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المعهد الوطني للبحث في الصيد البحري

Periodic, spatially structured matrix model for the study of sardine (*Sardina pilchardus*) population dynamics in the Moroccan Atlantic coasts

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Science and the Challenge of Managing Small Pelagic Fisheries, 11-14 Mars 2008 - Casablanca, Morocco

Outline

- Aims of the study
- Why matrix population models ?
- Acoustic surveys as data source
- Model formulation
- Model assumptions and parameter estimation
- Elaboration of the linear control functions
- Simulation results and management scenarios
- Conclusion and perspectives

Aims of the study

- Understanding the sardine dynamics within the stock area of distribution,
- Considering a mathematical modeling of a size structured periodic matrix : Two-stage model that represents immature and mature,
- Two phenomena are involved : the migration dynamics and the demographic process.

Why matrix population models ?

- Easy to contruct
- Simple to simulate
- Powerful software is available for their analysis

Continuous Time or Discrete time?

- Continuous Time: difficulties to model the temporal heterogeneity
- Discrete time: control of integral time on all scales : Demography and stages of life cycle

Biomass distribution of sardine at December 2004

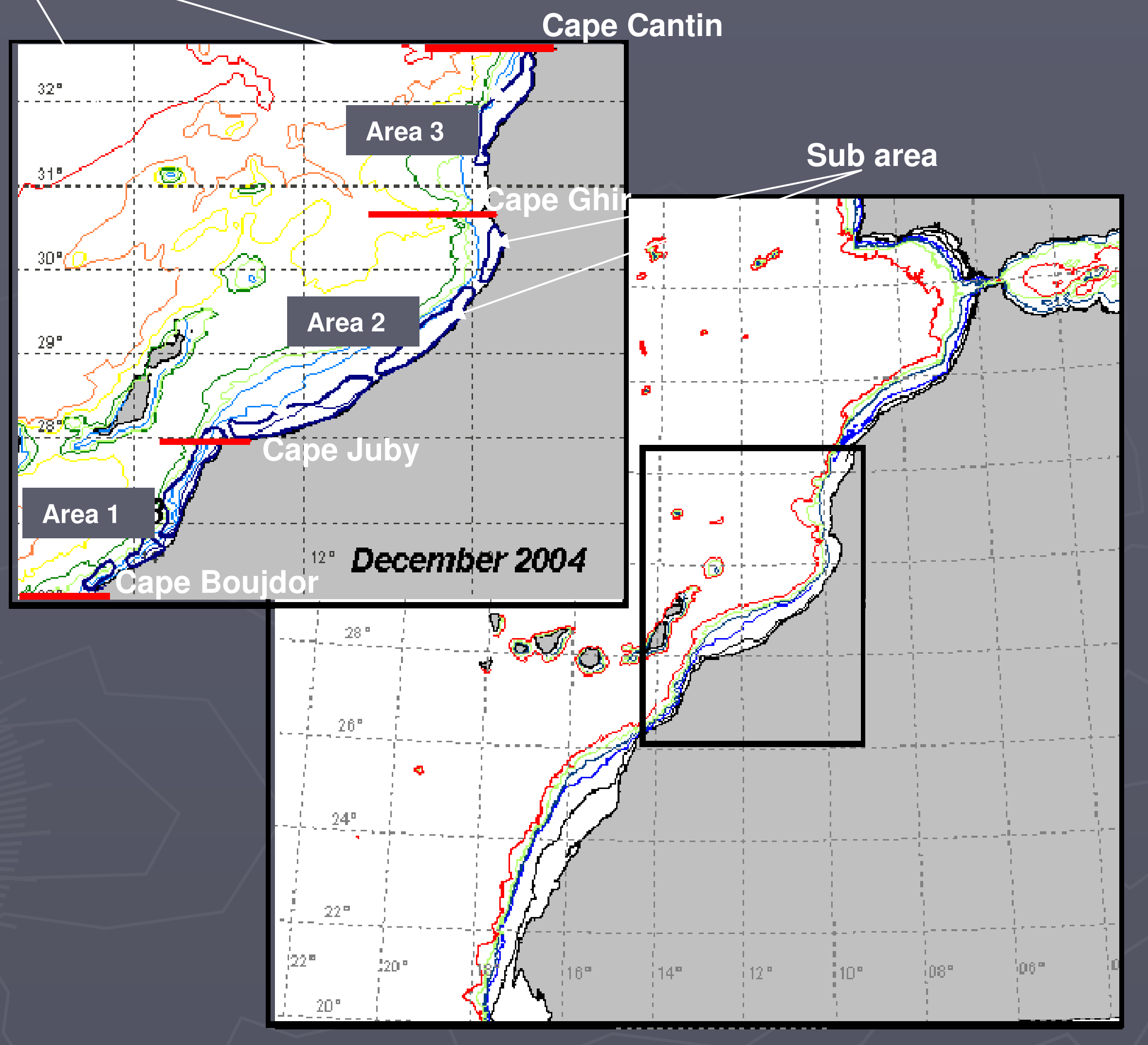
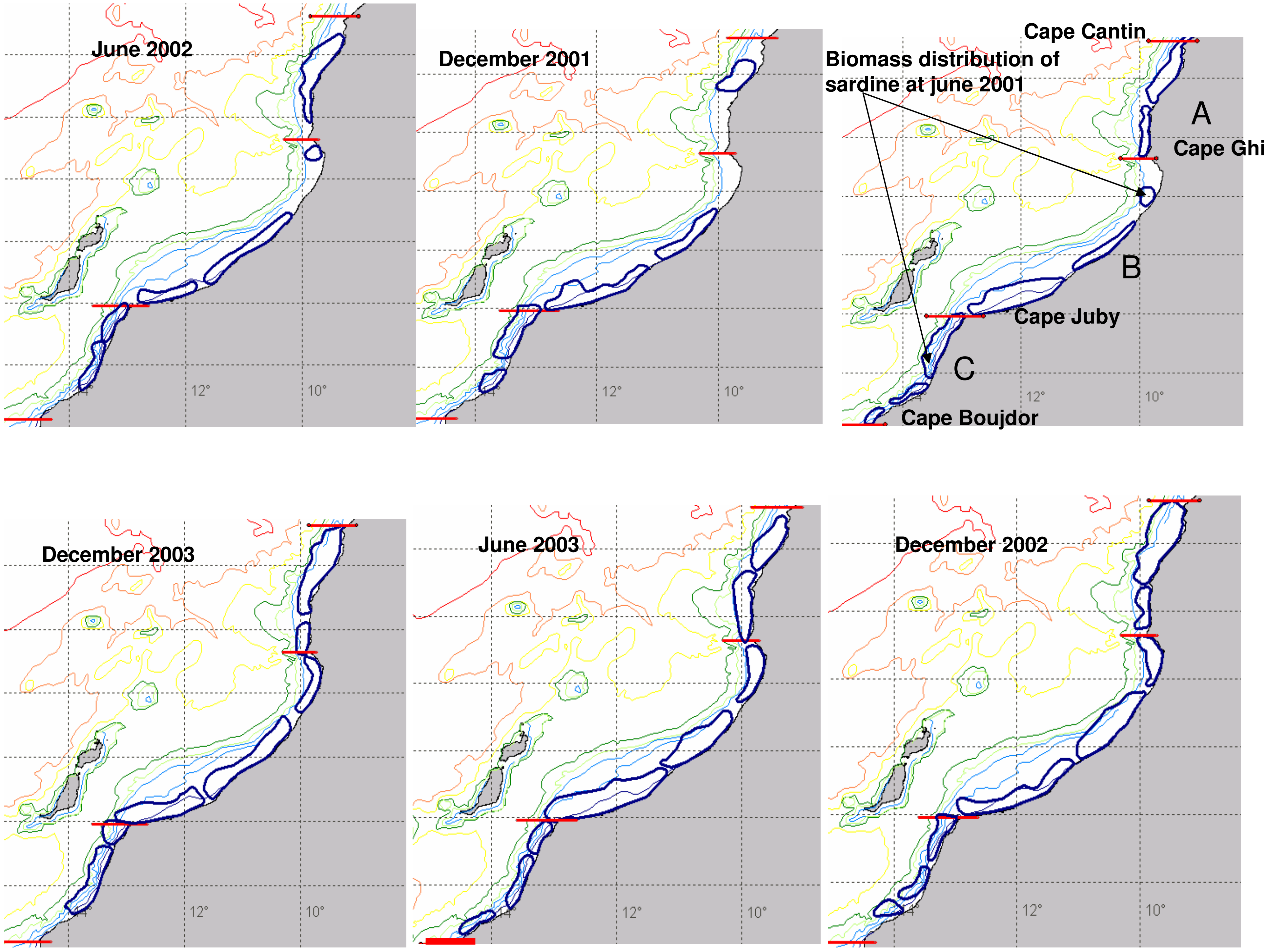


