} - I\gamma_{RPM}c_{12})a_{11} exp\left(-k_{rpm}\xi t\right) + \left(\alpha_1(C_{RPM}(0)k_{RPM}\xi - \gamma_{RPM}I)exp\left(-k_{rpm}\xi t\right)\right)\right]$$

(C.71)

$$\frac{(c_1(k_{rpm}\xi+a_{11})+b_{12})exp(-k_{rpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\Big[C(0)f_{RPM}c_{11}k_{rpm}\xi exp\big(-k_{rpm}\xi t\big) - I\gamma_{RPM}c_{12}k_{rpm}\xi exp\big(-k_{rpm}\xi t\big) + C(0)f_{RPM}c_{11}a_{11}exp\big(-k_{rpm}\xi t\big) - I\gamma_{RPM}c_{12}a_{11}exp\big(-k_{rpm}\xi t\big) + C(0)f_{RPM}k_{RPM}\xi\alpha_1 exp\big(-k_{rpm}\xi t\big) - \gamma_{RPM}I\alpha_1 exp\big(-k_{rpm}\xi t\big)\Big] \text{ (C.72)}$$

$$\frac{(c_1(k_{rpm}\xi+a_{11})+b_{12})exp(-k_{rpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\Big[C(0)\left(f_{RPM}c_{11}k_{rpm}\xi exp\big(-k_{rpm}\xi t\big) + f_{RPM}c_{11}a_{11}exp\big(-k_{rpm}\xi t\big) + f_{RPM}k_{RPM}\xi\alpha_1 exp\big(-k_{rpm}\xi t\big)\right) - I\gamma_{RPM}\left(c_{12}k_{rpm}\xi exp\big(-k_{rpm}\xi t\big) + c_{12}a_{11}exp\big(-k_{rpm}\xi t\big) + \alpha_1 exp\big(-k_{rpm}\xi t\big)\right)\Big] \text{ (C.73)}$$

$$\frac{(c_1(k_{rpm}\xi+a_{11})+b_{12})exp(-k_{rpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\Big[C(0)exp\big(-k_{rpm}\xi t\big)f_{rpm}\big(c_{11}k_{rpm}\xi + c_{11}a_{11} + k_{RPM}\xi\alpha_1\big) - I\gamma_{RPM}exp\big(-k_{rpm}\xi t\big)\big(c_{12}k_{rpm}\xi + c_{12}a_{11} + \alpha_1\big)\Big] \qquad \text{(C.74)}$$

Fifth term

$$\frac{(c_2(k_{dpm}\xi+a_{11})+b_{14})exp(-k_{dpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\Big[\, C(0)f_{DPM}c_{21}k_{dpm}\xi exp\big(-k_{dpm}\xi t\big) - I\gamma_{DPM}c_{22}k_{dpm}\xi exp\big(-k_{dpm}\xi t\big) + C(0)f_{DPM}c_{11}a_{11}exp\big(-k_{dpm}\xi t\big) - I\gamma_{DPM}c_{22}a_{11}exp\big(-k_{dpm}\xi t\big) + C(0)f_{DPM}k_{RPM}\xi\alpha_1 exp\big(-k_{rpm}\xi t\big) - \gamma_{RPM}I\alpha_1 exp\big(-k_{rpm}\xi t\big)\Big] \text{ (C.75)}$$

$$\frac{(c_2(k_{dpm}\xi+a_{11})+b_{14})exp(-k_{dpm}\xi t)}{a_{12}} = \frac{1}{a_{12}}\Big\lfloor C(0)exp\big(-k_{dpm}\xi t\big)f_{dpm}\big(c_{21}k_{dpm}\xi + c_{21}a_{11} + k_{DPM}\xi\alpha_1\big) - I\gamma_{DPM}exp\big(-k_{dpm}\xi t\big)\big(c_{22}k_{dpm}\xi + c_{22}a_{11} + \alpha_1\big)\Big\rfloor \qquad \text{(C.76)}$$

Which is finally:

$$C_2(t) = C(0)\frac{1}{a_{12}}\Big((\lambda_1 - a_{11})exp(\lambda_1 t)f_1\theta_{11} + (\lambda_2 - a_{11})exp(\lambda_2 t)f_1\theta_{21} - exp\big(-k_{rpm}\xi t\big)f_{rpm}\big(c_{11}k_{rpm}\xi + c_{11}a_{11} + k_{RPM}\xi\alpha_1\big) - exp\big(-k_{dpm}\xi t\big)f_{dpm}\big(c_{21}k_{dpm}\xi + c_{21}a_{11} + k_{DPM}\xi\alpha_1\big)\Big) + I\gamma_{DPM}\frac{1}{a_{12}}\Big(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{1_{DPM}} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2_{DPM}} - c_{02}a_{11} - \alpha_1 + exp\big(-k_{dpm}\xi t\big)\big(c_{22}k_{dpm}\xi + c_{22}a_{11} + \alpha_1\big)\Big) + I\gamma_{RPM}\frac{1}{a_{12}}\Big(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{1_{RPM}} +$$

$$(\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2_{RPM}} - c_{01}a_{11} - \alpha_1 + exp(-k_{rpm}\xi t)(c_{12}k_{rpm}\xi + c_{12}a_{11} + \alpha_1)\Big) +$$

$$I\gamma_{C2}\frac{1}{a_{12}}\left(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{2\_I} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2\_I} - c_{03}a_{11}\right) \qquad \text{(C.77)}$$