Figure 2.1.2: Seasonal distribution of *Sardina pilchardus* along Moroccan Atlantic coasts (Cape Cantin- Cape Boujdor).


June 2002
12°
10°
December 2001
12°
10°
Biomass distribution of sardine at june 2001
Cape Cantin
A
Cape Ghi
B
Cape Juby
C
Cape Boujdor
12°
10°
December 2003
12°
10°
June 2003
12°
10°
December 2002
12°
10°

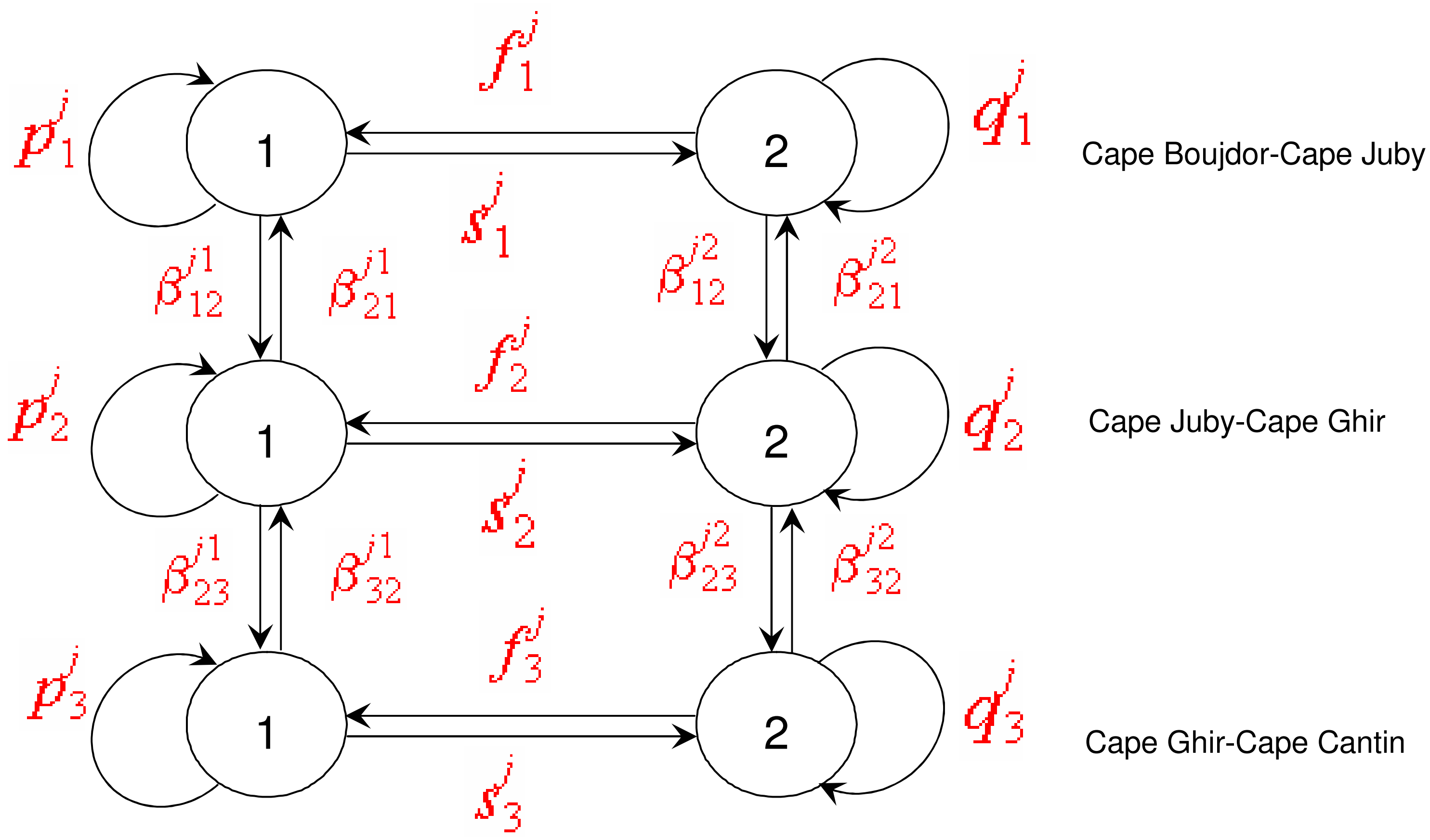


Figure 2 : The stage graph considered. The circles indicate the stages (1: juveniles and 2: adults), and the arrows represent the possible transitions (see the text) and migration flows of sardine between 3 patches (Cape Boujdor-Cape Juby, Cape Juby-Cape Ghir and Cape Ghir-Cape Cantin) in season j: aw (autumn winter) and ss (spring summer).

Model formulation

Deterministic discrete time model based on a recurrence equation $N^{ss}(t+1) = AN^{ss}(t)$, where N^{ss} is a vector of stage abundances in different patches and A is a constant projection matrix.

Individuals in each age class i may migrate between the three spatial patches.

We assume that n^{ji}_{m} (t) is the number of individuals in age class i living in patch m at season j and time t

Where (i = 1; 2; m = 1; 2; 3 and j = ss; aw) and $N^{ji}(t)$= $(n^{ji}_{1}(t); n^{ji}_{2}(t); n^{ji}_{3}(t))T$;

$N^{j}(t) = (N^{j1}(t), N^{j2}(t))^{T}$ (^{T} denotes transposition.)

Model formulation

We used an annual time step, but the matrix A was constructed by multiplying four matrices: $A = L_{ss}\bar{P}_{ss}L_{aw}\bar{P}_{aw}$, where L_{ss} and L_{aw} describes respectively demographic process at seasons ss and aw. The other matrices are similarly defined over 6-month intervals and describe migration phenomenon. This approach includes information on seasonal variation in vital rates into a model with an annual time step [2, 3].

$$L_{ss} = \begin{bmatrix} P^{ss} & F^{ss} \\ S^{ss} & Q^{ss} \end{bmatrix}; L_{aw} = \begin{bmatrix} P^{aw} & F^{aw} \\ S^{aw} & Q^{aw} \end{bmatrix}$$

Where $F^j = diag\{f_1^j, f_2^j, f_3^j\}$, $Q^j = diag\{q_1^j, q_2^j, q_3^j\}$, $P^j = diag\{p_1^j, p_2^j, p_3^j\}$, $S^j = diag\{s_1^j, s_2^j, s_3^j\}$.

f_m^j : proportion of stage 1 at season j coming from adult at season $j-1$ in patch m

q_m^j : proportion of adults surviving at season j in patch m,

p_m^j : proportion of young-of-the year surviving and remaining in the same stage at season j in patch m,

s_m^j : proportion of juvenile surviving and passes to stage 2 at season j in patch m.

Consider that $\bar{v}_m^{ji}$ is the proportion of stage $i = 1, 2$ at season $j = ss, aw$ in patch $m = 1, 2, 3$.

$$\bar{v}^{ji} = (\bar{v}_1^{ji}, \bar{v}_2^{ji}, \bar{v}_3^{ji})^T; \langle \bar{v}_m^{ji}, \bar{1}_3 \rangle = 1, \bar{1}_3 = (1, 1, 1)^T; \bar{P}_{ji} = (\bar{v}^{ji} \,|\, \bar{v}^{ji} \,|\, \bar{v}^{ji}); \bar{P}_j = diag\left\{\bar{P}_{j1}, \bar{P}_{j2}\right\}$$

Some simplifying assumptions

(1) The central stock is closed to external migrations,

(2) The migrations between patches are made so that, in each period, proportions of matures (respectively immature) remain constant on each patch over years; these proportions are estimated by averaged proportions from acoustic surveys data,

		Year 1		Year 2		.....	Year n		
		aw	ss	aw	ss	.....	aw	ss	Averaged proportions
Cape Boujdor - Cape Juby	Immature proprtion					.....			
	Mature proportion					.....			
Cape Juby - Cape Ghir	Immature proprtion					.....			
	Mature proportion					.....			
Cape Ghir - Cape Cantin	Immature proprtion					.....			
	Mature proportion					.....			

(3) Demographic rates are constant, in each period, over years.

Estimate of input parameters

Two stages : immature and mature

↓

Difficulty to estimate the inputs data

↓ Alternative

immature and mature will be structured in sub stages
immature : 2 sub stages and mature : 6 sub stages

↓ ??????

Each size sub stage in all study area (area 1+ area 2+ area 3), are transformed into intervals of age of six months

$$t(l) = t_0 - \frac{1}{k} * \ln\left(1 - \frac{l}{l_\infty}\right)$$

Von Bertalanffy reverse equation

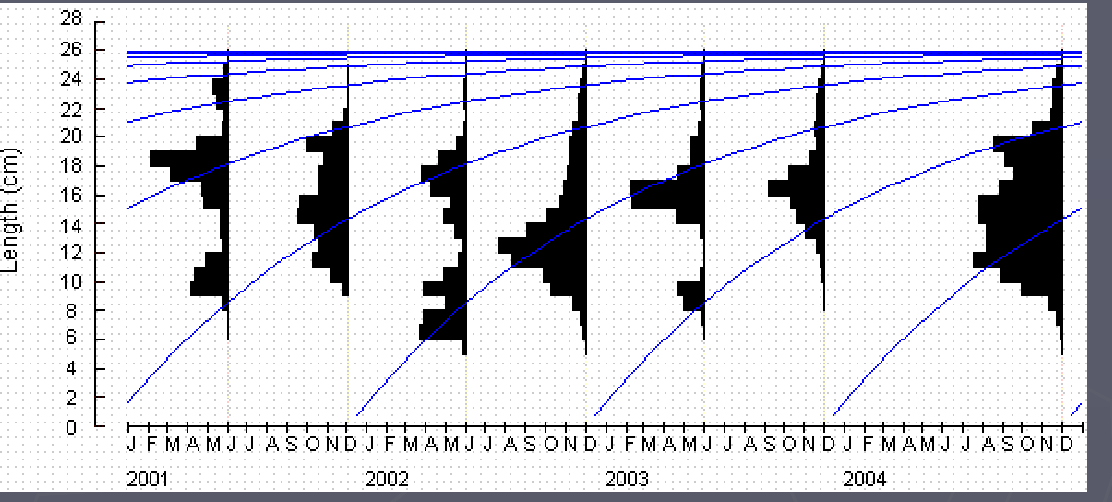


Figure 4: Curve resultant of the of growth Von Bertalanffy function (VBGF) covered by the histograms with frequency length oscillating at june and december.