This can be simplified by substituting coefficients and so we get

The final formula for the HUM pool

$$C_{HUM}(t) = C(0)s_{HUM} + I\gamma_{DPM}u_{HUM.DPM} + I\gamma_{RPM}u_{HUM.RPM} + I\gamma_{HUM}u_{HUM.HUM} \qquad \text{(C.78)}$$

with coefficients:

$$s_{HUM} = \frac{1}{a_{12}}\Big((\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{11} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{21}$$

$$- exp(-k_{rpm}\xi t)f_{rpm}(c_{11}k_{rpm}\xi + c_{11}a_{11} + k_{RPM}\xi\alpha_1)$$

$$- exp(-k_{dpm}\xi t)f_{dpm}(c_{21}k_{dpm}\xi + c_{21}a_{11} + k_{DPM}\xi\alpha_1)\Big)$$

$$u_{HUM.DPM} = \frac{1}{a_{12}}\Big(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{1_{DPM}} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2_{DPM}} - c_{02}a_{11} - \alpha_1$$

$$+ exp(-k_{dpm}\xi t)(c_{22}k_{dpm}\xi + c_{22}a_{11} + \alpha_1)\Big)$$

$$u_{HUM.RPM} = \frac{1}{a_{12}}\Big(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{1_{RPM}} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2_{RPM}} - c_{01}a_{11} - \alpha_1$$

$$+ exp(-k_{rpm}\xi t)(c_{12}k_{rpm}\xi + c_{12}a_{11} + \alpha_1)\Big)$$

$$u_{HUM.HUM} = \frac{1}{a_{12}}\left(-(\lambda_1 - a_{11})exp(\lambda_1 t)\theta_{2_I} + (\lambda_2 - a_{11})exp(\lambda_2 t)\theta_{2_I} - c_{03}a_{11}\right)$$

Based on equation C.26 we might write for the DPM and RPM pool:

$$C_{DPM}(t) = I\gamma_{DPM}u_{DPM} + C(0)\mathrm{s}_{\mathrm{DPM}} \qquad \text{(C.79)}$$

with coefficients

$$s_{DPM} = \mathrm{f}_{\mathrm{DPM}}\exp(-k_{DPM}\xi t)$$

$$u_{DPM} = \frac{1}{k_{DPM}\xi} - \frac{exp(-k_{DPM}\xi t)}{k_{DPM}\xi}$$

Based on equation C.27 we might write for the RPM pool:

$$C_{RPM}(t) = I\gamma_{RPM}u_{RPM} + C(0)\mathrm{s}_{\mathrm{DPM}} \qquad \text{(C.80)}$$

with coefficients

$$s_{RPM} = \mathrm{f}_{\mathrm{RPM}}\exp(-k_{RPM}\xi t)$$

$$u_{RPM} = \frac{1}{k_{RPM}\xi} - \frac{exp(-k_{RPM}\xi t)}{k_{RPM}\xi}$$

Adding equations C.78 –C.80 and the IOM pool we get the carbon stock at time t:

$$C(t) = C(0)\big(s_{DPM} + s_{rpm} + s_{hum} + s_{BIO}\big) + I[\gamma_{DPM}(u_{DPM.DPM}+u_{BIO.DPM} + u_{HUM.DPM}) + \gamma_{RPM}(u_{RPM.RPM}+u_{BIO.RPM} + u_{HUM.RPM}) + \gamma_{HUM}(u_{BIO.HUM} + u_{HUM.HUM})] + f_{IOM}C(0) \qquad \text{(C.81)}$$

which becomes:

$$C(t) = C(0)s_{impl} + I[\gamma_{DPM}u_{DPM} + \gamma_{RPM}u_{RPM} + \gamma_{HUM}u_{HUM}] + f_{IOM}C(0) \qquad \text{(C.82)}$$

with coefficients:

$$s_{impl} = s_{DPM} + s_{rpm} + s_{hum} + s_{BIO}$$

$$u_{DPM} = u_{DPM.DPM}+u_{BIO.DPM} + u_{HUM.DPM}$$

$$u_{RPM} = u_{RPM.RPM}+u_{BIO.RPM} + u_{HUM.RPM}$$

$$u_{HUM} = u_{BIO.HUM} + u_{HUM.HUM}$$

Amending organic carbon of variable degradability requires a reformulation of equation C.82:

$$C(t) = C(0)s_{impl} + \sum_{i=1}^{N} I_i\left[\gamma_{DPM\_i}u_{DPM} + \gamma_{RPM\_i}u_{RPM} + \gamma_{HUM\_i}u_{HUM}\right] + f_{IOM}C(0) \tag{C.82}$$