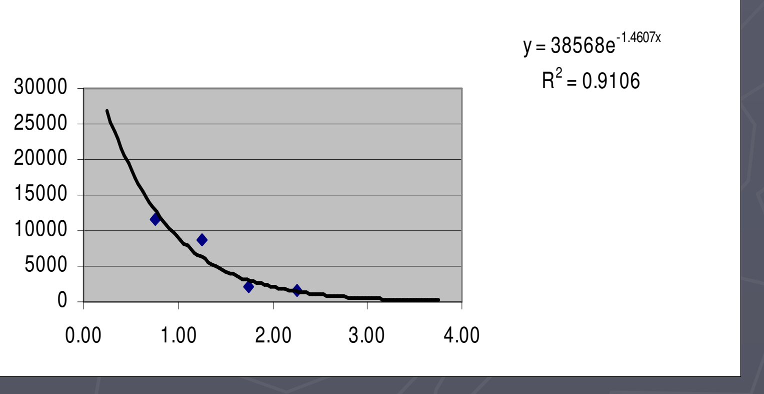
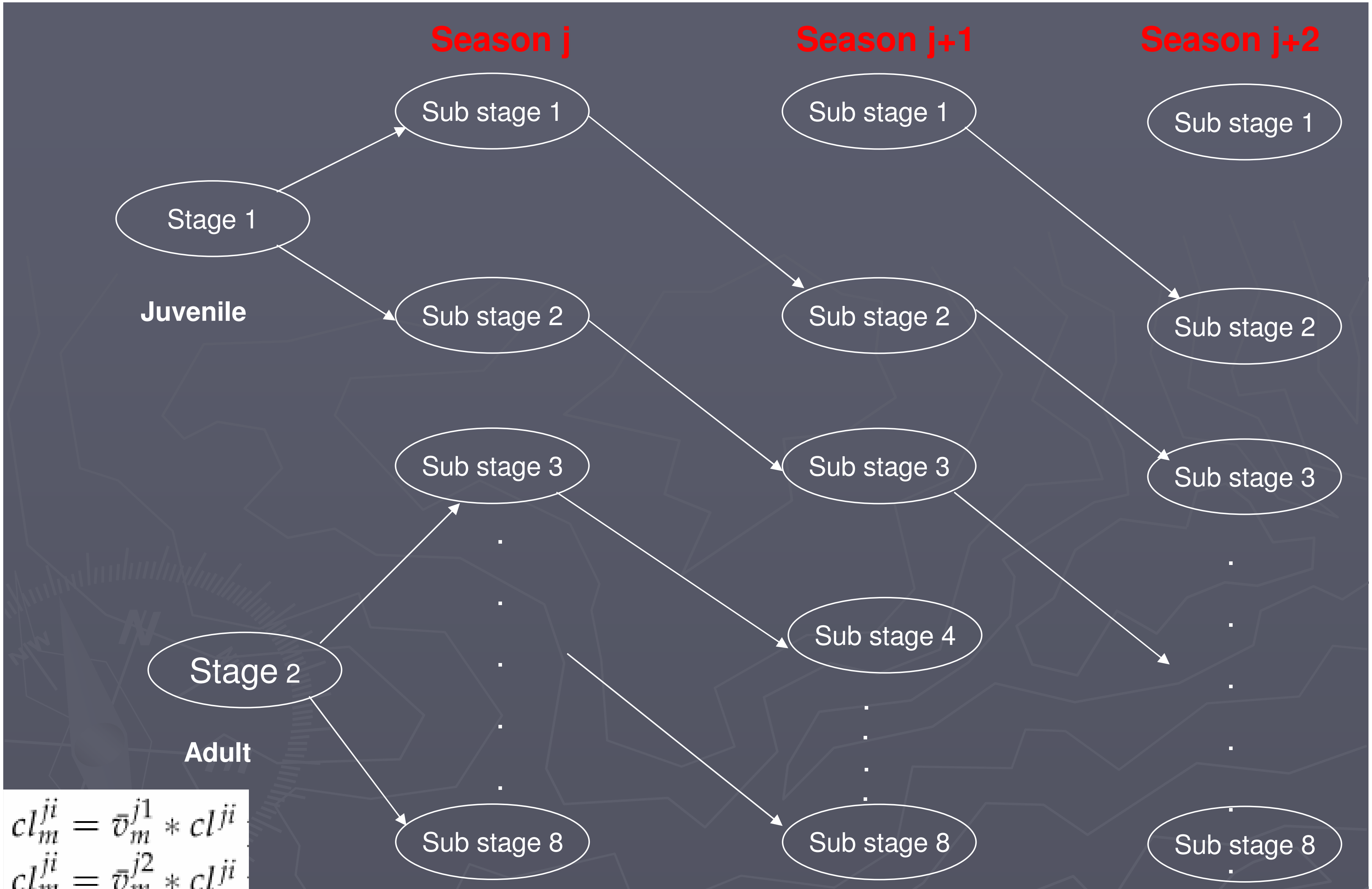


Figure 5: Decrease exponential model of cohort in december 2001 at June 2005.



$$cl_m^{ji} = \bar{v}_m^{j1} * cl^{ji}$$
$$cl_m^{ji} = \bar{v}_m^{j2} * cl^{ji}$$
$$n_m^{j1} = \sum_{i=1}^{2} cl_m^{ji}$$
$$n_m^{j2} = \sum_{i=3}^{8} cl_m^{ji}$$

$$p_m^j = \frac{cl_m^{j+1,2}}{\sum_{i=1}^{2} cl_m^{ji}},\ s_m^j = \frac{cl_m^{j+1,3}}{\sum_{i=1}^{2} cl_m^{ji}},\ q_m^j = \frac{\sum_{i=4}^{8} cl_m^{j+1,i}}{\sum_{i=3}^{8} cl_m^{ji}},\ f_m^j = \frac{cl_m^{j+1,1}}{\sum_{i=2}^{8} cl_m^{ji}}$$

Asymptotic Analysis

Asymptotically, the population size $N^{ss}(t)$ is such that $N^{ss}(t+1) \approx \lambda N^{ss}(t)$. The well-known Perron-Frobenius theorem ensures that λ is real and positive, In this example. $\lambda \approx 1$. The degenerate case $\lambda = 1$ leads to equilibrium of population.

Elaboration of the linear control functions

$$f(k) = a_{per}^{lm} = \left[a_{ob}^{lm} + \left(\frac{k * [\lambda' - \lambda - Sen(a_{ob}^{ij}) \Delta a^{ij}]}{k_{\infty} * Sen(a_{ob}^{lm})} \right) \right]$$

$$g(k) = a_{per}^{ij} = \left[a_{ob}^{ij} + \frac{(k * \Delta a^{ij})}{k_{\infty}} \right]$$

f and g dependent of the growth rate and of the sensitivity results

Soit $A = \left(a^{ij}\right)_{i,j}$ une matrice carrée

$$\lambda' = \lambda + \sum_{i,j} \frac{\partial \lambda}{\partial a^{ij}}$$

Supposons que seulement deux éléments a^{lm} et a^{ij} varient et les autres éléments restent constants

Donc

$$\lambda' = \lambda + \frac{\partial \lambda}{\partial a_{ob}^{lm}} \Delta a^{lm} + \frac{\partial \lambda}{\partial a_{ob}^{ij}} \Delta a^{ij}$$

Avec $\Delta a^{lm} = a_{per}^{lm} - a_{ob}^{lm}$ et $\Delta a^{ij} = a_{per}^{ij} - a_{ob}^{ij}$

D'où $\begin{cases} a_{per}^{lm} = a_{ob}^{lm} + \dfrac{\lambda' - \lambda - sen(a_{ob}^{ij})\Delta a^{ij}}{sen(a_{ob}^{lm})} \\ a_{per}^{ij} = a_{ob}^{ij} + \Delta a^{ij} \end{cases}$

Avec $\dfrac{\partial \lambda}{\partial a_{ob}^{lm}} = sen(a_{ob}^{lm})$ et $\dfrac{\partial \lambda}{\partial a_{ob}^{ij}} = sen(a_{ob}^{ij})$

Soit $1 \leq K \leq K_\infty$

$$\begin{cases} a_{per}^{lm} = a_{ob}^{lm} + \dfrac{K}{K_\infty} * \dfrac{(\lambda' - \lambda - sen(a_{ob}^{ij})\Delta a^{ij})}{sen(a_{ob}^{lm})} \\ a_{per}^{ij} = a_{ob}^{ij} + \dfrac{K}{K_\infty} * \Delta a^{ij} \end{cases}$$

Si $K = K_\infty$, c'est le cas trivial

Si on varie k dans l'intervalle $[1, K_\infty = 30]$ on obtient une fonction linéaire

Simulation results and management scenarios

- The adaptive strategy is expected to increase the growth rate
- spawning peak occurs at autumn-winter season in Cape Juby-Cape Ghir
- decrease of fishing effort in Cape Juby-Cape Ghir , which varies according to the function g
- During the spring-summer season, there is a weak reproduction
  and a predominance of the adults in the zone of Cape Ghir-Cape Cantin
- We proposed a fishing effort, which increase according to the function f

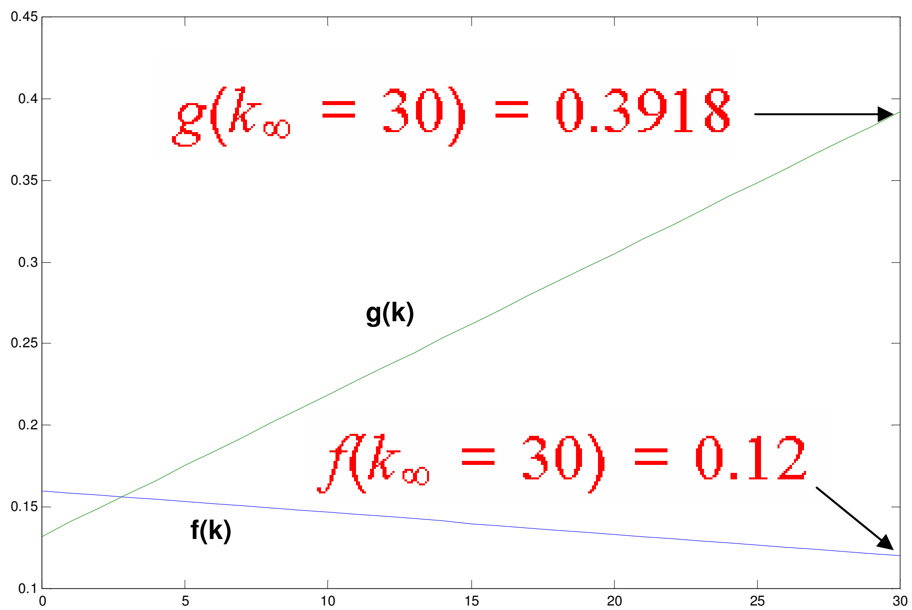


figure 2.3.4: Convergence des fonctions de contrôle f et g pour augmenter le taux de croissance de la population à la valeur 1.04.
f(k): Proportions des adultes qui survivent en saison pe à Cap Ghir-Cap Cantin
g(k): Proportions des adultes qui survivent en saison ah à Cap Juby-Cap Ghir

Simulation results and management scenarios

$g(k_\infty = 30) = 0.3918 = q^{aw}_{2per}$ and $f(k_\infty = 30) = 0.12 = q^{ss}_{3per}$: respectively perturbed proportion of adults surviving at season autumn winter in Cape Juby-Cape Ghir and at season spring summer in Cape Ghir-Cape Cantin ; $\lambda' = 1.04 > 1$; $Sen(q^{aw}_{2ob} = 0.1320) = 0.1926$ and $Sen(q^{ss}_{3ob} = 0,16) = 0.2508$: respectively sensitivity of population growth rate λ to change in observed q^{aw}_{12ob} and observed q^{ss}_{13ob}.

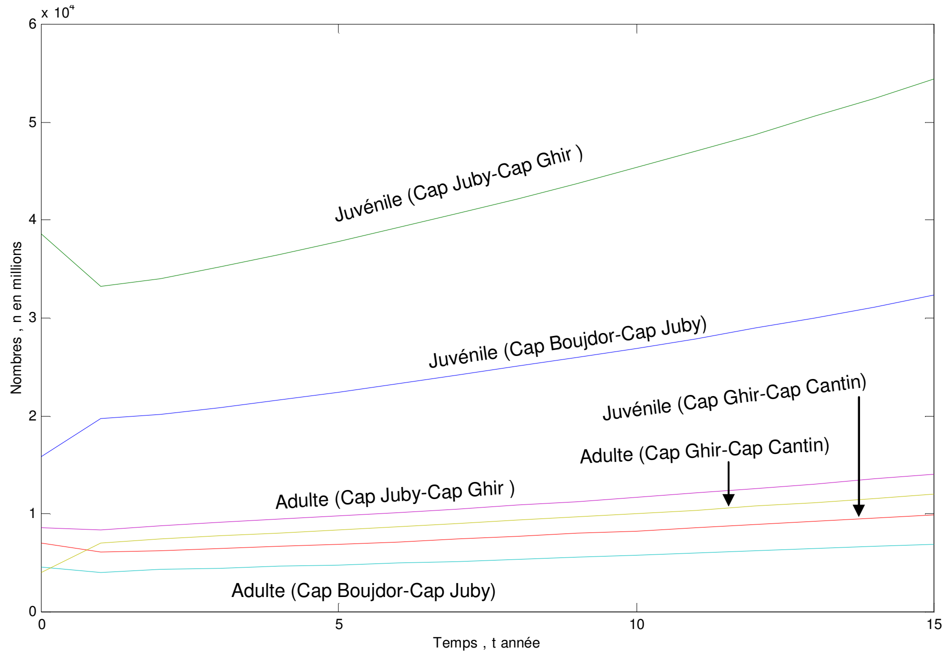
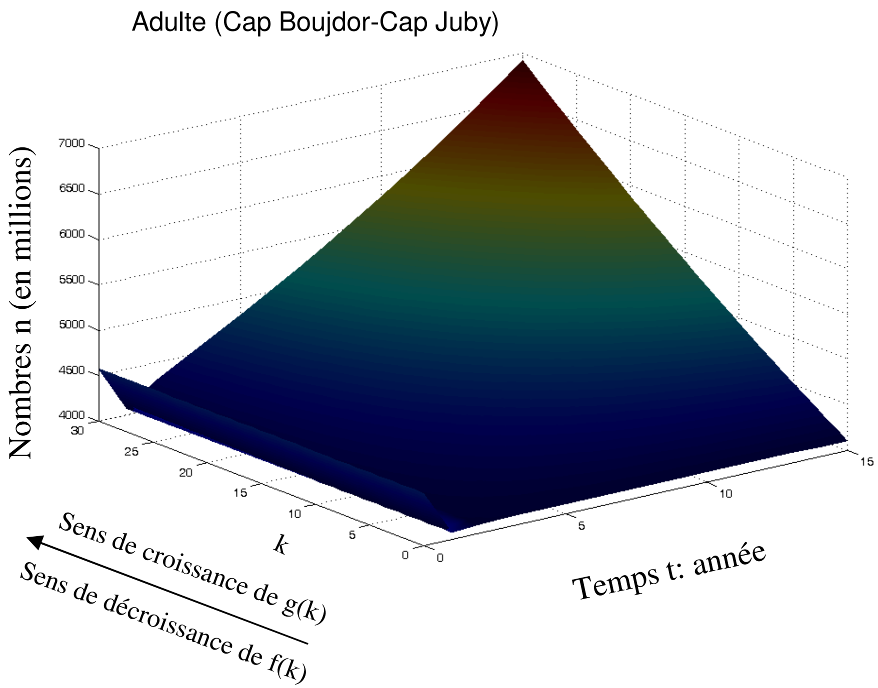
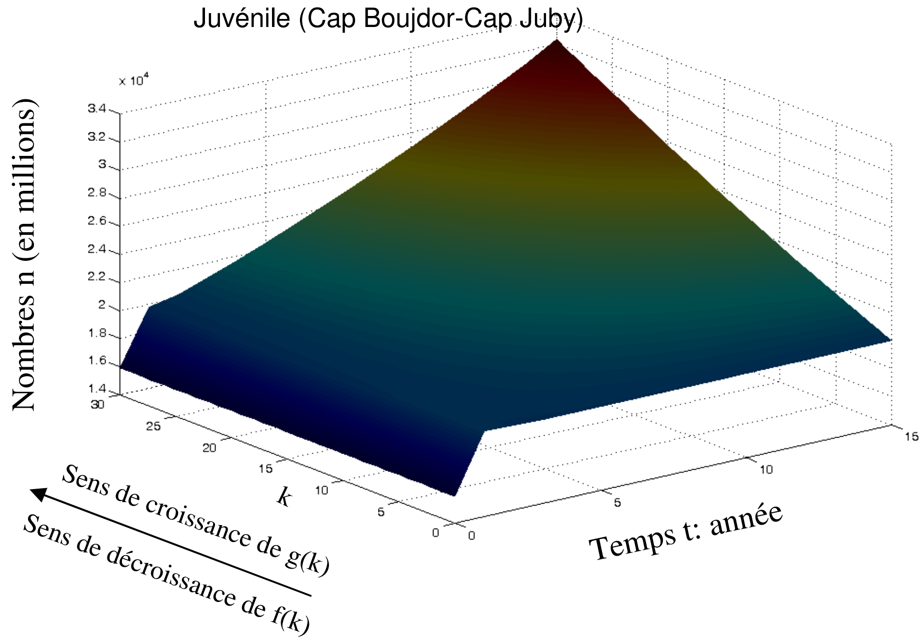
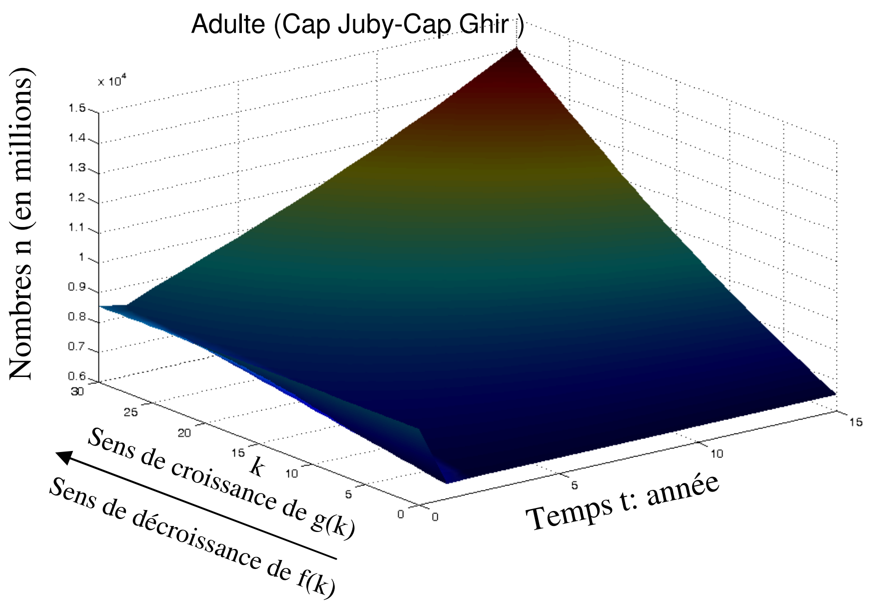
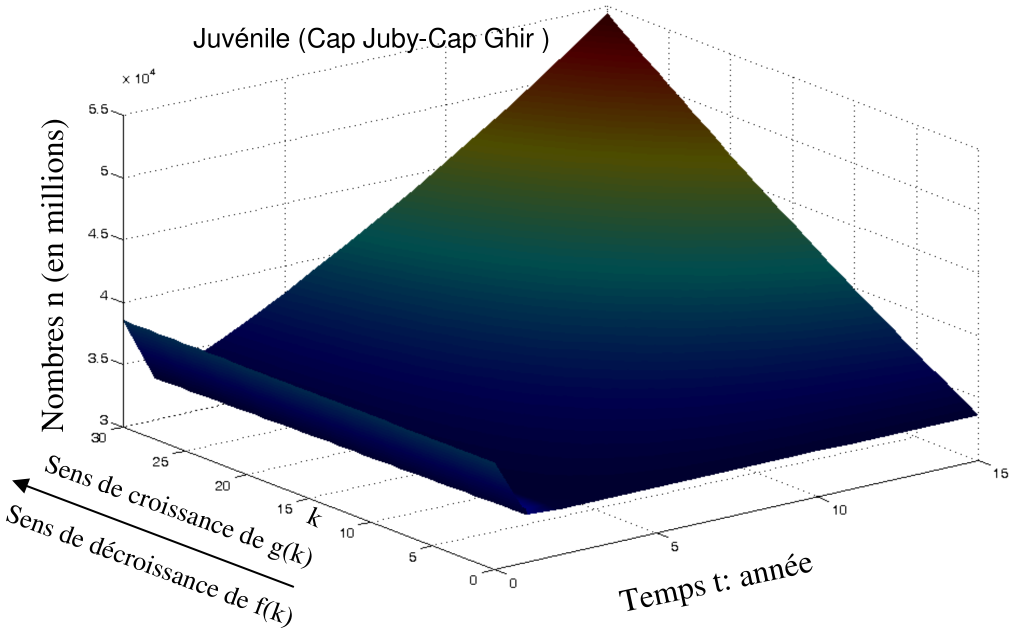


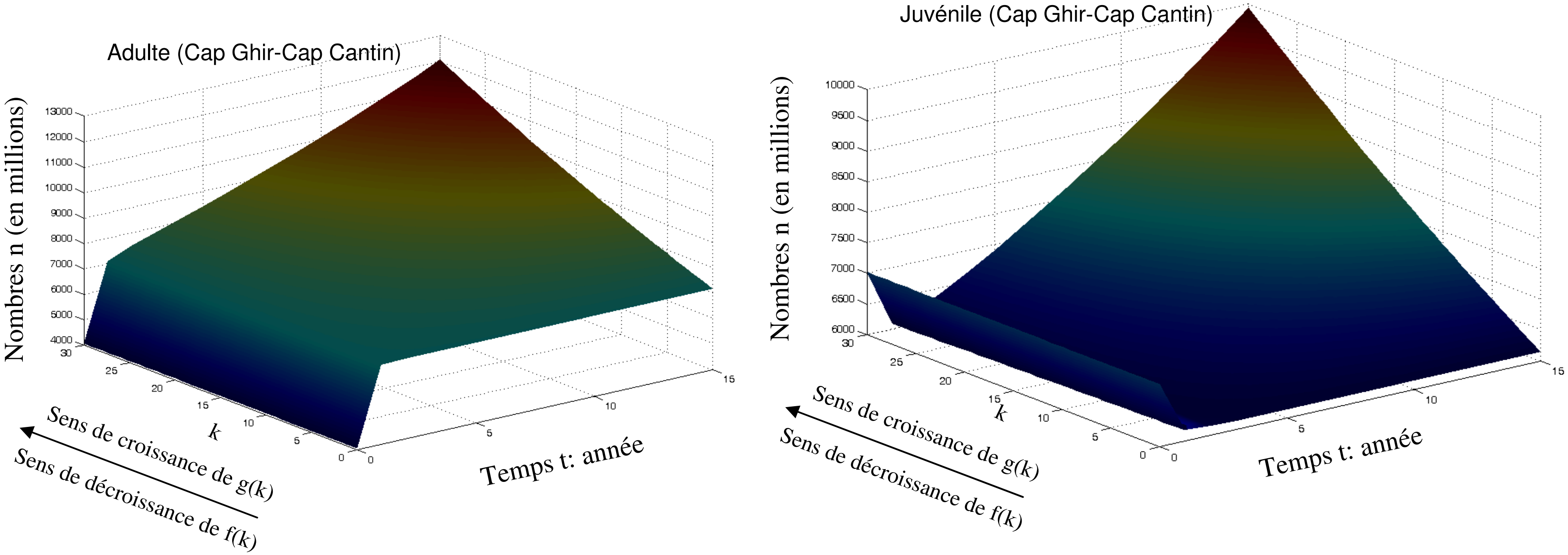
Figure 2.3.5: Tendance de la population à s'accroître pendant chaque années de la saison automne hiver selon les fonctions f et g.


Adulte (Cap Boujdor-Cap Juby)
Nombres n (en millions)
Sens de croissance de g(k)
Sens de décroissance de f(k)
k
Temps t: année


Juvénile (Cap Boujdor-Cap Juby)
x 10^4
Nombres n (en millions)
Sens de croissance de g(k)
Sens de décroissance de f(k)
k
Temps t: année


Adulte (Cap Juby-Cap Ghir)
x 10^4
Nombres n (en millions)
Sens de croissance de g(k)
Sens de décroissance de f(k)
k
Temps t: année


Juvénile (Cap Juby-Cap Ghir)
x 10^4
Nombres n (en millions)
Sens de croissance de g(k)
Sens de décroissance de f(k)
k
Temps t: année



Figures 2.3.6, 2.3.7,…, 2.3.11 : Evolution de la population selon les fonctions f et g dans les 3 sites pendant chaque année de la saison automne hiver

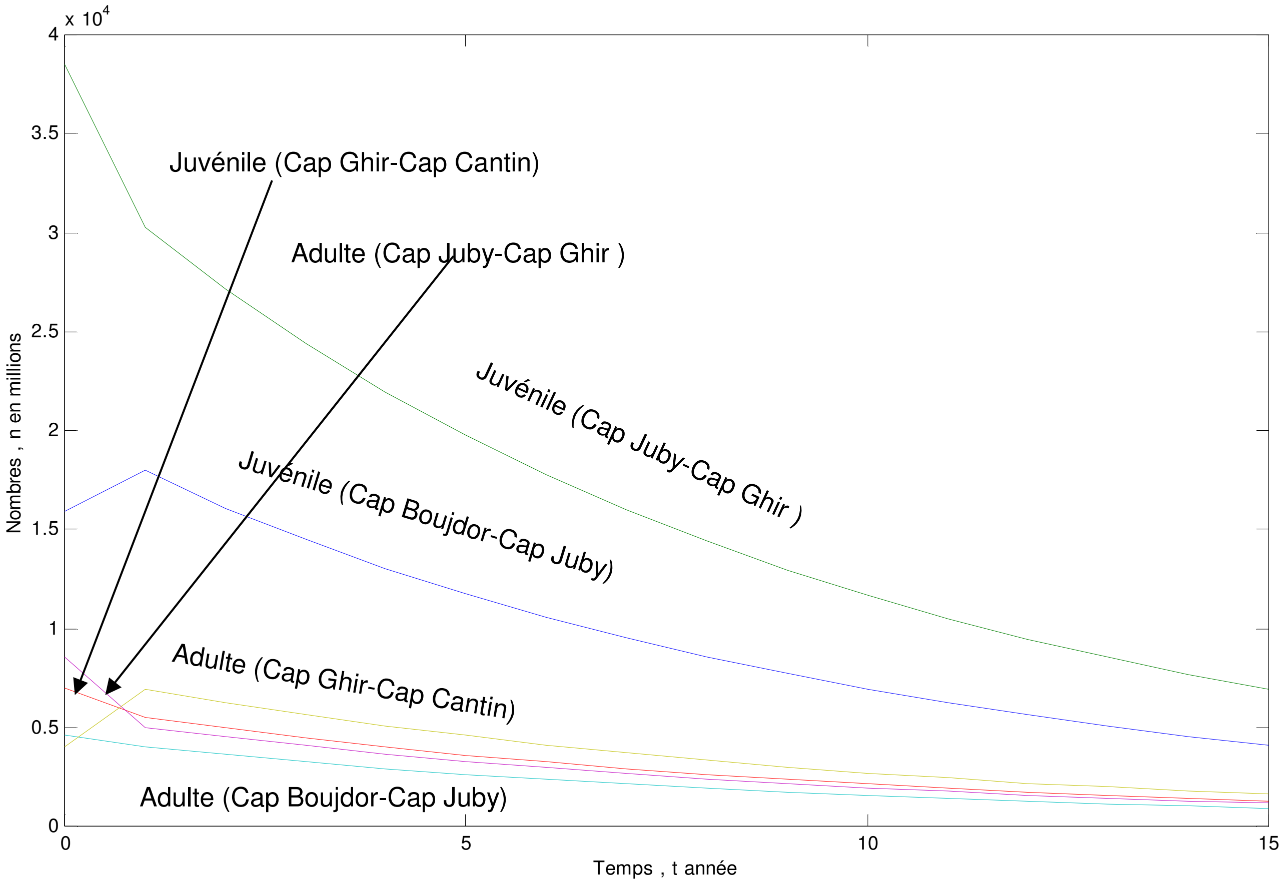


Figure 2.3.12: Décroissance exponentielle de la population pendant chaque année de la saison automne hiver après perturbation de la proportion des juvéniles qui survivent et passe à la classe adulte en ah dans la zone Cap Juby-Cap Ghir et de la proportion des juvéniles qui survivent et passe à la classe adulte en pe dans la zone Cap Juby-Cap Ghir (les deux proportions ont des sensibilités très fortes)

Conclusions and perspectives

- The model we presented is composed of two stages : immature and mature stages,
- We gave an approach to estimate some of the model parameters using available data from acoustic surveys,
- We have elaborated two control functions acting on two parameters of the projection matrix to lead the dominant eigenvalue to a desired value,
- For the sake of simplicity and data scarcity, we have made some assumptions that are questionable but somehow defendable as we argued along this work,
- Structured models dealing with the whole life cycle have several parameters and require enough data for parameters estimates,
- The model we proposed remains relatively simplified and should be developed into an age structured population model taking into account the whole life cycle of the sardine and its interaction with environment and fishing.